Physics 751: Final Study Guide 2008

Review all the answers to homework problem sets and read your own notes from class.

I will ask no questions on the historical stuff at the beginning, *except for the purely physics content*. For example, you should have a good understanding of the Bohr atom, including the classical limit of a very large atom and how that fixes the quantum of angular momentum, and know how the Uncertainty Principle can be used to estimate the size of bound states, such as the atom on the table.

You should know Schrödinger's equation by heart, the probability interpretation of the wavefunction, the conservation of probability current, and the expectation value of an operator.

Be able to find the time-dependent probability and current distribution for a superposition of states of different energies.

Know the definition of the delta function as a limit of a Gaussian wavepacket, and be able to Fourier transform a Gaussian wavepacket. (Know the integral you need by heart.) Also, know how to do Fourier series and transforms for simple functions, and be able to discuss the limitations of Fourier series for *discontinuous* functions.

You must know the bra and ket notation, be able to construct the first few elements of an orthogonal basis, the definitions of adjoint, Hermitian and unitary, and be able to diagonalize a Hermitian or a unitary matrix. Know the definition of determinant and trace, and how these relate to eigenvalues.

You should also feel comfortable going back and forth between the Schrödinger wavefunction notation and the Dirac bra ket notation.

Know the normalization conventions for the eigenkets of the momentum operator, and those of the position operator. Know the identity in terms of these kets, and be able to use it to express a Schrödinger wave function as an integral over $|x\rangle$ or $|k\rangle$ kets. Be able to express $|x\rangle$ in terms of $|k\rangle$'s.

I expect you to be able to solve simple boundary-matching problems for square barriers, steps, square wells, and delta function potentials.

Generalized Uncertainty Principle

Be able to derive the Generalized Uncertainty Principle: know the definition of expectation value and of the root mean square deviation in a set of measurements on identically prepared systems.

Energy-Time Uncertainty Principle

Be able to explain why the alpha particle emitted in alpha decay of a nucleus does not have a perfectly defined energy, but an energy spread related to the lifetime of the nucleus.

Simple Harmonic Oscillator: know

$$x=\sqrt{\hbar/2m\omega}\left(a^{\dagger}+a\right),\quad p=\sqrt{m\omega\hbar/2}\left(a^{\dagger}-a\right),\quad \langle n+1|a^{\dagger}|n\rangle=\sqrt{n+1},\quad \langle n-1|a|n\rangle=\sqrt{n}$$

and be able to use these in evaluating expectation values.

Know the definition of the **propagator**, and be able to derive the free particle propagator.

Be familiar with the **Heisenberg representation**, know how to find the equation of motion for an operator in that representation, and know the connection with Ehrenfest's theorem.

Coherent States

Know how a coherent state of a simple harmonic oscillator develops in time, what it is an eigenstate of, how it can be expressed in energy eigenstates, how they form a complete set.

Memorize the result $e^{A+B}=e^{A}e^{B}e^{-\frac{1}{2}[A,B]}$, know for what operators it is valid, and be able to use it for normalizing coherent states, etc.

We won't cover squeezed states.

Path Integrals

Know Feynman's formulation of a sum over paths, know the phase factor for a path, and be able to prove that for a particle in one dimension, it is equivalent to Schrödinger's equation. Know how to evaluate the integral for the free particle case.

Angular Momentum

From class notes: be able to show, if the operator $\vec{J}$ is defined by $|\psi\rangle\to e^{-\frac{i}{\hbar}\hat{\vec{J}}.\vec{\theta}}|\psi\rangle$ under a rotation defined by $\vec{\theta}$, then from our knowledge of *classical* rotations, $\left[J_i,J_j\right]=i\hbar\varepsilon_{ijk}J_k$, and know this formula by heart! Be able to derive from this the commutation relations among J^2, J_z, $J_\pm$. Be able to derive the matrix elements of these operators for the common eigenkets of J^2, J_z, and know how to prove that $2j$ is an integer. In fact, you should *memorize*:

$$J_{\pm}|j,m\rangle=\hbar\sqrt{j(j+1)-m(m\pm 1)}\,|j,m\pm 1\rangle.$$

Be able to use these to construct matrix representations of the components of angular momentum for given j.

2-D Orbital Angular Momentum

Be able to derive the angular part of the wave function in 2D, the allowed form of the radial function at the origin (at end of lecture), and the points covered in the homework #9 questions.

3-D Orbital Angular Momentum

Know the formulas for the components L_i in Cartesian coordinates.

You should be able to work with $l = 1$ and $l = 2$ eigenstates, (the expressions would be given) both in terms of θ, φ and x, y, z. For example, given that an angular one has $m = 1$ in the x-direction, you should be able to compute the probability that a measurement in the y-direction gives $m = 0$. Understand how the rotation operator for $j = 1$ relates to the rotation operator in ordinary three-dimensional space.

Spin

Be familiar with the Stern-Gerlach experiment, and be able to handle two in succession. Given the Pauli spin matrices, you need to be able to construct the rotation operator, and find a spinor pointing in any direction. Understand paramagnetic resonance.

Hydrogen Atom

Know why variables can be separated, know where the Bohr model goes wrong in linking energy and angular momentum, have some idea of the sizes and shapes of the orbits, both n and l dependence, and how the wave functions behave near the origin and far away, and how many radial nodes there are. Understand why the hydrogen atom energy level degeneracies do not all apply to bigger atoms.

Density Matrix

Be sure you understand my lecture notes, and the homework questions that were asked.