Physics 751 Homework #5

Due October 1, 11:00 am.

1. (a) For the finite square well potential $V = 0$ for $x < 0$, $V = -V_0$ for $0 < x < L$, $V = 0$ for $L < x$, prove that the transmission amplitude $S(k)$ for an electron coming in from the left with momentum k is

$$S(k) = \frac{2kk_1}{2kk_1 \cos k_1 L - i(k_1^2 + k^2)\sin k_1 L},$$

where k_1 is the momentum inside the well. Compare this with the transmission through a square *barrier* in http://www.phys.virginia.edu/classes/751.mf1i.fall02/OneDimSchr.htm and comment on the similarities and differences.

(b) Sketch the probability of transmission carefully as a function of energy, or plot it with *Maple* or *Mathematica*. For what energies is there perfect transmission? Can you give any physical explanation?

(c) Now regard the incoming momentum k as a *complex* variable. Note that $S(k)$ become *infinite* when

$$\tan k_1 L = \frac{2\alpha k_1}{k_1^2 - \alpha^2}, \quad \text{with } k = i\alpha$$

and use the formula $\tan 2\theta = 2\tan\theta/(1-\tan^2\theta)$ to show that this is equivalent to the formulas for bound states in the finite square well. Give a physical interpretation of why an infinite value of $S(k)$ would correspond to a bound state.

Shankar problems: 5.2.1, 5.2.2.