Note on Spin Precession

Consider first a magnetized classical object spinning about its center of mass, with angular momentum $\vec{L}$ and parallel magnetic moment $\vec{\mu}$, $\vec{\mu} = \gamma \vec{L}$. Now add a magnetic field $\vec{B}$, say in the z-direction. This will exert a torque $\vec{T} = \vec{\mu} \times \vec{B} = \gamma \vec{L} \times \vec{B} = d\vec{L}/dt$, easily solved to find the angular momentum vector $\vec{L}$ precessing about the magnetic field direction with angular velocity of precession $\vec{\omega}_0 = -\gamma \vec{B}$.

(*Proof*: from $d\vec{L}/dt = \gamma \vec{L} \times \vec{B}$, take $L_+ = L_x + iL_y, \quad dL_+/dt = -i\gamma B L_+, \quad L_+ = L_+^0 e^{-i\gamma Bt}$. Of course, $dL_z/dt = 0$, since $d\vec{L}/dt = \gamma \vec{L} \times \vec{B}$ is perpendicular to $\vec{B}$, which is in the z-direction.)

The exact same result comes from a quantum analysis: for $\vec{\mu} = \gamma \vec{S}$, the Hamiltonian for the interaction with the magnetic field is $H_{\text{int}} = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{S} \cdot \vec{B}$, so the time development is

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$

with

$$U(t) = e^{-iEt/\hbar} = e^{i\gamma\vec{\sigma}\cdot\vec{B}t/2}$$

but this is exactly the rotation operator (as shown earlier) through an angle $-\gamma Bt$ about $\vec{B}$!
So $\vec{\omega}_0 = -\gamma \vec{B}$, just as for the classical case.

Paramagnetic Resonance

Note that the spin precession frequency is independent of the angle of the spin to the field. Consider how all this looks in a frame of reference which is *itself* rotating about the z-axis. Let's call the magnetic field $\vec{B}_0 = B_0 \hat{\vec{z}}$, because we will soon be adding another one. In the rotating frame, the observed precession frequency is $\vec{\omega}_r = -\gamma\left(\vec{B}_0 + \vec{\omega}/\gamma\right)$, so there is a different effective field in the rotating frame. Obviously, if the frame rotates exactly at the precession frequency, spins pointing in any direction will remain *at rest* in that frame—there's no effective field at all.

Now suppose we add a small rotating magnetic field in the x,y plane, so

$$\vec{B} = B_0 \hat{\vec{z}} + B_1\left(\hat{\vec{x}} \cos \omega t - \hat{\vec{y}} \sin \omega t\right)$$

and in the rotating frame

$$\vec{B}_r = \left(B_0 - \omega/\gamma\right)\hat{\vec{z}} + B_1 \hat{\vec{x}}$$

Now, if we tune the angular frequency of the small rotating field so that it exactly matches the precession frequency in the original static magnetic field, all the magnetic moment will see in the rotating frame is the small field in the x-direction! It will therefore precess about the x-direction at the slow angular speed γB_1.

If the spins are lined up preferentially in the z-direction by the static field, and the small oscillating field is switched on for a time such that $\gamma B_1 t = \pi/2$, the spins will be preferentially in the y direction in the rotating frame, so in the lab they will be rotating in the x,y plane, and a coil will pick up an ac signal from the induced emf.