Physics 751 Homework #4

Due Friday October 3, 11:00 am.

1. (a) Define the derivative of the delta function by

$$\delta'(x) = \lim_{\Delta\to 0} \frac{d}{dx} \frac{1}{\left(4\pi\Delta^2\right)^{1/2}} e^{-x^2/4\Delta^2}.$$

Sketch this function for small Δ. What is the value of $<\delta'(x-a)\,|\,\psi(x)>$?

(b) What function has the delta function for its derivative? Explain briefly.

2. On the interval (−1, 1) the polynomials $1, x, x^2, x^3, \ldots$ are a basis. Use the Gram-Schmidt orthogonalization procedure to find the first four members of an *orthonormal* basis constructed from the polynomials.

3. Suppose we define the inner product for real functions defined on the infinite line to be:

$$<f\,|\,g> = \int_{-\infty}^{\infty} f(x)g(x)e^{-x^2}\,dx.$$

Starting with the polynomials $1, x, x^2, x^3, \ldots$, construct the first four members of an orthonormal basis.

4. Solve the time-independent Schrödinger equation in one dimension for an attractive delta function potential $V(x) = \lambda\delta(x)$ to find the energy of a bound state. Can there be more than one bound state? Explain.

5. For a one-dimensional general time-dependent solution of Schrödinger's equation, prove:

(a) $\frac{d}{dt}\int_{-\infty}^{\infty} |\psi(x,t)|^2\,dx = 0.$ How would the result change if the limits of integration were finite? Can you write an equation for that case?

(b) If $<x> = \int x|\psi|^2\,dx,$ show that $\frac{d}{dt}<x> = \frac{<p>}{m}.$

(c) Prove $\frac{d}{dt}<p> = -\left\langle \frac{dV}{dx} \right\rangle.$