Physics 751 Homework #5

Due October 17, 11:00 am.

1. (a) For the finite square well potential $V = 0$ for $x < 0$, $V = -V_0$ for $0 < x < L$, $V = 0$ for $L < x$, prove that the transmission amplitude $S(k)$ for an electron coming in from the left with momentum k is

$$S(k) = \frac{2kk_1}{2kk_1 \cos k_1 L - i(k_1^2 + k^2)\sin k_1 L},$$

where k_1 is the momentum inside the well. Compare this with the transmission through a square *barrier* in http://www.phys.virginia.edu/classes/751.mf1i.fall02/OneDimSchr.htm and comment on the similarities and differences.

(b) Sketch the probability of transmission carefully as a function of energy, or plot it with *Maple* or *Mathematica*. For what energies is there perfect transmission? Can you give any physical explanation?

(c) Now regard the incoming momentum k as a *complex* variable. Note that $S(k)$ become *infinite* when

$$\tan k_1 L = \frac{2\alpha k_1}{k_1^2 - \alpha^2}, \quad \text{with } k = i\alpha$$

and use the formula $\tan 2\theta = 2\tan\theta/(1-\tan^2\theta)$ to show that this is equivalent to the formulas for bound states in the finite square well. Give a physical interpretation of why an infinite value of $S(k)$ would correspond to a bound state.

2. (a) Consider an electron in one-dimension in the potential $V(x) = -\lambda(\delta(x-b) + \delta(x+b))$. What can you say about possible symmetries of bound state wave functions? Find (graphically) the energies of possible bound states, give the values of binding energies in the limits of small and large λ, and comment on how these relate to the binding energy of a single delta function potential.

(b) Draw rough graphs of possible bound-state wavefunctions for *three* equally-spaced attractive delta functions.

Also do: Shankar, page 163, Exercises 5.2.1, 5.2.2.