Physics 861 – Fall 01
Problem set 7 - Due Tuesday, Oct 30

1.

Problem 2, page 239, of Ashcroft-Mermin

2.

Problem 3, page 239, of Ashcroft-Mermin. Also,

(c) Compute the electrical conductivity of n-type Silicon with $n = 10^{22}\,\mathrm{m}^{-3}$. Take into account that there are six electron pockets, each with its own anisotropic inverse mass tensor. Assume that the relaxation time is $\tau = 10^{-14}$ s.

3.

Problem 2, page 585, of Ashcroft - Mermin. To solve this problem, it may help to look at problem 1 on the same page.

Motion in an electric field

Take motion in one dimension to keep the writing simple. In empty space, this is all we need, because the motion perpendicular to the field is free. Let E be the electric field and $-e$ the charge on the electron.

Classically, the momentum is $p(t) = p_0 - eEt$ and the energy is $w(t) = p(t)^2/2m$. Quantally, we look for a wave function of the form

$$\psi(x;t) = c(t)\exp[ip(t)x/\hbar]$$

where $c(t)$ is to be determined. Since $|c|^2$ is fixed by normalization, we put $c = \exp(-i\phi)$. We find

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + eEx\psi = (w(t) + eEx)\,\psi$$
$$i\hbar\frac{d\psi}{dt} = \left(\hbar\frac{d\phi}{dt} + eEx\right)\psi$$

Then the time-dependent Schrödinger equation gives $\hbar d\phi/dt = w(t)$, or

$$\hbar\phi = \int w(t)dt = \frac{1}{2m}\left(p_0^2 t - p_0 eEt^2 + \frac{1}{3}e^2E^2t^3\right)$$

If we want energy eignfunctions with energy E, all we have to do is take a Fourier transform. Choosing $p_0 = 0$, we obtain

$$\psi(x;E) = \int_{-\infty}^{\infty} \exp(iEt/\hbar)\psi(x;t)dt = \int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar}\left(Et - \frac{1}{6m}e^2E^2t^3 - \frac{eEx}{2m}t\right)\right)dt$$

The integral can be expressed in terms of the Airy function

$$\mathrm{Ai}\left((3a)^{-1/3}\xi\right) = \frac{1}{\pi}(3a)^{1/3}\int_0^{\infty}\cos(at^3 + \xi t)\,dt$$