# Problem set 3

1. Consider a quantum particle in a $d$-dimensional shallow quantum well ($U_0 \ll \hbar^2/2ma^2$, where $a$ is the radius of the potential). Using the integral equation for the scattering amplitude (see lecture notes), find the energy of the weakly bound state (if any) for $d = 2$ and $d = 3$.

   *Hint:* Follow your lecture notes. Model a shallow quantum well with a $\delta$-function potential. Write the integral equation for the scattering amplitude. Assume that the amplitude depends only on the energy. The spectrum is determined by the poles of the scattering amplitude.

2. A charged particle of mass $m$ is moving in a 1-dimensional parabolic potential $U(x) = m\omega^2 x^2/2$. At $t = 0$, a weak electric field $E$ is suddenly turned on for a short period of time $\tau$ and then turned off. Find the probability of the transition between the ground state $|0>$ and the $n$-th excited state $|n>$.

   *Solve this problem using the interaction representation,* with $\mathcal{H}_0 = \hbar\omega\left(\hat{a}^\dagger \hat{a} + 1/2\right)$ and $\mathcal{H}_{\rm int} = -eE\hat{x} \propto \left(\hat{a} + \hat{a}^\dagger\right)$. Treat the interaction in perturbation theory.

   *Difficult problem (optional): Find the exact $S$-matrix.

3. Consider a fermionic chain (see also your lecture notes and homework 2):

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int},$$

   where

$$\mathcal{H}_0 = \sum_{n=-\infty}^{\infty} \left(J_1 a_n^\dagger a_{n+1} - B a_n^\dagger a_n + \text{h. c.}\right)$$

   and

$$\mathcal{H}_{\rm int} = \sum_{n=-\infty}^{\infty} \left(J_2 a_n a_{n+1} + \text{h. c.}\right)$$

   Find the Green's function $G_0(\varepsilon, p)$ of the "non-interacting" problem described by the Hamiltonian $\mathcal{H}_0$. Draw the Feynmann graphs, corresponding to the exact Green's function. Consider two types of interaction vertices corresponding to the hermitian conjugate terms $2iJ_2 \sin q$ and $-2iJ_2 \sin q$. Note that only even orders of perturbation theory with respect to $\mathcal{H}_{\rm int}$ give non-zero contributions. Sum up the corresponding series and find the spectrum of quasiparticles which is given by the poles of the exact Green's function.

*Due Thursday, September 22 (in class)*