Phys 742 – Assignment 1 – Due 30 Jan 96

1. (Related to Jackson's problem 1.1)

(a) A point charge q is located at the center of a conducting spherical shell having inner radius R_1 and outer radius R_2. Find the electric field everywhere and the surface charge on the inner and outer surfaces of the shell, when the shell is grounded.

(b) Same questions as in (a), when the shell is insulated and carries no net charge.

(c) If the point charge q is located off-center at $x = R_1/2$, what is the electric field for $r > R_1$ when the shell is grounded? Consider separately the two regions $R_1 < r < R_2$, $R_2 < r$ and compute the field in each region.

(d) Same questions as in (c), when the shell is insulated and carries no net charge.

2. (Similar to Jackson's problem 1.4)

Each of three charged cylinders of radius a and length $>> a$, one conducting, one having a uniform charge density within its volume, and one having a cylindrically symmetric charge density which varies radially as ρ^n $(n > -2)$, has a total charge λ per unit length. Use Gauss's theorem to obtain the electric fields both inside and outside each cylinder. Sketch the behavior of the fields as a function of radius for the first two cylinders, and for the third with $n = -1$.

3.

(a) Jackson's problem 1.2 gives a possible representation of the Dirac δ function. List (at least) three other representations of δ, correctly normalized. You are likely to have encountered already the square box, the lorentzian and the sinc function representations, for instance. But you can also be creative.

(b) Using the result of Jackson's problem 1.2, express the three-dimensional δ function in cylindrical coordinates (ρ, ϕ, z) and in spherical coordinates (r, θ, ϕ).

(c) Check that in both cases of part (b) the result is of the form

$$\delta(\mathbf{x} - \mathbf{x}') = \frac{1}{J}\,\delta(u - u')\delta(v - v')\delta(w - w')$$

where J is a Jacobian. The Jacobian of what, precisely? (Of course, $1/J$ is the inverse Jacobian). Is this equation valid for nonorthogonal coordinates *(u,v,w)* as well?

4.

Do Jackson's problem 1.5 in reverse. Start with the time-averaged electron charge density

$$\rho(r) = -q\,\frac{\alpha^3}{2\pi}\,e^{-\alpha r}$$

and a point charge q at the origin, compute the radial electric field E by Gauss's theorem, and then the potential $\Phi(r)$ by integration. You should get Jackson's Φ.

What is the numerical value of E in Volts per centimeter at the typical atomic distance of $r = 1$ Ångstrom? Supply the value of α for Hydrogen from the store of your knowledge.