Phys 742 – Assignment 3 – Due 13 Feb 96

1. (Similar to Jackson's 2.6; see also 2.14d)

A parallel plate capacitor has a small hemispherical boss of radius a on one of its plates. Assume that A is the area of each plate, d the separation between them, with $\sqrt{A} >> d >> a$. To be definite, assume that the plates are flat circular disks perpendicular to the z axis and put the boss at the center of the lower plate. However, the result should hold more generally, as long as a is much smaller than the distance of the boss from the closest edge.

(a) Where on the lower plate does the maximum electric field occur, and how much larger than average is it? Comment on the chances of being struck by lightning in hilly country.

(b) Show that the presence of the boss does not change the capacitance, to order a^2/A. Do you expect higher order corrections to increase or decrease C?

2. (Similar to Jackson's 2.5)

(a) Solve the problem of an uncharged conducting cylindrical shell of radius a in a uniform field E_0 perpendicular to its axis. You can use the method of images if you wish. More simply, you can superimpose the external field and that of an induced line dipole, so that the shell is an equipotential.

(b) If the cylinder is cut into half by a plane perpendicular to the field, what force per unit length is required to keep the half-cylinders from separating?

(c) Same question as in (b), when the charge on the shell is λ per unit length.

3.

Jackson's 2.4, but you can take parts (b) and (d) together and look at Estevez and Suen, page 42, for guidance. Be sure to correct the misprints, however

4.

(a) For the problem of Fig. 2.10, what is the behavior of $\Phi(x, y)$ near the origin, i.e., for $\rho << a$? Sketch equipotentials and field lines in this (two-dimensional) corner.

(b) For the same geometry as in (a), put $\Phi = 0$ on the planes $x = 0$ and $y = 0$, $\Phi = V$ on the plane $x = a$. What is now the behavior of $\Phi(x, y)$ near the origin? Sketch equipotentials and field lines in this (two-dimensional) corner.

(c) For the problem of Fig. 2.9 with $a = b = c$ (for a cube), what is the behavior of $\Phi(x, y, z)$ near the origin? What are the charge densities on the three faces as they meet in this (three-dimensional) corner?