General Rotating Black Holes & Their Microscopics

Recent efforts: w/ Finn Larsen 1106.3341 & 1112.4856
w/ Gary Gibbons 1201.0601

[Earlier work: w/ Donam Youm '94-'96:multi-charged rotating asympt.Mink. BH's
w/ Finn Larsen '97-'99,'10: greybody factors; special(BPS) microsc.
w/ Chong, Lü & Pope '06-'08: (AdS) rotating black hole solutions
w/ Chow, Lü & Pope '09: special (Kerr/CFT) microsc.]

Highlight: progress to extract from geometry (mesoscopic approach) the underlying conformal symmetry & promoting it to two-dimensional conformal field theory

→

governing microscopic structure of four and five dimensional asymptotically flat general rotating charged black holes

Main issue in Black Hole Microscopics how to relate

Thermodynamic (Bekenstein) Entropy $=\frac{1}{4}$ $\text{Area}_{\text{horizon}}$

to

Statistical Entropy = logNi

[by identifying number N_i of -micrcoscopic degrees of freedom]

In String Theory such a connection via:

AdS/CFT (Gravity/Field Theory) correspondence

[A string theory on a specific Curved Space-Time (in D-dimensions) related to specific Field Theory (in (D-1)- dimensions) on its boundary]

Maldacena'97

Microscopics of black holes in string theory, in particular relation to 2d-dim CFT (via AdS_3/CFT_2 correspondence) extensively explored over past 10-15 years

Shown in specific/special cases (AdS/CFT):

- BPS (supersymmetric) limit ($m \to 0$) [M=Q]

Strominger&Vafa'96

- near-BPS limit ($m << 1$) . . .Maldacena&Strominger'97

- near-BPS multi-charged rotating black holes w/Larsen'98

Recently:

- (near-)extreme rotating black holes ($m – l << 1$)
  Kerr/CFT correspondence Guica,Hartman,Song,Strominger 0809.4266…

- extreme AdS charged rotating black holes in diverse dim.

. . . . . w/Chow,Lü & Pope arXiv:0812.2918

Another approach: internal structure of black holes
via probes such as scalar wave equation
in the black hole background (greybody factors)

If certain terms in the wave equation omitted →
$SL(2,R)^2$ symmetry & radial solution hypergeometric functions

Omission justified for special backgrounds:
- near-BPS limit ($m \ll 1$) Maldacena-Strominger'97
- near-extreme Kerr limit ($m - l \ll 1$) w/Larsen'97
- low-energy probes ($\omega \ll 1$) Das-Mathur'96…

Recently:
-super-radiant limit ($\omega - n\Omega \ll 1$) .
D=4 Kerr Bredberg,Hartman,Song&Strominger 0907.3477
D=4,5 multi-charged rotating w/Larsen 0908.1136

On the other hand for general black hole backgrounds there is NO $SL(2,R)^2$ symmetry

This would seem to doom a CFT interpret. of the general BH's

Recent proposal dubbed "hidden conformal symmetry"

Castro, Maloney &Strominger 1004.0096

asserts conformal symmetry suggested by
certain terms of the massless wave equation is there,
just that it is spontaneously broken…
pursued by many researchers…

In this talk a different perspective:

Program to quantify ``conventional wisdom'' that general (asymptotically flat) black holes might have microscopic explanation In terms of 2D CFT

w/Larsen '97-'99

But such black holes typically specific heat $c_p < 0$ due to the coupling between the internal structure of the black hole and modes that escape to infinity

Should focus on the black hole "by itself" → one must necessarily enclose the black hole in a box, thus creating an equilibrium system.

[Must be taken into account in any precise discussion of black microscopics.]

I. Quantified geometry of a black hole in a box:

w/Larsen 1106.3341 & 1112.4856

II. Sources supporting this geometry (as a scaling limit of certain BH's) & ``Deconstructing'' origin of conformal symmetry: w/Gibbons 1201.0601

Summary

Employing mesoscopic approach to deduce microscopics from classical geometry for general asymptotically Minkowski black holes in D=5 [& D=4]

w/Larsen 1106.3341,1212.4856

Main technical results:

I. Construct the explicit geometry

whose wave equation exhibits $SL(2,R)^2$ symmetry

[geometrical counterpart to the omission of terms violating $SL(2,R)^2$ in the wave equation.]

→ "subtracted geometry" by ONLY removing certain terms in an overall warp factor of the original metric

→ Physical interpretation – enclosure of the ``black hole in a box''
(subtracted asymptotic Minkowski space-time)

→ asymptotic metric of Lifshitz-type (time & radial coordinate scale differently]

→ Properties of subtraction:

- preserves conformal invariance & consistent with separation of variables
- same thermodynamic potentials and entropy as the full geometry!

II. Further Geometric/Microscopic Interpretation:

lifting the subtracted geometry from D=5 to D=6

→ locally $AdS_3 x S^3$ geometry, w/global identification
S^3 fibered over BTZ black hole

→ $SL(2,R)^2$ conformal symmetry promoted to Virasoro by standard techniques of AdS_3/CFT_2 à la Brown-Henneaux

→ quantitative match of microscopic entropy

III. ``Deconstruction'' of Subtracted Geometry:

w/Gibbons 1201.06018

→Full solution (with sources) of subtracted solution as a scaling limit of another black hole (reminiscent of near-supersymmetric limit)

→Further insights into geometric origin of $SL(2,R)^2/Z_2 x SO(4)$

[Analogous analysis carried out also for general D=4 BH's]

For the case study choose: most general black holes of D=5 N=4 (or N=8) un-gauged supergravity, actually its generating solution

N=4 (N=8) supersymmetric ungauged SG in D=5 can be obtained as a toroidal reduction of Heterotic String (Type IIA String) on $T^{(10\text{-}D)}$ (D=5). Former D=5, N=4 SG, w/ global symmetry O(5,21) xO(1,1). The relevant subsector for generating solutions can also be viewed as D=5 N=2 SG coupled to three vector super-multiplets:

$$e^{-1}\mathcal{L} = R - \frac{1}{2}\delta\vec{\varphi}^2 - \frac{1}{4}\sum_{i=1}^{3} X_i^{-2}\left(F^i\right)^2 + \frac{1}{24}|\epsilon_{ijk}|\,\epsilon^{\mu\nu\rho\sigma\lambda}F^i_{\mu\nu}\,F^j_{\rho\sigma}\,A^k_\lambda$$

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}.$$

Gravity with two scalar fields & three U(1)-gauge fields
[special case: when U(1) gauge fields identified→ Maxwell-Einstein Theory in D=5]

Such three charge rotating solutions were obtained by employing solution generating techniques c.f., Ehlers,… Gibbons, Sen

a) Reduce D=5 stationary solution-
Kerr BH (with mass m and two angular momenta l_1 and l_2)
to D=3 on t and one angular direction

b) D=3 Largrangian has O(3,3) symmetry

c) Acting with an $O(1, 1)^3$ subgroup of O(3, 3) transformations on t
the dimensionally reduced solution to generate
generate new solutions with three parameters δ_i

d) Upon lifting back to D = 5, arrive at spinning solutions
with two angular momenta & three charges parameterised by the three δ_i

w/Youm hep-th/9603100

D=5 Kerr Solution:

$$
\begin{aligned}
ds^2 = & -\frac{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta - 2m}{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta}dt^2 + \frac{r^2(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)}{(r^2 + l_1^2)(r^2 + l_2^2) - 2mr^2}dr^2 \\
& + (r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)d\theta^2 + \frac{4ml_1l_2\sin^2\theta\cos^2\theta}{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta}d\phi d\psi \\
& + \frac{\sin^2\theta}{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta}[(r^2 + l_1^2)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2ml_1^2\sin^2\theta]d\phi^2 \\
& + \frac{\cos^2\theta}{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta}[(r^2 + l_2^2)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2ml_2^2\cos^2\theta]d\psi^2 \\
& - \frac{4ml_1\sin^2\theta}{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta}dtd\phi - \frac{4ml_2\cos^2\theta}{r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta}dtd\psi.
\end{aligned}
$$

m-mass; l_{12}=two angular momenta

Myers&Perry'86

Metric:

$$
\begin{aligned}
ds_E^2 = \bar{\Delta}^{\frac{1}{3}} \Bigg[& -\frac{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta - 2m)}{\bar{\Delta}} dt^2 \\
& + \frac{r^2}{(r^2 + l_1^2)(r^2 + l_2^2) - 2mr^2} dr^2 + d\theta^2 + \frac{4m\cos^2\theta\sin^2\theta}{\bar{\Delta}} [l_1 l_2 \{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& - 2m(\sinh^2\delta_{e1}\sinh^2\delta_{e2} + \sinh^2\delta_e\sinh^2\delta_{e1} + \sinh^2\delta_e\sinh^2\delta_{e2})\} + 2m\{(l_1^2 + l_2^2) \\
& \times \cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2l_1 l_2\sinh^2\delta_{e1}\sinh^2\delta_{e2}\sinh^2\delta_e\}] d\phi d\psi \\
& - \frac{4m\sin^2\theta}{\bar{\Delta}} [(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e) \\
& + 2ml_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e] d\phi dt - \frac{4m\cos^2\theta}{\bar{\Delta}} [(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& \times (l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e) + 2ml_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e] d\psi dt \\
& + \frac{\sin^2\theta}{\bar{\Delta}} [(r^2 + 2m\sinh^2\delta_e + l_1^2)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} \\
& + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2m\sin^2\theta\{(l_1^2\cosh^2\delta_m - l_2^2\sinh^2\delta_m)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& + 4ml_1 l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
& \times (l_1^2\cosh^2\delta_e + l_2^2\sinh^2\delta_e) - 2ml_2^2\sinh^2\delta_e(\sinh^2\delta_{e1} + \sinh^2\delta_{e2})\}] d\phi^2 \\
& + \frac{\cos^2\theta}{\bar{\Delta}} [(r^2 + 2m\sinh^2\delta_e + l_2^2)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} \\
& + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2m\cos^2\theta\{(l_2^2\cosh^2\delta_e - l_1^2\sinh^2\delta_e)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& + 4ml_1 l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
& \times (l_1^2\sinh^2\delta_e + l_2^2\cosh^2\delta_e) - 2ml_1^2\sinh^2\delta_e(\sinh^2\delta_{e1} + \sinh^2\delta_{e2})\}] d\psi^2 \Bigg],
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\Delta} \equiv & (r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& \times (r^2 + 2m\sinh^2\delta_e + l_1^2\cos^2\theta + l_2^2\sin^2\theta),
\end{aligned}
$$

Scalar and gauge fields:

$$g_{11} = \frac{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$e^{2\varphi} = \frac{(r^2 + 2m\sinh^2\delta_e + l_1^2\cos^2\theta + l_2^2\sin^2\theta)^2}{(r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)},$$

$$A^{(1)}_{t\,1} = \frac{m\cosh\delta_{e1}\sinh\delta_{e1}}{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(1)}_{\phi\,1} = m\sin^2\theta\frac{l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e}{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(1)}_{\psi\,1} = m\cos^2\theta\frac{l_1\cosh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - l_2\sinh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e}{r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(2)}_{t\,1} = \frac{m\cosh\delta_{e2}\sinh\delta_{e2}}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(2)}_{\phi\,1} = m\sin^2\theta\frac{l_1\cosh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$A^{(2)}_{\psi\,1} = m\cos^2\theta\frac{l_1\sinh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e - l_2\cosh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e}{r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta},$$

$$\begin{aligned} B_{t\phi} = &-2m\sin^2\theta(l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2 + l_1^2\cos^2\theta \\ &+ l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})/[(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1}) \\ &\times(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})], \end{aligned}$$

$$\begin{aligned} B_{t\psi} = &-2m\cos^2\theta(l_2\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_1\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2 + l_1^2\cos^2\theta \\ &+ l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})/[(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1}) \\ &\times(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})], \end{aligned}$$

$$B_{\phi\psi} = \frac{2m\cosh\delta_e\sinh\delta_e\cos^2\theta\sin^2\theta(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})}{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1})(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})},$$

Solution specified by three charges, mass, two angular momenta:

$$Q_1^{(1)} = 2m\cosh\delta_{e1}\sinh\delta_{e1}, \quad Q_1^{(2)} = 2m\cosh\delta_{e2}\sinh\delta_{e2}, \quad Q = 2m\cosh\delta_e\sinh\delta_e,$$

$$\begin{aligned} M &= 2m(\cosh^2\delta_{e1} + \cosh^2\delta_{e2} + \cosh^2\delta_e) - 3m \\ &= \sqrt{m^2 + (Q_1^{(1)})^2} + \sqrt{m^2 + (Q_1^{(2)})^2} + \sqrt{m^2 + Q^2}, \\ J_\phi &= 4m(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e), \\ J_\psi &= 4m(l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e). \end{aligned}$$

Special cases: all δ_I equal — Reissner-Nordström BH in D=5

$m \rightarrow 0$ $\delta_i \rightarrow \infty$ w/ Q_i finite — Supersymmetric (BPS) limit

$(l_1^2 - l_2^2)^2 + 4m(m - l_1^2 - l_2^2) = 0$ — Extreme -Kerr limit

We shall employ a bit more compact form w/ a warp factor Δ_o (as U(1) fibration over 4d base):

w/Chong,Lü&Pope: hep-th/06006213

Metric: $$ds_5^2 = -\Delta_0^{-2/3} G(dt+\mathcal{A})^2 + \Delta_0^{1/3} ds_4^2 \ ,$$

$$ds_4^2 = \frac{dx^2}{4X} + \frac{dy^2}{4Y} + \frac{U}{G}(d\chi - \frac{Z}{U}d\sigma)^2 + \frac{XY}{U}d\sigma^2$$

$$\Delta_0 = (x+y)^3 H_1 H_2 H_3 \qquad H_i = 1 + \frac{\mu \sinh^2 \delta_i}{x+y} \ , \ (i=1,2,3)$$

$$X = (x+a^2)(x+b^2) - \mu x \ ,$$

Horizon X=0

$$Y = -(a^2-y)(b^2-y) \ ,$$

$$U = yX - xY \ ,$$

$$Z = ab(X+Y)$$

$$G = (x+y)(x+y-\mu) \ ,$$

Ergosphere G=0

$$\mathcal{A} = \frac{\mu \Pi_c}{x+y-\mu}[(a^2+b^2-y)d\sigma - ab\,d\chi] - \frac{\mu \Pi_s}{x+y}(ab\,d\sigma - y\,d\chi)$$

$$x = r^2 \ ,$$

$$y = a^2\cos^2\theta + b^2\sin^2\theta$$

$$\sigma = \frac{1}{a^2-b^2}(a\phi - b\psi)$$

$$\chi = \frac{1}{a^2-b^2}(b\phi - a\psi)$$

$$\Pi_c \equiv \prod_{i=1}^{3}\cosh\delta_i \ , \quad \Pi_s \equiv \prod_{i=1}^{3}\sinh\delta_i$$

Sources:

two scalars: $X_i = H_i^{-1}(H_1H_2H_3)^{1/3}$ i=1,2,3 w/ $X_1X_2X_3=1$

three gauge potentials:

$$
\begin{aligned}
A^1 = & \frac{2m}{(x+y)H_1}\{\sinh\delta_1\cosh\delta_1 dt \\
+ & \sinh\delta_1\cosh\delta_2\cosh\delta_3[abd\chi + (y-a^2-b^2)d\sigma] \\
+ & \cosh\delta_1\sinh\delta_2\sinh\delta_3(abd\sigma - yd\chi)\} \quad (
\end{aligned}
$$

A^2, A^3 via cyclic permutations

Thermodynamics - Suggestive of weakly interacting 2-dim CFT w/ ``left-'' & ``right-moving'' excitations [noted already w/Youm'96]

Not only entropy:

$$S = S_L + S_R$$

$$S_L = 2\pi\sqrt{\frac{1}{4}\mu^3(\prod_i \cosh\delta_i + \prod_i \sinh\delta_i)^2 - J_L^2} = \pi\mu(\prod_i \cosh\delta_i + \prod_i \sinh\delta_i)\sqrt{\mu - (l_1 - l_2)^2}\,.$$

$$S_R = 2\pi\sqrt{\frac{1}{4}\mu^3(\prod_i \cosh\delta_i - \prod_i \sinh\delta_i)^2 - J_R^2} = \pi\mu(\prod_i \cosh\delta_i - \prod_i \sinh\delta_i)\sqrt{\mu - (l_1 + l_2)^2}\,.$$

Also (inverse) Hawking temperature:

$$\beta_H \equiv \frac{1}{2}(\beta_L + \beta_R)\,,$$

$$\beta_R = \frac{2\pi}{\kappa_+} + \frac{2\pi}{\kappa_-} = \frac{2\pi\mu^2}{\sqrt{\mu^2 - l^2}}\left(\prod_i \cosh\delta_i + \prod_i \sinh\delta_i\right)\,,$$

$$\beta_L = \frac{2\pi}{\kappa_+} - \frac{2\pi}{\kappa_-} = 2\pi\mu\left(\prod_i \cosh\delta_i - \prod_i \sinh\delta_i\right)\,.$$

Two angular velocities:

$$\beta_H\Omega_L = \frac{2\pi(l_1 - l_2)}{\sqrt{\mu - (l_1 - l_2)^2}}$$

$$\beta_H\Omega_R = \frac{2\pi(l_1 + l_2)}{\sqrt{\mu - (l_1 + l_2)^2}}$$

Shown, all independent of the warp factor Δo ! w/ Larsen'11

Subtracted geometry obtained by changing warp factor $\Delta_o \rightarrow \Delta$ such that the scalar wave eq. preserves precise $SL(2,R)^2$

Wave eq. written for a metric with an implicit warp factor Δ:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) = 0$$

Equation separable: $\Phi \sim e^{-i\omega t + im_R(\phi+\psi) + im_L(\phi-\psi)}$ η(x) ζ(y)

$$\Bigg[4\partial_x X\partial_x + \frac{x_+ - x_-}{x - x_+}\left(\frac{\beta_R\omega}{4\pi} - m_R\frac{\beta_H\Omega_R}{2\pi} + \frac{\beta_L\omega}{4\pi} - m_L\frac{\beta_H\Omega_L}{2\pi}\right)^2$$

$$-\frac{x_+ - x_-}{x - x_-}\left(\frac{\beta_R\omega}{4\pi} - m_R\frac{\beta_H\Omega_R}{2\pi} - \frac{\beta_L\omega}{4\pi} + m_L\frac{\beta_H\Omega_L}{2\pi}\right)^2 + \boxed{\mu\omega^2(1+\sum_i \sinh^2\delta_i) + x\omega^2 + y\omega^2 + \frac{\Delta-\Delta_0}{G}\omega^2}\Bigg]\Phi$$

$$= j(j+2)\Phi$$

Adjust Δ to cancel → $SL(2,R)^2$ restored!

S^3 Laplacian eigenvalues

$$\Delta_0 = \prod_{i=1}^{3}(x + y + \mu\sinh^2\delta_i) \quad \rightarrow \quad \Delta = \mu^2\left[(x+y)(\Pi_c^2 - \Pi_s^2) + \mu\Pi_s^2\right]$$

$$\Pi_c \equiv \prod_{i=1}^{3}\cosh\delta_i\ , \quad \Pi_s \equiv \prod_{i=1}^{3}\sinh\delta_i$$

Remarks:

Subtracted geometry does not satisfy Einstein's equation →

Subtraction that results in exact conformal symmetry ↔

black hole in a box, which has to be supported by sources (return to them later)

Asymptotic geometry of a Lifshitz-type w/ a deficit angle

$$ds_5^2 = -\left(\frac{R}{R_0}\right)^8 dt^2 + 12 dR^2 + R^2 d\Omega_3^2$$

→ black hole in an `` asymptotically conical box''

Lift to auxiliary 6-dimensions:

$$ds_6^2 = \Delta(\frac{1}{\mu}d\alpha + \mathcal{B})^2 + \Delta^{-1/3}ds_5^2$$
$$= \Delta(\frac{1}{\mu}d\alpha + \mathcal{B})^2 - \Delta^{-1}G(dt + \mathcal{A})^2 + ds_4^2$$

where the KK-field along α is

$$\mathcal{B} = \frac{1}{\Delta}\left[\mu((a^2+b^2-y)\Pi_s - ab\Pi_c)d\sigma + \mu(y\Pi_c - ab\Pi_s)d\chi - \frac{\Pi_s\Pi_c}{\Pi_c^2 - \Pi_s^2}dt\right]$$

Massless 6D fields independent of α satisfy precisely the same wave equation as massless 5D fields.

↓

Geometry factorizes: locally AdS_3 x S^3,
globally S^3 fibered over BTZ black hole

→ conformal symmetry promoted to Virasoro algebra
& quantitative (standard) microscopic calculation (AdS_3/CFT_2)
à la Brown-Hennaux

[long spinning string interpretation]

w/Gibbons 1201.0601

Sources supporting subtracted geometry is obtained as a scaling limit of a black-hole w/ two large charges (denoted w/ ``tiilde'' variables and two equal (large) charges $\tilde\delta_1 = \tilde\delta_2 \equiv \tilde\delta$):

$$\epsilon \to 0 \quad \tilde{x} = x\epsilon, \quad \tilde{t} = t\epsilon^{-1}, \quad \tilde{y} = y\epsilon, \quad \tilde{\sigma} = \sigma\epsilon^{-1/2}, \quad \tilde{\chi} = \chi\epsilon^{-1/2},$$
$$\tilde{m} = m\epsilon, \quad \tilde{a}^2 = a^2\epsilon, \quad \tilde{b}^2 = b^2\epsilon,$$
$$2\tilde{m}\sinh^2\tilde{\delta} \equiv Q = 2m\epsilon^{-1/2}(\Pi_c^2 - \Pi_s^2)^{1/2}, \quad \sinh^2\tilde{\delta}_3 = \frac{\Pi_s^2}{\Pi_c^2 - \Pi_s^2}$$

``Untilded'' variables are those of the subtracted geometry metric w/ three charges δ_1,$\delta_{,2}\delta_3$, and subtracted warp factor

$$\Delta = (2m)^2\left[(x+y)(\Pi_c^2 - \Pi_s^2) + 2m\Pi_s^2\right]$$

Fully determined sources: Scalars: $X_1 = X_2 = X_3^{-\frac{1}{2}} = \frac{\Delta^{\frac{1}{3}}}{2m}$

Gauge potentials: $A^1 = A^2 = -\frac{x+y}{2m}\,dt + y\Pi_c\,d\sigma - y\Pi_s\,d\chi$

$$A^3 = \frac{(2m)^4\Pi_s\Pi_c}{(\Pi_c^2 - \Pi_s^2)\Delta}\,dt + \frac{\Pi_s}{\Delta}[ab\,d\chi + (y - a^2 - b^2)d\sigma] + \frac{\Pi_c}{\Delta}(ab\,d\sigma - y\,d\chi)$$

Comments:

a) Scaling limit (resulting in subtracted geometry) is
reminiscent of near-BPS limit, but with
two (equal) charges $\rightarrow \infty$ & third one $\rightarrow 0$

b) Infinite charges can be gauged away (by rescaling the scalars).
However, the asymptotic metric is of Lifshitz type
(``softer'' than AdS)

c) In retrospect the lift to D=6 as $AdS_3 \times S^3$ expected (due to
BPS-like nature of the scaling limit)

d) Subtracted geometry can also be obtained as a Harrison
transformation (with an infinite boost) on the original solution
[explicitly shown on a static solution]

Further Remarks:

Rotating Asymptotically Minkowski BH's in D=4, parameterized by mass, angular momentum and (four-)charges

Subtracted geometry prescription works in D=4 for general (four-) charge rotating black hole! w/Larsen 1112.4856

Metric written in terms of a warp factor; termodyn. again indep. of warp factor

Allows for restoration of $SL(2,R)^2$ in the wave eq.

Lift to D=5: locally $AdS_3 \times S^2$; globally S^2 fibered over BTZ

-- quantitative microscopics again à la Brown-Henneaux

-- FULL solution with subtracted geometry obtained as a scaling limit on a black hole with three (large) charges, again reminiscent of near-BPS black hole! w/Gibbons 1201.0601

General AdS Black Holes?

Not 2D CFT ←→ more than two horizons

Intriguingly, the product of areas associated with all horizons quantized w/Gibbons&Pope 1011.008 (PRL)

All known D=5 solutions written with warp factors → possible subtracted geometry that points to underlying (higher dim) conformal symmetry

→ FURTHER STUDY