• The Hubble constant

• Weinberg, chapters I and II


The Hubble constant

Now that we know the universe is expanding, the first obvious question to ask is: what’s the center? Usually when we think of something like a balloon being blown up. This obviously has a central point which is not moving. But if the expansion has a center, then our assumption of homogeneity is obviously wrong: the center is a special point.

There’s only one way of having an expansion and homogeneity at the same time. This is if the relative velocity between two objects is proportional to the distance in between them. By looking at the figure in Weinberg, you can see how only in this case is no point special: everybody agrees that the relative velocity of two objects is proportional to the distance between them. In an equation,

\[ v = H d \]

or equivalently, the redshift \( z \) means non-relativistically,

\[ d = \frac{z c}{H} \]

The constant \( H \) is called the “Hubble constant”. As we will see, it is not constant in time; it is constant in space.

This means that now from the redshifts, we know relative distances: twice the \( z \) means twice the distance (and twice as old, because the light will have taken twice as long to reach us). Note this is an average relation: a given galaxy will be moving in some random direction as well, changing the value of \( z \). For Andromeda, \( z \) is negative, but that does not mean it is a negative distance away. But now, they have seen hundreds of thousands of galaxies with \( z \) large enough so that we can neglect this random motion. For example, in one sky survey I looked up they restricted to galaxies with \( 0.01 \leq z \leq 0.17 \).

The much harder thing to do is to determine absolute distance. In other words, what is the Hubble constant \( H \)? To do this, we need to figure out the absolute distance to something. Then
we can fit $H$ by using this formula, because we measure $v$ by the Doppler shift and $d$ directly. You can measure the absolute distance to something if you know its absolute luminosity, i.e. how bright it actually is. Then by measuring how bright it is here, you can compare the two and then know the distance (assuming there's no intergalactic dust that absorbs the light, etc).

The first way of measuring absolute distance cosmologically was an observation by Henrietta Levitt that a certain kind of star, called a Cepheid, has an unusual property. Its brightness oscillates in time, and (the important part) the period of oscillation is proportional to its absolute brightness! Determine the precise constant of proportionality is quite tricky (you have to use yet another method of measuring an absolute distance): the original number Hubble used was off by a factor of 10 or so. Even when I was a graduate student, the Hubble constant was known only to about a factor of 2. But just in the last 5 years, the data have gotten much better. The current best-fit value Hubble constant is

$$H = 21 \pm 1 \frac{km/s}{\text{million light years}}$$

The higher the $z$, the farther away. The farther away, the older what we’re seeing is. This is because the speed of light is finite. The $z$ of 1.7 corresponds to light emitted about 11 billion years ago.

If we assume that the galaxies aren’t accelerating or decelerating, we can use the Hubble constant to estimate the age of the universe. We do this by tracing backwards in time to where there was no distance between galaxies. If a galaxy is a distance $d$ from us and traveling at a constant relative velocity $v$, that means a time $t = d/v$ in the past, it was on top of us. This time was the time the Big Bang happened. From our Hubble formula, this means the age of the universe at approximately $t_{universe} = d/v = H^{-1}$. Putting the Hubble constant in more useful units for this purpose, we get

$$t_{universe} = H^{-1} = \frac{10^6ly \times 3.0 \times 10^5 \text{ km/sec}}{21 \text{ km/s}} = 14 \text{ billion years}$$

The velocity is not constant over time, so this is only an approximation. By luck it agrees with more accurate methods, which give the age of the universe to be $14 \pm 1$ billion years.