P. Q. Hung

University of Virginia

Hue, July 25, 2011

P. Q. Hung Dynamical Electroweak Symmetry Breaking



Fresh from The News of the World: The Higgs boson has been discovered at the LHC!

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Just kidding:))



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and/or perhaps...

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a session of Karaoke?

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Part I

• A brief review of the Hierarchy Problem of the Standard Model and its "standard" solutions.

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- Models of DESB: $\langle \overline{F}_L F_R \rangle$ coming from a heavy 4th generation. *F*: 4th generation fermions.

Part II

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- Phenomenological constraints and implications.
- Speculation: SM4 merges into a theory with no mass scales \rightarrow Conformally invariant theory above the condensation scale.

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- Even if $m_H \ll M_P$ at tree level, one-loop radiative corrections to the Higgs mass squared give $\delta m_H^2 \sim O(m_H^2 m_f^2)(\Lambda/\nu)^2$ with Λ being a physical cutoff scale.

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- If Λ ~ M_P ⇒ precise cancellation between boson and fermion masses ⇒ Extreme fine tuning such that m_H ≪ M_P. Higher order corrections will destroy this ⇒ Back to the same problem.

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- However, no elementary scalar field has been found ⇒ SUSY has to be broken, not only broken but broken spontaneously so as to preserve the stability of the result at higher orders.
- It is not yet clear what mechanism is responsible for the spontaneous breakdown of SUSY: gravity-mediated, gauge-mediated,...? One thing that most workers in that field agree on is that the spontaneous breakdown of SUSY is in a Hidden Sector which is then mediated to the supersymmetric SM.

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ARE THERE SIMPLER ALTERNATIVES? In particular, we are interested in scenarios in which the electroweak breaking scale of O(TeV) is DYNAMICAL \rightarrow Natural physical cutoff scale.

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- Chiral symmetry breaking in Quantum Chromodynamics (QCD) as an example.

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- The Lagrangian has a global chiral symmetry: $SU(2)_L \times SU(2)_R$ with $q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ and $q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$ transforming as doublets under $SU(2)_L$ and $SU(2)_R$ respectively.

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- The running coupling g_3 grows strong at low energy and become large at roughly $E \sim 300 \text{ MeV} \Rightarrow$ chiral symmetry breaking and quark confinement.

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- Notice that the composite scalars have exactly the same EW quantum numbers as the SM Higgs: $\phi = \begin{pmatrix} \pi^+ \\ \sigma + i \pi_3 \end{pmatrix}$.

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- Problems with Electroweak Precision Parameters!

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- S, T and U enter electroweak measurables as $M_Z^2 = M_{Z0}^2 \frac{1-\hat{\alpha}(M_Z)T}{1-G_F M_{20}^2 S/2\sqrt{2\pi}}$, etc...

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• A fit to the data $\rightarrow S = 0.01 \pm 0.10$, $T = 0.03 \pm 0.11$ and $U = 0.06 \pm 0.10$.

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- The simplest QCD-like one-family of Technifermions gives $S \sim 0.25 N_D \frac{N_{TC}}{3} \sim 1$ for $N_D = 4$ and for example $N_{TC} = 3$ (number of $SU(2)_L$ doublets: 3 for techniquarks and 1 for technileptons). Clearly in contradiction with the data!

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- More problems to come when one looks at how fermions get masses.
- In the SM, fermions get masses by coupling to the Higss (see Ling-Fong Li's lecture). In TC models, The Higgs is a composite of Technifermions. Question: For standard fermions (t, b, c, s, u, d,...) to get masses in TC, they should somehow couple to TC fermions.

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Extended Technicolor

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- So what are the problems with this?

• TC: scaled-up version of QCD \Rightarrow Anomalous dimension $\gamma_m(\mu) \ll 1$ $\Rightarrow \langle \overline{T} T \rangle_{ETC} \approx \langle \overline{T} T \rangle_{TC} \sim O(F_{\pi}^3).$

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- ETC interactions of the type $q \to T \to q'$ induce Flavor changing-neutral current interactions e.g. $\frac{g_{ETC}^2 \theta_{sd}^2}{M_{erc}^2} \bar{s} \Gamma^{\mu} d \bar{d} \Gamma'_{\mu} s.$

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- Constraints on $K_L K_S$ mass difference $\Rightarrow M_{ETC} > 1000 \ TeV \Rightarrow m_q < 100 \ MeV!$
- What to do next is a Big Question for the ETC community!

• Enhance $\langle \overline{T} T \rangle_{ETC}$. How? Making $\gamma_m(\mu)$ large. How? By bringing $\alpha_{TC}(\mu)$ close to a critical coupling α_C such that $\gamma_m(\mu) = 1 - \sqrt{1 - \frac{\alpha_{TC}(\mu)}{\alpha_C}} \rightarrow \gamma_m(\Lambda_{TC}) \approx 1 \rightarrow \beta(\alpha_{TC}(\mu)) \approx 0$ for $\mu > \Lambda_{TC}$.

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- With $\gamma_m(\Lambda_{TC}) \approx 1$ for $\mu > \Lambda_{TC}$, one obtains $\langle \bar{T} T \rangle_{ETC} \approx \langle \bar{T} T \rangle_{TC} \frac{M_{ETC}}{\Lambda_{TC}} \Rightarrow m_q \sim \frac{g_{ETC}^2}{M_{ETC}^2} \langle \bar{T} T \rangle_{TC} \frac{M_{ETC}}{\Lambda_{TC}}$. A big enhancement!

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- It is however quite complicated to find a model which can do all these things! In particular the aforementioned phenomena occur in a nonperturbative regime.

Is there a simpler alternative to Extended Technicolor?



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Is there a simpler alternative to Extended Technicolor? The Standard Model with 4 generations.

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• Can a pair of heavy fermions in the SM form a bound state by the exchange of the Higgs boson?

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where m_H is the Higgs mass and $\alpha_Y = \frac{m_1 m_2}{4\pi v^2}$ with v = 246 GeV, with m_1, m_2 being masses of the 2 fermions. Reduced mass: $M = m_1 m_2 / (m_1 + m_2)$.

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• Rayleigh-Ritz variational method with the trial wave function $u(y, r) = 2y^{\frac{3}{2}} e^{-yr}$. y: variational parameter.

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• Numerical integration of the Schrödinger equation gives

$K_f > 1.68$

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- 1st remark: $m_t = 173 \text{ GeV}$ and $m_H > 115 \text{ GeV}$ gives $K_t < 0.06 \Rightarrow$ The top quark cannot form a bound state by exchanging the Higgs boson! (Remember the condition $K_f > 1.68$)

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- To satisfy $K_f > 1.68$, we need heavy fermions! This is where the 4th generation comes in. It was found by Hung and Xiong that the bound states are loose (the binding energy is small compared with the mass) when the the fermion mass is comparable with the Higgs mass.

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- Relativistic corrections to the Yukawa potential by Ishiwata and Wise came to similar conclusions that, for $m_{q'} > 350 \ GeV$, the Higgs Yukawa coupling plays a crucial role in the formation of bound states. The bound state is loose when the 4th generation quark mass is comparable to the Higgs mass and the binding energy increases as the Higgs mass becomes smaller than the 4th generation mass.

Dynamical Symmetry Breaking: Another look at QCD

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- Pagels and Stokar: Step 1: No current quark mass; Step 2: Dynamically generated quark mass from the self-energy Σ(p); Step 3: Pion decay constant f_π computed in terms of Σ(p).
- The information about Dynamical Chiral Symmetry Breaking encoded in $\Sigma(p)$ and f_{π} .

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- Explicitly, take e.g. the 4th generation quarks (t', b') and assuming $g_{t'} = g_{b'} = g_{4Q} \rightarrow$

 $g_{f}\bar{\psi}_{L}\phi\psi_{R}+H.c.=g_{4Q}\{\bar{q}_{L}\begin{pmatrix}\phi^{+}\\\phi^{0}\end{pmatrix}b_{R}^{'}+\bar{q}_{L}\begin{pmatrix}\phi^{0*}\\-\phi^{-}\end{pmatrix}t_{R}^{'}+H.c.\}$

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• Schwinger-Dyson equation for the fermion self-energy $\Sigma(p)$ in the ladder approximation for such a Higgs-Yukawa sector. (See Fukuda and Kugo; Leung, Love and Bardeen;..)

• Contribution to $\Sigma(p)$ in the ladder approximation:



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• $\Sigma(p) = \frac{+2g_{4Q}^2}{(2\pi)^4} \int d^4q \frac{1}{(p-q)^2} \frac{\Sigma(q)}{q^2 + \Sigma^2(q)}$
• Integral equation converted into differential equation:

$$u^{''} + 4u^{'} + 3u + (\frac{\alpha_{4Q}}{\alpha_{4}^{c}})\frac{u}{1+u^{2}} = 0$$

where $\Sigma(p) = e^t u(t + t_0)$, $t_0 = \ln(\Sigma(0))$, $t = \ln(p)$, $\alpha_{4Q} = \frac{g_{4Q}^c}{4\pi}$, and $\alpha_4^c = \frac{\pi}{2} \approx 1.57$ is the critical coupling.

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- Above differential equation + Boundary conditions \equiv Integral equation

• $\alpha_{4Q} < \alpha_4^c$: No solution. Does not satisfy the boundary conditions.



• $\alpha_{4Q} > \alpha_4^c$: Non trivial solution satisfying the boundary conditions



• Asymptotic solution:

$$\Sigma_{4Q}(p) \sim p^{-1} \sin[\sqrt{rac{lpha_{4Q}}{lpha_4^c}} - 1(\ln p + \delta)], \quad ext{for } lpha_{4Q} > lpha_4^c$$



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•
$$\delta = \ln(\frac{\Lambda}{\Sigma_{4Q}(0)}) + \frac{\pi}{\sqrt{\frac{\alpha}{\alpha_4^c} - 1}} - 1$$

• Condensates with electroweak quantum numbers:

 $\langle ar{t'}_L t'_R
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• The condensates break the electroweak symmetry: Requirement $\propto O(-\Lambda_{EW}^3)$

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$$\Lambda(\sqrt{rac{lpha_{4Q}}{lpha_c}}-1)\sim O(\Lambda_{EW}) \Rightarrow$$

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 for $\Lambda \sim 10^{16} \ GeV$.

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- No such fine tuning is required if Λ ~ O(TeV) as is the case with a heavy fourth generation.
- Possible solution to the hierarchy problem!

• At the electroweak scale, the fundamental scalar also obtains an induced negative mass squared $\frac{1}{2} \{ \frac{2g_{4Q}^2}{\Sigma_{4Q}(0)} \langle \bar{t'}_L t'_R \rangle + \frac{2g_{4L}^2}{\Sigma_{4L}(0)} \langle \bar{L}_L L_R \rangle \} |\phi^0|^2 \Rightarrow$ It also develops a VEV.

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- Three Higgs doublets: Two composites and one fundamental. The Goldstone bosons eaten up by *W* and *Z* are three combinations of these scalars. Rich spectrum of scalars!

So from the SD equation, we know that $\alpha_{4Q} > \frac{\pi}{2} \sim 1.57$ for condensates to form. But at what energy scale?

• Run the 2-loop RG equations:

 $16\pi^2 \frac{dY}{dt} = \beta_Y$

from Λ_{EW} on, with $Y = \lambda, g_t^2, g_q^2, g_l^2$ (quartic, top, 4th quark, 4th lepton couplings).

Run the 2-loop RG equations:

 $16\pi^2 \frac{dY}{dt} = \beta_Y$

from Λ_{EW} on, with $Y = \lambda, g_t^2, g_q^2, g_l^2$ (quartic, top, 4th quark, 4th lepton couplings).

• Start with initial values of the couplings at Λ_{EW} translated into the naive masses by $m_f = g_f v / \sqrt{2}$ ($v = 246 \ GeV$).



Quasi-fixed point at O(TeV) for heavy 4th generation. The (ruled out) light case is shown for comparison.



Quasi-fixed point at O(TeV) for heavy 4th generation.



The Landau pole (at one loop) is shown for comparison. Λ_{FP} or a scale close to it could play the role of a physical cut off scale.



Possibilities: Region II is characterized by a scale invariant theory both at the classical and quantum levels. Near the boundary between Regions I and II, a fermion-antifermion bound state can get formed by the exchange of a massless scalar \Rightarrow Dynamical electroweak symmetry breaking at O(TeV)! (P. Q. Hung and Chi Xiong, arXiv:1012.4479 [hep-ph], NPB)

A possible scenario:



• Region I: SM4, broken scale invariance.

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• EW precision data: The 4th generation is not ruled out at 6σ by the S-parameter as claimed by PDG which applies only to degenerate doublets, e.g. $\Delta S = \frac{2}{3\pi} - \frac{1}{3\pi} (\ln \frac{m_{t'}}{m_{b'}} - \ln \frac{m_N}{m_E})$. Adjusting the mass splitting in both S and $T \Rightarrow OK$ with EW precision data \Rightarrow can accommodate a heavier Higgs. (Gfitter)

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- Advantage: Conformality at $\Lambda_{FP} \sim O(TeV)$ is as competitive as SUSY. Much fewer parameters.

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- SM4 may merge into a conformally invariant theory at the TeV scale.

Mixings:





Back-up slides: Experimental constraints on the 4th generation

• Bounds on mixings between the 4th generation and the other three: Directly measured matrix elements and ϵ_K , Δm_u , Δm_s , with $m_{t'} > 300 \ GeV$ (Lacker)

	$\left V_{CKM}^{4\times4}\right =$	0.97418 0.224	0.2253 0.973	0.0043 0.041	<0.046 <0.20	Limits
:		< 0.038	< 0.123 > 0.021	>0.78	< 0.63	@ ~20
		< <u>0.074</u>	< 0.20	< 0.63	>0.78	

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Phenomenology of the 4th generation

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- Buras et al also investigated the effects of SM4 on mixing and CP violation in the Charm system: Large effects in K, D and B_s systems possible, disfavoured in B_d system. $S_{\psi \phi}(B_s) > 0.2$ plus measured $\frac{\epsilon'}{\epsilon}$: Significant reduction of SM4 effects in the D system.

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- Soni et al also studied rare K and B decays: Current data favor $m_{t'} \sim 400 600 \text{ GeV}$, $|V_{t'b}^*V_{t's}| \sim (0.05 1.4) \times 10^{-2}$, large CP-odd associated phase.

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t' mass:



Phases:











√s = 10 TeV and 1 fb⁻¹
m_{q'} = 600 GeV



• LHC studies of Higgs compositeness by Soni and Bar-shalom