Introduction to Spontaneous Symmetry Breaking

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2011 BCVSPIN, 25, July 2011

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Introduction

Fundamental Interactions in nature

1. **Strong interactions**–Quantum Chromodynamics(QCD) : gauge theory based on $SU(3)$ symmetry

2. **Electromagnetic interaction** / **Weak interaction** } **Electroweak interaction**–gauge theory based on $SU(2) \times U(1)$ symmetry with spontaneous symmetry breaking

3. **Gravitational interaction**: gauge theory of local Lorentz symmetry

Symmetry and Conservation Law

Noether's Theorem: Any continuous transformation which leaves the action

$$S = \int d^4x \mathcal{L}$$

invariant, will give a conserved current

$$\partial^\mu J_\mu = 0$$

which gives conserved charge

$$\frac{dQ}{dt} = 0, \qquad Q = \int d^3x \, J_0$$

Symmetry Transformation	Conserved Charge
time translation $\quad t \to t + a$	Energy
space translation $\quad \vec{x} \to \vec{x} + \vec{b}$	Momentum
rotation	Angular momentum
...	...

Other conserved quantities: Electric charge, Baryon number,...

Remarks

- Conservation laws $\Longrightarrow$ stability of particles
  e.g. Baryon number conservation $\Longrightarrow$ proton is stable, Charge conservation $\Longrightarrow$ electron is stable, Dark Matter ?$\cdots$
- Conservation laws might change as we gain more knowledge
  e.g. muon number is violated only when ν oscillations were observed
  Parity violation was discovered only in late 50's, CP violation ...
- In quantum system, symmetries $\Longrightarrow$ degenercies of energy levels
  e.g. rotational symmetry $\Longrightarrow$ $(2l+1)$ degeneracies

2) Symmetry Breaking

Most of the symmetries in nature are approximate symmetries.

(a) **Explicit breaking**–

$$H = H_0 + H_1, \qquad H_1 \quad \text{does not have the symmetry of } H_0$$

Example: Hydrogen atom in external magnetic field $\vec{B}$

$$H_0 = \frac{\vec{p}^{\,2}}{2m} - \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r}, \qquad H_1 = -\vec{\mu}\cdot\vec{B}$$

H_0 is invariant under all rotations while H_1 is invariant only for rotation along direction of $\vec{B}$
Add small non-symmetric terms to the Hamiltonian$\Longrightarrow$ degeneracies reduced or removed

(b) **Spontaneous breaking**:

Hamiltonian has the symmetry, $[Q, H] = 0$ but ground state does not, $Q\,|0\rangle \neq 0$ (Nambu 1960, Goldstone 1961)
Example: Ferromagnetism
$T > T_C$ (Curie Temp) all magnetic dipoles are randomly oriented–rotational symm
$T < T_C$ all magnetic dipoles are in the same direction-not invariant under rotation

Ginsburg-Landau theory:

Write the free energy u as function of magnetization $\overrightarrow{M}$ in the form,

$$u(\overrightarrow{M}) = (\partial_t \overrightarrow{M})^2 + \alpha_1 (T) (\overrightarrow{M} \cdot \overrightarrow{M}) + \alpha_2 (\overrightarrow{M} \cdot \overrightarrow{M})^2,$$

where

$$\alpha_2 > 0, \qquad \alpha_1 = \alpha (T - T_C) \qquad \alpha > 0$$

Here u has $O(3)$ rotational symmetry. The ground state is at

$$\overrightarrow{M}(\alpha_1 + 2\alpha_2 \overrightarrow{M} \cdot \overrightarrow{M}) = 0$$

$T > T_C$ minimum at $\overrightarrow{M} = 0$.
$T < T_C$ minimum at $\left|\overrightarrow{M}\right| = \sqrt{\frac{-\alpha_1}{2\alpha_2}} \neq 0$. If we choose $\overrightarrow{M}$ to be in some direction the rotational symmetry is broken.

Goldstone Theorem

Noether theorem: continuous symmetry $\Longrightarrow$ conserved current,

$$\partial_\mu J^\mu = 0, \qquad Q = \int d^3x J^0(x), \qquad \frac{dQ}{dt} = 0$$

Suppose $A(x)$ and $B(x)$ are some local operators and transform into each other under the symmetry charge Q,

$$[Q, A(0)] = B(0)$$

Suppose

$$\langle 0 | [Q, A(0)] | 0 \rangle = \langle 0 | B(0) | 0 \rangle = v \neq 0, \qquad \text{symmetry breaking condition}$$

This implies

$$Q|0\rangle \neq 0,$$

and Q is a broken charge. Then

$$E_n = 0, \qquad \text{as} \qquad \vec{p}_n = 0, \quad \text{for some state } n$$

This means zero energy excitations.

To show this, write

$$\langle 0 \left| [Q, A(0)] \right| 0 \rangle = \int d^3x \left\langle 0 \left| \left[J^0(x), A(0) \right] \right| 0 \right\rangle$$

Inserting a complete set of states and integating over x

$$\begin{aligned} & \langle 0 \left| [Q, A(0)] \right| 0 \rangle \\ = \quad & \sum_n (2\pi)^3 \delta^3(\vec{p}_n) \left\{ \left\langle 0 \left| J^0(0) \right| n \right\rangle \langle n \left| A \right| 0 \rangle e^{-iE_n t} - \langle 0 \left| A \right| n \rangle \left\langle n \left| J^0(0) \right| 0 \right\rangle e^{iE_n t} \right\} = v \neq 0 \end{aligned}$$

RHS is time independent while LHS depends explicitly on time from $e^{\pm iE_n t}$. This relation can be satisfied only if there exists an intermediate state $|n\rangle$ for which

$$E_n = 0, \qquad \text{for} \qquad \vec{p}_n = 0$$

For relativistic system, energy momentum relation yields

$$E_n = \sqrt{\vec{p}_n^{\,2} + m_n^2} \qquad \Rightarrow \qquad m_n = 0, \quad \text{Goldstone boson}$$

Remarks

1. State $|n\rangle$ with property

$$\langle 0 | A(0) | n \rangle \neq 0,$$

   is the massless Goldstone boson while local field B corresponds to some massive particle. This state will also have the property

$$\langle n | Q | 0 \rangle \neq 0$$

   This means $|n\rangle$ can connect to vacuum through broken charge Q.

2. Spectrum
   Here Q is a broken symmetry charge. In general, each broken charge will have a Goldstone boson,

$$\# \text{ of Goldstone bosons} = \ \# \text{ of broken generators}$$

   In many cases, there are symmetry charges which remain unbroken,

$$Q_i |0\rangle = 0, \qquad i = 1, 2, \cdots, l$$

   These unbroken charges $Q_1, Q_2, \cdots Q_l$ form a group H, a subgroup of original symmetry group G. As a consequence, the particles will form multiplets of symm group H. For example, if symmetry $SU(2) \times SU(2)$ is spontaneously broken to $SU(2)$, particles will form $SU(2)$ multiplets

3. In relativistic field theory, massless particle $\Longrightarrow$ long range force. In real world , no massless particls except photon.
   In strong interaction, we have approximate spontaneous symmetry breaking. $\Longrightarrow$ *almost* Goldtone bosons

4. Goldstone bosons can either be elementary fields or composite fields
   a) Elementary fields
   Here local operators A, B are fields in the Lagrangian–this is the case in Standard electroweak theory
   b) Composite fields
   In this case A, B correspond to some bound states of elementary fields.
   For example in QCD the flavor chiral symmetry is broken by quark condensates. Here we have

$$\left[Q^5, \bar{q}\gamma_5 q\right] = \bar{q}q$$

   Q^5 some chiral charge, q quark field
   If $\left\langle 0 \left| \bar{q}q \right| 0 \right\rangle \neq 0$ (quark condensate), then $\bar{q}\gamma_5 q$ will correspond to a Goldstone boson.
   This is usually referred to as **dynamical symmetry breaking**.

Global symmetry

We now discuss the spontaneou symmetry in some simple relativistic field theory. As an example, consider

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2] - V(\sigma^2 + \pi^2)$$

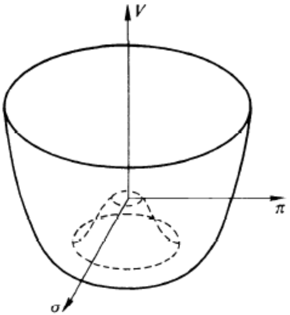
with

$$V(\sigma^2 + \pi^2) = -\frac{\mu^2}{2}(\sigma^2 + \pi^2) + \frac{\lambda}{4}(\sigma^2 + \pi^2)^2 \qquad (1)$$

This potential has $O(2)$ symmetry

$$\begin{pmatrix} \sigma \\ \pi \end{pmatrix} \rightarrow \begin{pmatrix} \sigma' \\ \pi' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$

where α is a constant, global symm



To get the minimum we solve the equations

$$\frac{\partial V}{\partial \sigma} = \left[-\mu^2 + \lambda(\sigma^2 + \pi^2)\right] \sigma = 0$$

$$\frac{\partial V}{\partial \pi} = \left[-\mu^2 + \lambda(\sigma^2 + \pi^2)\right] \pi = 0$$

Solution

$$\sigma^2 + \pi^2 = \frac{\mu^2}{\lambda} = v^2 \qquad \text{circle in } \sigma - \pi \text{ plane}$$

Choose $\sigma = v$, $\pi = 0$ which are fields configurations for classical vacuum. New quantum fields

$$\sigma' = \sigma - v \quad , \quad \pi' = \pi$$

correspond to oscillations around the minimum. Note that

$$\langle \sigma \rangle = v \neq 0$$

is the symmetry breaking condition in Goldstone's theorem. The new Lagrangian is

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \sigma'^2 + (\partial_\mu \pi)^2] - \mu^2 \sigma'^2 - \lambda v \sigma' (\sigma'^2 + \pi'^2) - \frac{\lambda}{4}(\sigma'^2 + \pi'^2)^2$$

There is no π'^2 term, $\Rightarrow$ π' massless Goldstone boson

Local symmetry: **gauge symmetry**
Maxwell Equations:

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0}, \qquad \vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0, \qquad \frac{1}{\mu_0}\vec{\nabla}\times\vec{B}=\varepsilon_0\frac{\partial\vec{E}}{\partial t}+\vec{J}$$

Source free equations can be solved by introducing scalar and vector potentials, ϕ, $\vec{A}$,

$$\vec{B}=\vec{\nabla}\times\vec{A}, \qquad \vec{E}=-\vec{\nabla}\phi-\frac{\partial\vec{A}}{\partial t}$$

These solutions are not unique because of **gauge invariance**,

$$\phi\longrightarrow\phi-\frac{\partial\alpha}{\partial t}, \qquad \vec{A}\longrightarrow\vec{A}+\vec{\nabla}\alpha$$

Or

$$A_\mu\longrightarrow A_\mu-\partial_\mu\alpha\left(x\right)$$

Classically, it is not so clear what the physical role is played by gauge invariance.

In quantum mechanics, Schrodinger eq for charged particle is,

$$\left[\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - e\vec{A}\right)^2 - e\phi\right]\psi = i\hbar\frac{\partial\psi}{\partial t}$$

This requires transformation of wave function,

$$\psi \longrightarrow \exp\left(i\frac{e}{\hbar}\alpha\left(x\right)\right)\psi$$

to get same physics. Thus gauge invariance is now connected to **symmetry** (local) transformation.

Spontaneous symmetry breaking in local symmetry

Local symmetry : symm parameter depends on space-time.
Consider a field theory with $U(1)$ local symmetry,

$$L = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where

$$D_\mu \phi = \left(\partial_\mu - igA_\mu\right)\phi, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

This local $U(1)$ phase transformation is of the form,

$$\phi(x) \longrightarrow \phi'(x) = e^{-i\alpha}\phi(x), \qquad \alpha = \alpha(x)$$

The derivative transforms as

$$\partial_\mu \phi(x) \longrightarrow \partial_\mu \phi'(x) = e^{-i\alpha}\left[\partial_\mu \phi(x) - i\partial_\mu \alpha\right]$$

which is not a phase $U(1)$ transformation.

Introduce A_μ to form **covariant derivative**,

$$D_\mu \phi = \left(\partial_\mu - igA_\mu\right)\phi$$

which will a simple $U(1)$ transformation

$$\left(D_\mu \phi\right)' = e^{-i\alpha}\left(D_\mu \phi\right)$$

if we require

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

which is the gauge transformation in Maxwell's equations.
Important features of local symmetry: gauge coupling is universal. Also gauge field (photon) is massless(long range force), because mass term $A_\mu A^\mu$ is not gauge invariant.

Symmetry Breaking

When $\mu^2 > 0$, minimum of potential

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

is

$$\phi^\dagger \phi = \frac{v^2}{2}, \qquad \text{with} \qquad v^2 = \frac{\mu^2}{\lambda}$$

Write

$$\phi = \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2)$$

Quantum fields are the oscillations around the minimum,

$$\langle 0 | \phi_1 | 0 \rangle = v, \qquad \langle 0 | \phi_2 | 0 \rangle = 0$$

As before ϕ_2 is a Goldstone boson.
New feature : covariant derivative produces mass term for gauge boson,

$$\left| D_\mu \phi \right|^2 = \left| \left(\partial_\mu - igA_\mu \right) \phi \right|^2 \simeq \ldots \frac{g^2 v^2}{2} A^\mu A_\mu + \ldots$$

with mass

$$M = gv$$

Write the scalar field in polar coordinates

$$\phi = \frac{1}{\sqrt{2}} \left[v + \eta \left(x \right) \right] \exp(i\xi / v)$$

We can now use gauge transformation to transform away ξ. Define

$$\phi^{''} = \exp \left(-i\xi / v \right) \phi = \frac{1}{\sqrt{2}} \left[v + \eta \left(x \right) \right]$$

and

$$B_\mu = A_\mu - \frac{1}{gv} \partial_\mu \xi$$

ξ disappears. In fact ξ becomes the longitudinal component of B_μ.

massless gauge boson + Goldstone boson=massive vector meson

all long range forces disappear. This was discovered in the 60$'s$ by Higgs, Englert & Brout, Guralnik, Hagen & Kibble independently and is usually called **Higgs phenomena**

- The extension to non-abelian local symmetry(Yang-Mills fields) is straightforward and no new features emerge.

Connection with superconductivity

Equation of motion for scalar field interacting with em field,

$$\vec{\nabla} \times \vec{B} = \vec{J}$$

with

$$\vec{J} = ie\left[\phi^\dagger\left(\vec{\nabla} - ie\vec{A}\right)\phi - \left(\vec{\nabla} + ie\vec{A}\right)\phi^\dagger\phi\right]$$

Spontaneous symmetry breaking $\Rightarrow \phi = v$ and

$$\vec{J} = e^2 v^2 \vec{A}$$

This is London equation. Then

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \vec{\nabla} \times \vec{J}, \qquad \Longrightarrow \qquad \nabla^2 \vec{B} = e^2 v^2 \vec{B}$$

This gives Meissner effect.

For the static case, $\partial_0 \vec{A} = 0$, $A_0 = 0$, we get $\vec{E} = 0$ and from Ohm's law

$$\vec{E} = \rho \vec{J}, \qquad \rho \ \text{resistivity}$$

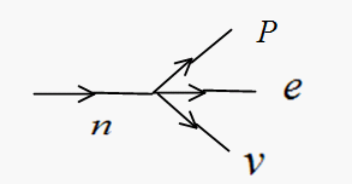
we get $\rho = 0$, i.e. superconductivity.

Standard Model of Electroweak Interactions

Important features of Weak Interactions :

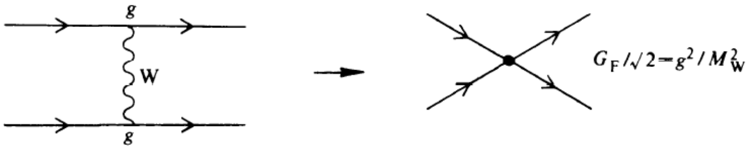
- Universality of coupling strength
  e.g. $\beta-$decay and $\mu-$decay can be described by same coupling $G_F \simeq \frac{10^{-5}}{M_p^2}$ in 4-fermion theory,

$$\mathcal{L}_{wk} = \frac{G_F}{\sqrt{2}} \left(J^{\mu} J_{\mu}^{\dagger} + h.c. \right) \qquad \text{where } J_{\mu} = \left[\bar{\nu} \gamma_{\mu} \left(1 - \gamma_5 \right) e \right] + \cdots$$



- Short range interaction
  If weak interaction is mediated by a massive vector bosons W

$$\mathcal{L}_{wk} = g \left(J_{\mu} W^{\mu} + h.c. \right), \qquad \frac{g^2}{M_w^2} = \frac{G_F}{\sqrt{2}}$$



Then W boson is massive

These features suggest a theory gauge interaction with massive gauge boson.

Standard Model (Weinberg 1967, 't Hooft 1971)
This is a gauge theory with spontaneous symmetry breaking.
Gauge group: $SU(2) \times U(1)$ gauge bosons: $\vec{A}_\mu, B_\mu$ –includes both weak and em interaction
Spontaneous Symmetry Breaking :$SU(2) \times U(1) \longrightarrow U(1)_{em}$
Simple choice for scalar fields

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

The Lagrangian containing ϕ is

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

where

$$D_\mu \phi = \left(\partial_\mu - \frac{ig}{2} \vec{\tau} \cdot \vec{A}_\mu - \frac{ig'}{2} B_\mu \right) \phi$$

$$V(\phi) = -\mu^2 \left(\phi^\dagger \phi \right) + \lambda \left(\phi^\dagger \phi \right)^2$$

Spontaneous symmetry breaking: $SU(2) \times U(1) \longrightarrow U(1)_{em}$
Minimum

$$\frac{\partial V}{\partial \phi_i} = \left[-\mu^2 + 2(\phi^\dagger \phi) \right] \phi_i = 0$$

$\Longrightarrow$

$$-\mu^2 + 2(\phi^\dagger \phi) = 0$$

Simple choice

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \qquad \nu = \sqrt{\frac{\mu^2}{\lambda}}$$

Expand the quanutm fields around the minimum

$$\phi = \phi' + \langle\phi\rangle_0 = \begin{pmatrix} \phi'^{+} \\ \phi'^{0} + \frac{v}{\sqrt{2}} \end{pmatrix}$$

the quadratic terms in V are

$$V_2(\phi) = -\mu^2 \phi'^{\dagger}\phi' + \frac{\lambda v^2}{2}\phi'^{\dagger}\phi' + 2\left(\mathrm{Re}\,\phi'^{0}\right)^2 v^2 = 2\left(\mathrm{Re}\,\phi'^{0}\right)^2 v^2$$

Thus ϕ'^{+} and $\mathrm{Im}\,\phi'^{0}$ are Goldstone bosons and the symmetry breaking is

$$SU(2) \times U(1) \longrightarrow U(1)_{em}$$

From covariant derivative in $\mathcal{L}_\phi$

$$\mathcal{L}_\phi = \frac{v^2}{2}\chi^{\dagger}(g\frac{\vec{\tau}\cdot\vec{A}'_{\mu}}{2} + \frac{g' B'_{\mu}}{2})(g\frac{\vec{\tau}\cdot\vec{A}'^{\mu}}{2} + \frac{g' B'^{\mu}}{2})\chi + \cdots, \qquad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

we get the mass terms for the gauge bosons,

$$\begin{aligned}\mathcal{L}_\phi &= \frac{v^2}{8}\{g^2[(A_\mu^{\mathbf{1}})^2+(A_\mu^{\mathbf{2}})^2]+(gA_\mu^{\mathbf{3}}-g^{'}B_\mu)^2\}+\cdots \\ &= M_W^2 W^{+\mu}W_\mu^- + \frac{1}{2}M_Z^2 Z^\mu Z_\mu + \cdots\end{aligned}$$

where

$$W_\mu^+ = \frac{1}{\sqrt{2}}(A_\mu^{\mathbf{1}} - iA_\mu^{\mathbf{2}}), \qquad M_W^2 = \frac{g^2v^2}{4}$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g^{'}A^{\mathbf{3}} - gB_\mu), \quad M_Z^2 = \frac{g^2+g'^2}{4}v^2$$

The field

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g^{'}A_\mu^{\mathbf{3}} + gB_\mu)$$

is massless photon.
We can write the scalar fields in the form

$$\phi = \exp\left(i\overrightarrow{\tau}\cdot\overrightarrow{\xi}(x)/v\right)\begin{pmatrix} 0 \\ v+\eta(x)\end{pmatrix}$$

where $\vec{\xi}(x)$ are Goldstone bosons. We can make a " gauge transformation"

$$\phi' = U(\vec{\xi})\phi = \begin{pmatrix} 0 \\ v + \eta(x) \end{pmatrix}$$

$$\frac{\vec{\tau} \cdot \vec{A}'_\mu}{2} = U(\vec{\xi})\frac{\vec{\tau} \cdot \vec{A}_\mu}{2} U^{-1}(\vec{\xi}) - \frac{i}{g}\left[\partial_\mu U(\vec{\xi})\right] U^{-1}(\vec{\xi})$$

$$B'_\mu = B_\mu$$

where

$$U(\vec{\xi}) = \exp\left(-i\vec{\tau} \cdot \vec{\xi}(x)/v\right)$$

Again the Goldstone bosons $\vec{\xi}(x)$ will be eaten up by gauge bosons to become massive. The left over field $\eta(x)$ is usually called **Higgs Particle.**
Massive gauge bosons:

$$\begin{aligned} W^\pm_\mu &= \tfrac{1}{\sqrt{2}}\left(A^1_\mu \mp iA^2_\mu\right) \\ Z_\mu &= \cos\theta_W A^3_\mu - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W A^3_\mu + \cos\theta_W B_\mu \end{aligned} \qquad \begin{aligned} M^2_W &= \tfrac{g^2 v^2}{4} \\ M^2_Z &= \frac{g^2}{4\cos^2\theta_W} v^2 \\ M_\gamma &= 0 \end{aligned}$$

where

$$\tan\theta_W = \frac{g'}{g}$$

Note that

$$\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

The mass for the Higgs particle is

$$m_\eta^2 = 2\lambda v^2$$

From comparing Standard Model with 4-fermion theory, we get

$$v = \frac{1}{\sqrt{\sqrt{2}G_F}} \simeq 246\ Gev$$

Since magnitude of Higgs self coupling λ is not known, we do not know the mass of Higgs particle.

Fermion mass and spontaneous symmetry breaking

Another role played by spontaneous symmetry breaking is to give masses to the fermions.
Fermions:
a) Leptons

$$L_i = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \qquad R_i = e_R, \mu_R, \tau_R,$$

b) Quarks (Glashow, Iliopoupos, and Maiani, Kobayashi and Maskawa)

$$q_{iL} = \begin{pmatrix} u' \\ d \end{pmatrix}_L, \begin{pmatrix} c' \\ s \end{pmatrix}_L, \begin{pmatrix} t' \\ b \end{pmatrix}_L, \qquad U_{iR} = u_R, c_R, t_R, \quad D_{iR} = d_R, s_R, b_R$$

All left-handed fermions are in $SU(2)$ doublets and right-handed fermions are all singlets.
Yukawa coupling:

$$\mathcal{L}_Y = f_{ij} \bar{L}_i R_j \phi + h.c. + \cdots$$

Fermions get their masses from spontaneous symmetry breaking through Yukawa couplings,

$$m_{ij} = f_{ij} v$$

This implies that the Yukawa couplings $\propto$ masses. This an example of spontaneous breaking of global symmetry(chiral symmetry),

$$L_i \longrightarrow L_i' = \exp(-i\frac{\overrightarrow{\tau}\cdot\overrightarrow{\alpha}}{2})L_i, \qquad R_i \longrightarrow R_i' = R_i$$

Remark:
The scalar particles ϕ plays 2 important roles in Standard Model:

1. It breaks the gauge symmetry through the universal gauge coupling
2. It give mass to fermions through Yukawa couplings which are not universal