Massive Neutrinos, Neutrino Mixing, Oscillations, Leptonic CP Violation and Beyond

S. T. Petcov

SISSA/INFN, Trieste, Italy, and
IPMU, University of Tokyo, Japan

BCVSPIN Summer School
on Particle Physics and Cosmology
Hue, Vietnam, July 27, 2011

Plan of Lectures

1. Introduction.

2. Massive Neutrinos, Neutrino Mixing and Oscillations.

3. Three Neutrino Mixing: Current Status.

4. Open Questions.

5. Determining the Type of Neutrino Mass Spectrum.

6. Dirac and Majorana CP Violation in the Lepton Sector.

7. The Nature of Massive Neutrinos.

6. Dirac and Majorana Leptonic CP-Violation and Leptogenesis.

8. Conclusions.

3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active $\nu's$ and $\tilde{\nu}'s$
- The notion of "type" ("flavour") - dynamical;

ν_e: $\nu_e + n \rightarrow e^- + p$; ν_μ: $\pi^+ \rightarrow \mu^+ + \nu_\mu$; etc.

- $\nu_l \neq \nu_{l'}$, $\tilde{\nu}_l \neq \tilde{\nu}_{l'}$, $l \neq l' = e, \mu, \tau$; $\nu_l \neq \tilde{\nu}_{l'}$, $l, l' = e, \mu, \tau$.

The states must be orthogonal (within the precision of the corresponding data): $\langle \nu_l' | \nu_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}_l' | \tilde{\nu}_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}_l' | \nu_l \rangle = 0$.

- Data (relativistic ν's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).

Standard Model: ν_l, $\tilde{\nu}_l$ - $\nu_{lL}(x)$;

$\nu_{lL}(x)$ form doublets with $l_L(x)$, $l = e\mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix} \quad l = e, \mu, \tau \, .$$

- No (compelling) evidence for existence of (relativistic) ν's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$.)

If ν_R, $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l, $\tilde{\nu}_l$: ν_R, $\tilde{\nu}_L$ - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the SM, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the SM as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the SM, notably in $SO(10)$ GUT's.

The RH ν's can play crucial role
i) in the generation of $m(\nu) \neq 0$,
ii) in understanding why $m(\nu) \ll m_l, m_q$,
iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

$\nu_{lR}(x)$ in the SM: $Q_{el} = 0$, $T_W = 0$, thus $Y_W = 0$; $\nu_{lR}(x)$ - have no gauge couplings.

SM + $m(\nu) = 0$: $L_l = const.$, $l = e, \mu, \tau$;
$L \equiv L_e + L_\mu + L_\tau = const.$

Compelling Evidences for ν−Oscillations

−ν_{atm}: SK UP-DOWN ASYMMETRY

θ_Z−, L/E− **dependences of μ−like events**

Dominant $\nu_\mu \longrightarrow \nu_\tau$ K2K, MINOS; CNGS (OPERA)

−$\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**
Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \longrightarrow \nu_{\mu,\tau}$ **BOREXINO**

− LSND: **Dominant** $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$;

MiniBOONE 2010: $\nu_\mu \longrightarrow \nu_e$ incompatible, $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$ compatible (!?)

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\ m_j \neq 0\,; \quad l = e, \mu, \tau\,.$$

B. Pontecorvo, 1957; 1958; 1967;
Z. Maki, M. Nakagawa, S. Sakata, 1962;

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{n} U_{lj}\,\nu_{j\mathrm{L}} \quad l = e, \mu, \tau; \qquad n \geq 3\,.$$

$n > 3$: possible, e.g. if $\nu_{lR}(x)$ present in the SM;

at least 3 ν_j "light", choose: ν_1, ν_2, ν_3, $m_{1,2,3} \lesssim 1$ eV.

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

ν-mixing: flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_\tau) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U, m_j^2 - m_k^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}}\ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathrm{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1, \quad C^{-1}\gamma_\mu C = -\gamma_\mu^T; \ C^T = -C, \ C^{-1} = C^\dagger$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1): \ \chi_k(x) \rightarrow e^{iQ\alpha}\chi_k(x) - \ \text{impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0: \ Q_{\mathrm{el}} = 0, \ L_l = 0, \ L = 0, \ldots$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.

$\Delta m^2_{ji} \equiv m^2_j - m^2_i$.

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.6 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.26$ (3σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.35 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.041$ (0.050) 2σ (3σ) (see further).

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

Neutrino Oscillations in Vacuum

Suppose at $t = 0$ in vacuum

$$|\nu_e > \ = |\nu_1 > \cos\theta + |\nu_2 > \sin\theta,$$

$$|\nu_{\mu(\tau)} > \ = -|\nu_1 > \sin\theta + |\nu_2 > \cos\theta; \quad \nu_{1,2}: \quad m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e >_t \ = e^{-iE_1 t}|\nu_1 > \cos\theta + e^{-iE_2 t}|\nu_2 > \sin\theta, \ E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$

$$A(\nu_e \to \nu_\mu; t) = < \nu_\mu|\nu_e >_t = \frac{1}{2}\sin 2\theta \ (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta \ (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \to \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \to \nu_\mu; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L,\ E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$

$(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}},\ L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$

$P(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta\ (1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}),\ L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$

$L_{osc}^{vac} \cong 2.5\ m\ \frac{E[MeV]}{\Delta m^2[eV^2]}$

$E \cong 3$ MeV, $\Delta m^2[eV^2] \cong 8 \times 10^{-5}$: $L_{osc}^{vac} \cong 100$ km

$E \cong 1$ GeV, $\Delta m^2[eV^2] \cong 2.5 \times 10^{-3}$: $L_{osc}^{vac} \cong 1000$ km

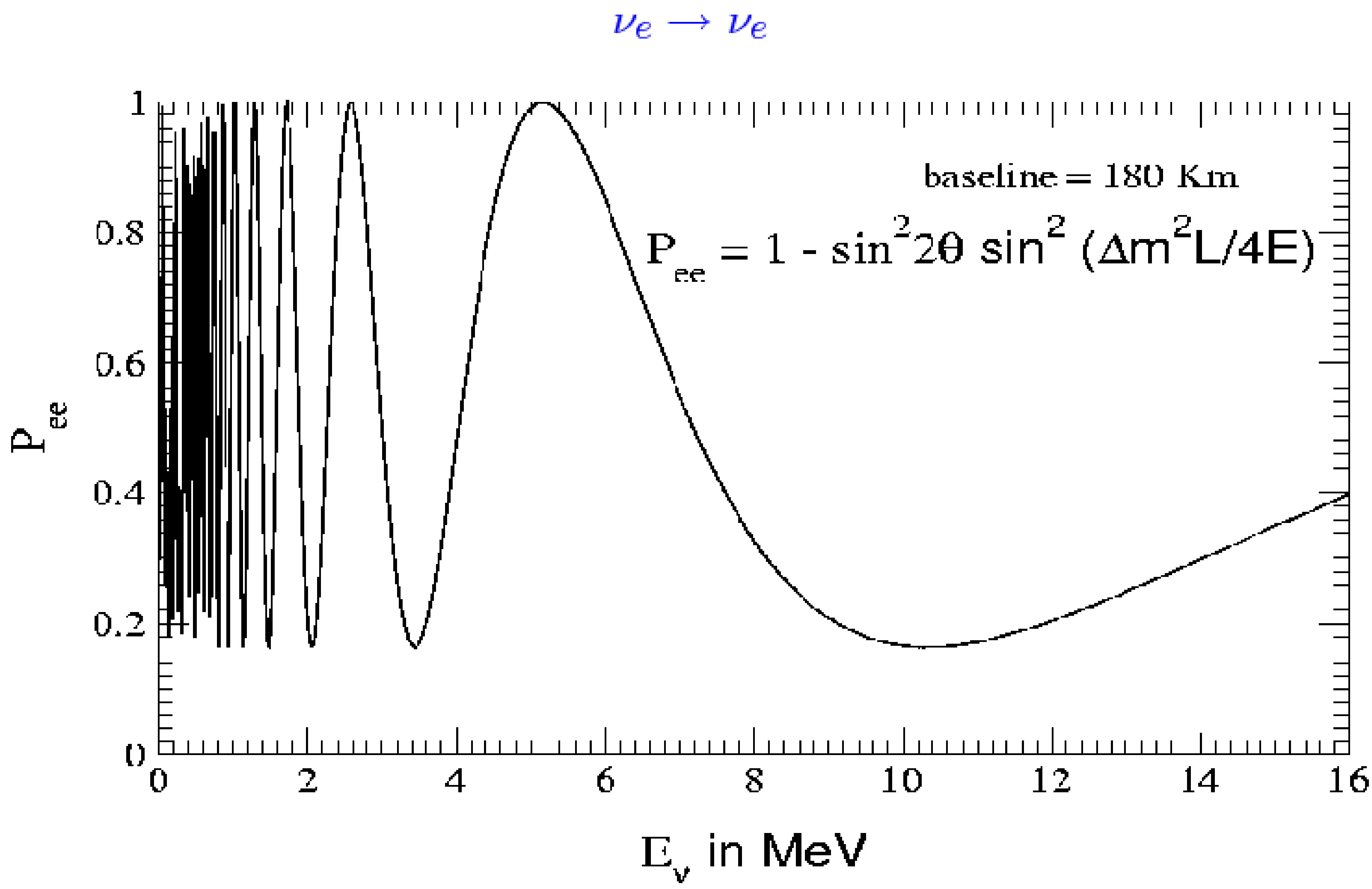
Effects of oscillations observable if

$\sin^2 2\theta - sufficiently\ large, \quad L \gtrsim L_{osc}^{vac}$

Two basic parameters: $\sin^2 2\theta,\ \Delta m^2$

SK, K2K, MINOS; CNGS (OPERA): dominant $\nu_\mu \to \nu_\tau$

KamLAND: $\bar{\nu}_e \to \bar{\nu}_e$; $\bar{\nu}_e \to (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$



$\nu_e \rightarrow \nu_e$
baseline = 180 Km
$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L/4E)$
P_{ee}
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
E_ν in MeV

Source	**Type of ν**	$\bar{E}$[MeV]	L[km]	min(Δm^2)[eV^2]
Reactor	$\tilde{\nu}_e$	$\sim$ **1**	**1**	$\sim 10^{-3}$
Reactor	$\tilde{\nu}_e$	$\sim$**1**	**100**	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	**1**	~ 1
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	**1000**	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{\mu,e}, \tilde{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Correspond to: CHOOZ ($L \sim 1$ km), KamLAND ($L \sim 100$ km), $\tilde{\nu}_e$ disappearnce; $E = (1.8 \div 8.0)$ MeV;

to accelerator experiments - past ($L \sim 1$ km); recent, current: K2K ($L \sim 250$ km), MINOS ($L \sim 730$ km), ν_μ disappearnce; OPERA ($L \sim 730$ km), $\nu_\mu \rightarrow \nu_\tau$;

T2K ($L \sim 250$ km), future NOνA ($L \sim 800$ km), ν_μ disappearnce, $\nu_\mu \rightarrow \nu_e$; $E \sim 1$ GeV;

SK experiment studying atmospheric ν_μ, $\tilde{\nu}_\mu$, ν_e, $\tilde{\nu}_e$ ($E \cong 0.1 \div 100$ GeV), and solar ν_e ($E \cong 5 \div 14$ MeV) oscillations, and to the solar ν experiments ($E \cong 0.29 \div 14$ MeV).

$|\nu_l\rangle = \sum_j U^*_{lj}\,|\nu_j;\tilde{p}_j\rangle, \quad l = e, \mu, \tau$

$\pi^+ \to \mu^+ + \nu_\mu$ decay at rest:

$E_j = E + m_j^2/(2m_\pi)$, $p_j = E - \xi m_j^2/(2E)$, $E = (m_\pi/2)(1 - m_\mu^2/m_\pi^2) \cong 30$ MeV, $\xi = (1 + m_\mu^2/m_\pi^2)/2 \cong 0.8$.

Taking $m_j = 1$ eV: $E_j \cong E\,(1 + 1.2 \times 10^{-16})$,
$p_j \cong E\,(1 - 4.4 \times 10^{-16})$.

Problem avoided if one uses the fact that the ν_j state is entagled with the μ^+ state.

$$A(\nu_l \to \nu_{l'}) = \sum_j U_{l'j}\, D_j\, U^\dagger_{jl}\,, \quad l, l' = e, \mu, \tau\,,$$

$$D_j = e^{-i\tilde{p}_j\,(x_f - x_0)} = e^{-i(E_j T - p_j L)}\,, \qquad p_j \equiv |\mathbf{p}_j|\,.$$

$$\delta\varphi_{jk} = (E_j - E_K)T - (p_j - p_k)L$$
$$= (E_j - E_K)\left[T - \frac{E_j + E_K}{p_j + p_k}\,L\right] + \frac{m_j^2 - m_k^2}{p_j + p_k}\,L\,;$$

First term - negligible:

- L and T related: $T = (E_j + E_k)\,L/(p_j + p_k) = L/\bar{v}$,
  $\bar{v} = (E_j/(E_j + E_k))v_j + (E_k/(E_j + E_k))v_k$ - the "average" velocity of ν_j and ν_k, $v_{j,k} = p_{j,k}/E_{j,k}$;
- $E_j = E_k = E_0$;
- $p_j = p_k = p$
  (additionally suppressed by $(m_j^2 + m_k^2)/p^2$: $L = T$ up to $\sim m_{j,k}^2/p^2$);
- $E_j \neq E_k$, $p_j \neq p_k$, $j \neq k$: the same conclusion
  (neutrinos are relativistic, $L \cong T$ up to corrections $\sim m_{j,k}^2/E_{j,k}^2$).

$$\delta\varphi_{jk} \cong \frac{m_j^2 - m_k^2}{2p} L = 2\pi \frac{L}{L_{jk}^v} \text{sgn}(m_j^2 - m_k^2)\,, \;\; p = (p_j + p_k)/2,$$

$$L_{jk}^v = 4\pi \frac{p}{|\Delta m_{jk}^2|} \cong 2.5 \text{ m } \frac{p[MeV]}{|\Delta m_{jk}^2|[eV^2]}$$

is the neutrino oscillation length associated with Δm_{jk}^2.

- One can safely neglect the dependence of p_j and p_k on the masses m_j and m_k and consider p to be the zero neutrino mass momentum, $p = E$.

- The phase $\delta\varphi_{jk}$ is Lorentz invariant.

$$\sigma_{m^2} = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2}$$

Condition for producing coherently $\nu_1, \nu_2, \ldots$:

$$\sigma_{m^2} > |\Delta m_{jk}^2|$$

The equation used above corresponds to a plane wave description of the propagation of neutrinos ν_j. It accounts only for the movement of the center of the wave packet describing ν_j. In the wave packet treatment of the problem, the interference between the states of ν_j and ν_k is subject to a number of conditions, the localisation condition (in space and time) and the condition of overlapping of the wave packets of ν_j and ν_k at the detection point being the most important. For relativistic neutrinos, the localisation condition in space reads: $\sigma_{xP}, \sigma_{xD} < L^{v}_{jk}/(2\pi)$, $\sigma_{xP(D)}$ being the spatial width of the production (detection) wave packet. Thus, the interference will not be suppressed if the spatial width of the neutrino wave packets detetermined by the neutrino production and detection processes is smaller than the corresponding oscillation length in vacuum. In order for the interference to be nonzero, the wave packets describing ν_j and ν_k should also overlap in the point of neutrino detection. This requires that the spatial separation between the two wave packets at the point of neutrinos detection, caused by the two wave packets having different group velocities $v_j \neq v_k$, satisfies $|(v_j - v_k)T| \ll \max(\sigma_{xP}, \sigma_{xD})$. If the interval of time T is not measured, T in the preceding condition must be replaced by the distance L between the neutrino source and the detector.

Examples

- Spatial localisation condition

ΔL - dimensions of the ν- source (and/or detector):

$$2\pi\Delta L/L^{v}_{jk} \lesssim 1.$$

- Time localisation condition

ΔE - detector's energy resolution:

$$2\pi(L/L^{v}_{jk})(\Delta E/E) \lesssim 1.$$

If $2\pi\Delta L/L^{v}_{jk} \gg 1$, and/or $2\pi(L/L^{v}_{jk})(\Delta E/E) \gg 1$,

$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2\, |U_{lj}|^2$$

Atmospheric Neutrinos ν_μ, $\bar{\nu}_\mu$, ν_e, $\bar{\nu}_e$, $E \sim$1 GeV (0.20 - 100 Gev)

$$\nu_\mu + N \to \mu^- + X, \quad \tilde{\nu}_\mu + N \to \mu^+ + X$$

$$\nu_e + N \to e^- + X, \quad \tilde{\nu}_e + N \to e^+ + X$$

K2K, MINOS, T2K, ν_μ ($\bar{\nu}_\mu$), $E \sim$1 GeV

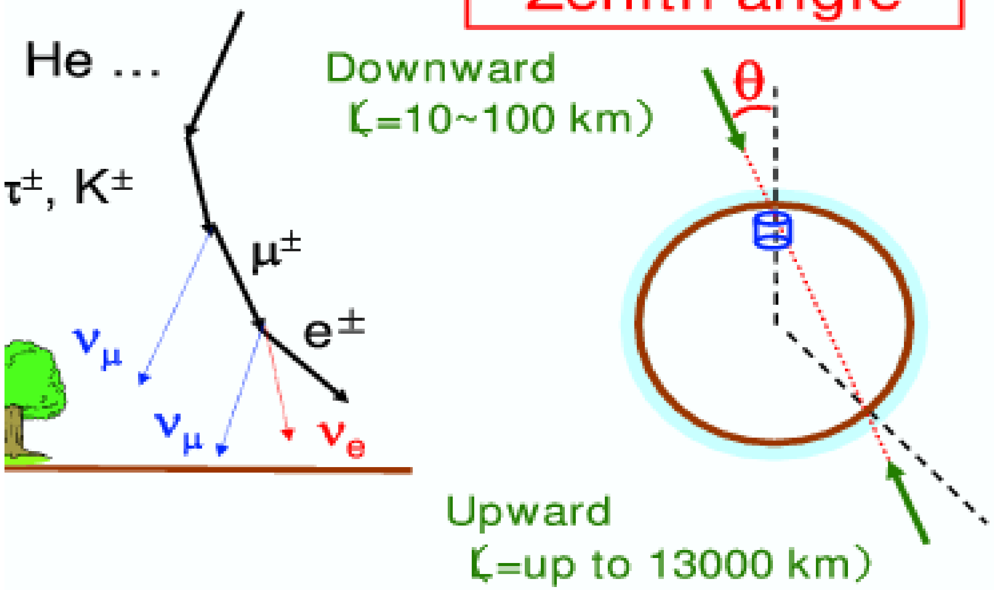
$$\nu_\mu + N \to \mu^- + X \quad (\nu_e + N \to e^- + X)$$

Reactor $\bar{\nu}_e$, E $\sim$2 MeV: CHOOZ,KamLAND, Double Chooz, RENO, Daya Bay ($E \cong 2 - 8$ MeV)

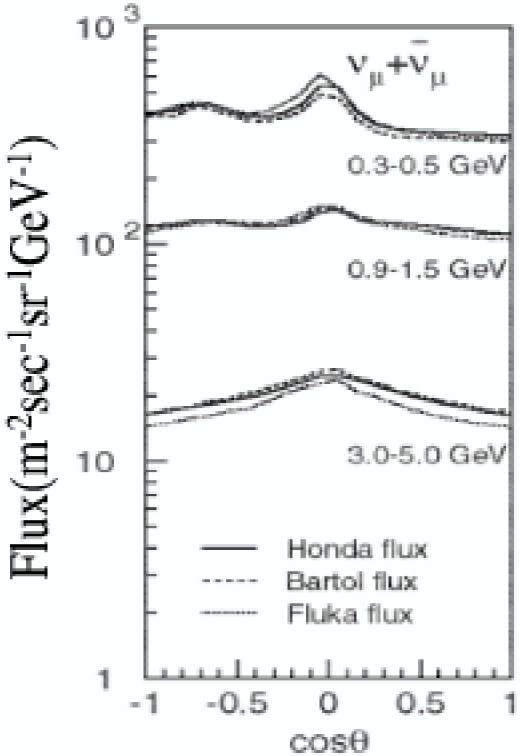
$$\bar{\nu}_e + p \to e^+ + n$$

tmospheric neutrinos

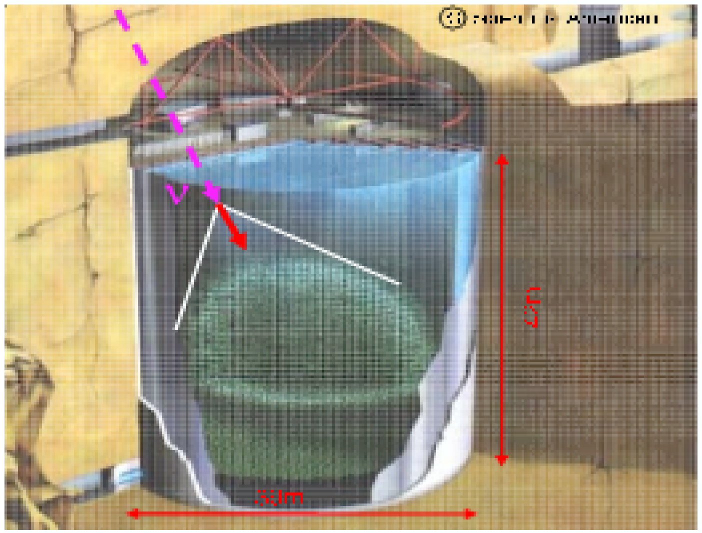
Zenith angle



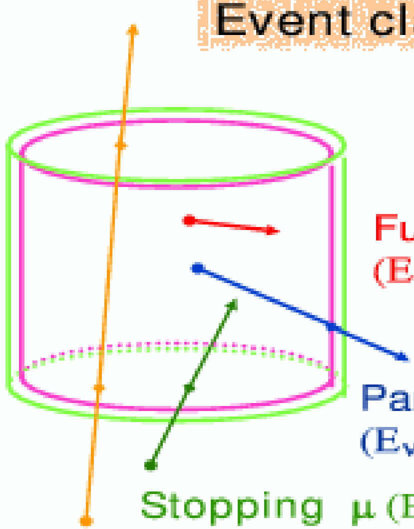
Zenith angle dist. of Atmospheric ν flux



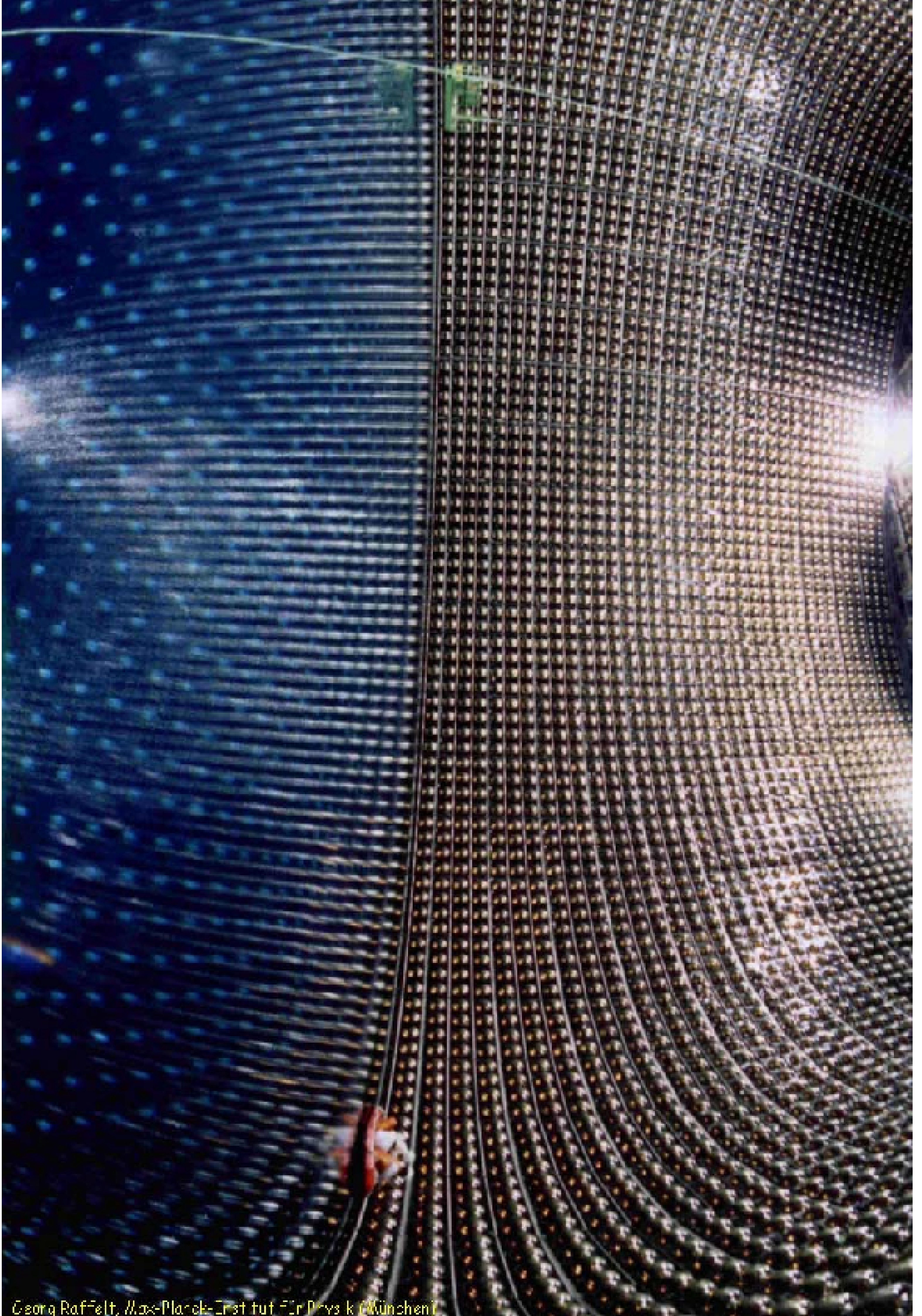
Eν > a few GeV
Up/Down Symmetry



Water Cherenkov detector



1000 m underground
50,000 ton (22,500 ton fid.)
nner-detector(ID): 11,146 20 inch PMTs(SK-I)
outer-detector(OD): 1,885 8 inch PMTs

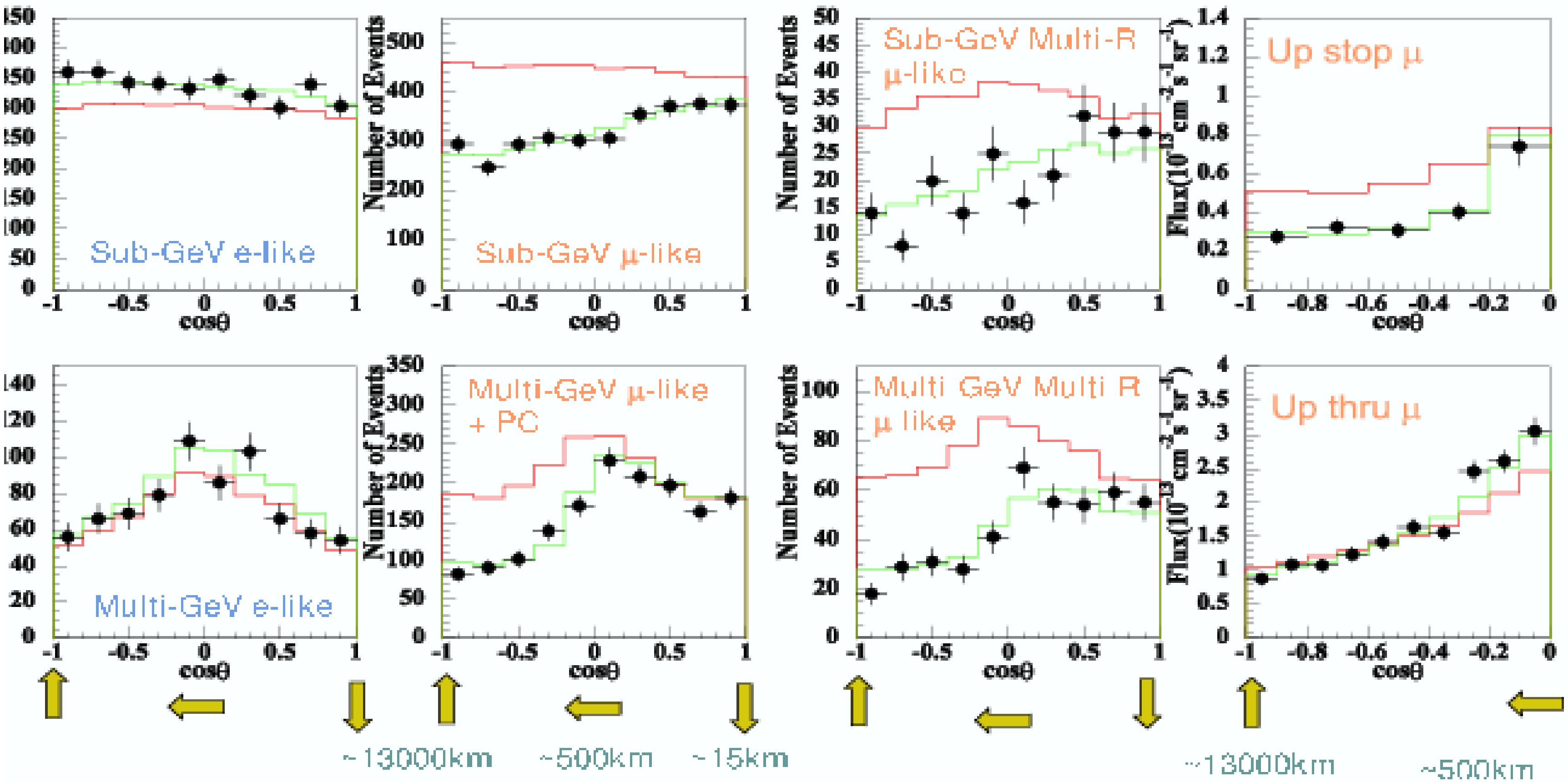

Georg Raffelt, Max-Planck-Institut für Physik (München)

Zenith angle distributions

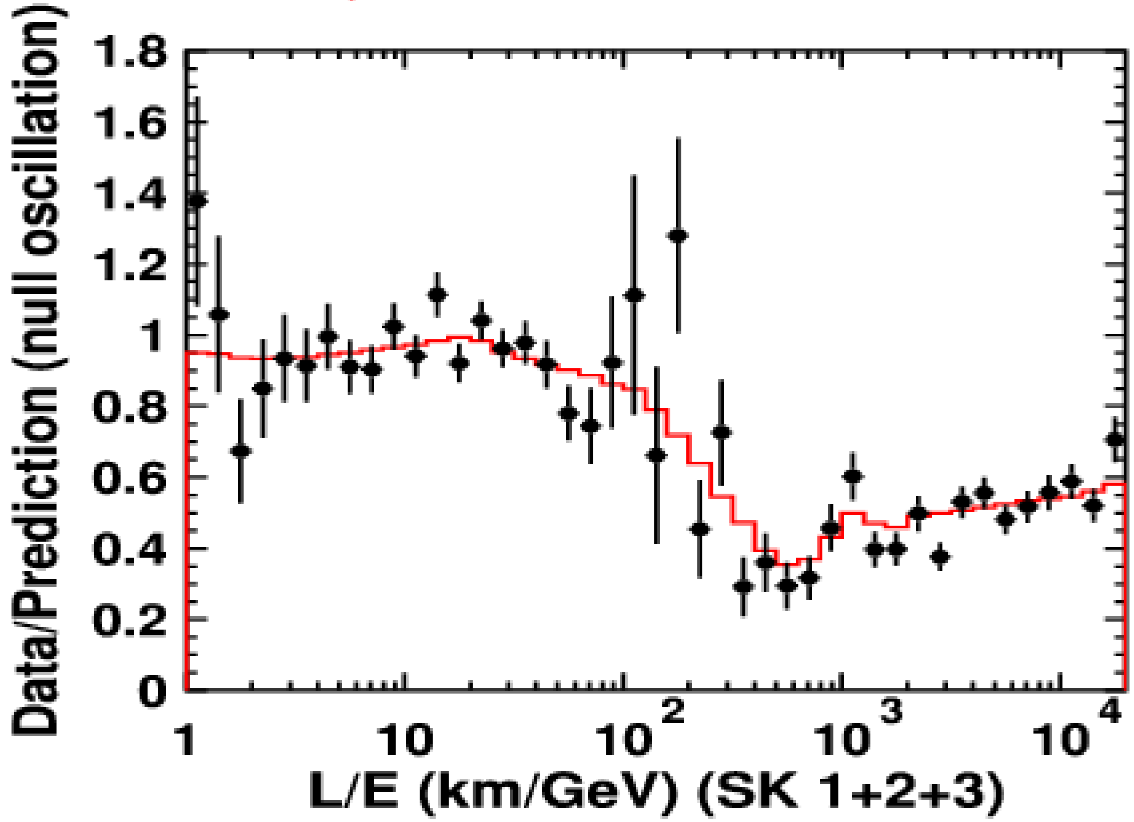
$\nu_\mu \leftrightarrow \nu_\tau$
2-flavor oscillations

Best fit
$\sin^2 2\theta = 1.0$, $\Delta m^2 = 2.0 \times 10^{-3}$ eV^2

Null oscillation



SK: L/E Dependence, μ−Like Events

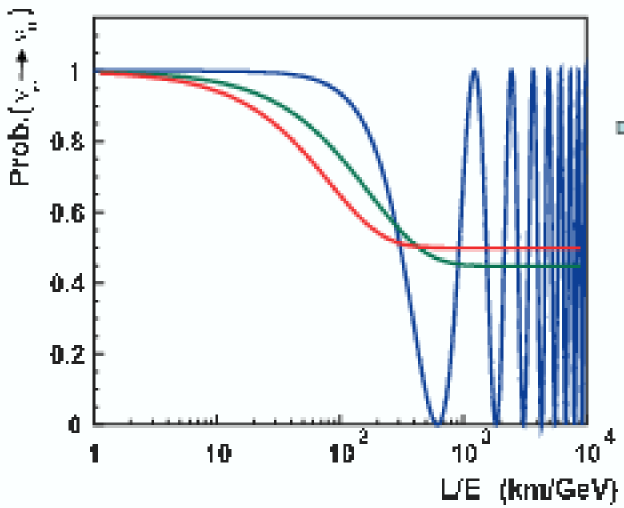


L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2\theta + \sin^2\theta \times \exp(-\frac{m}{2\tau}\frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2}\sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



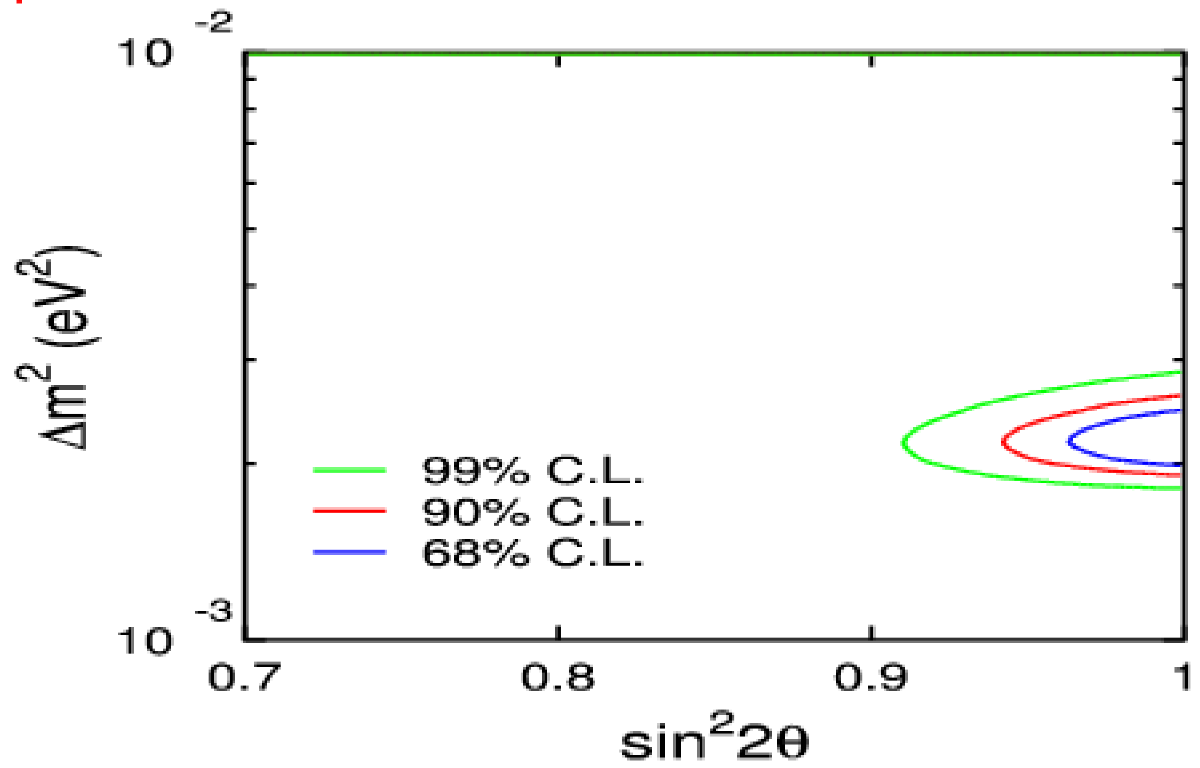
Use events with high resolution in L/E

The first dip can be observed

→ Direct evidence for oscillations

→ Strong constraint to oscillation parameters, especially Δm^2 value

SK: Atmospheric ν Data



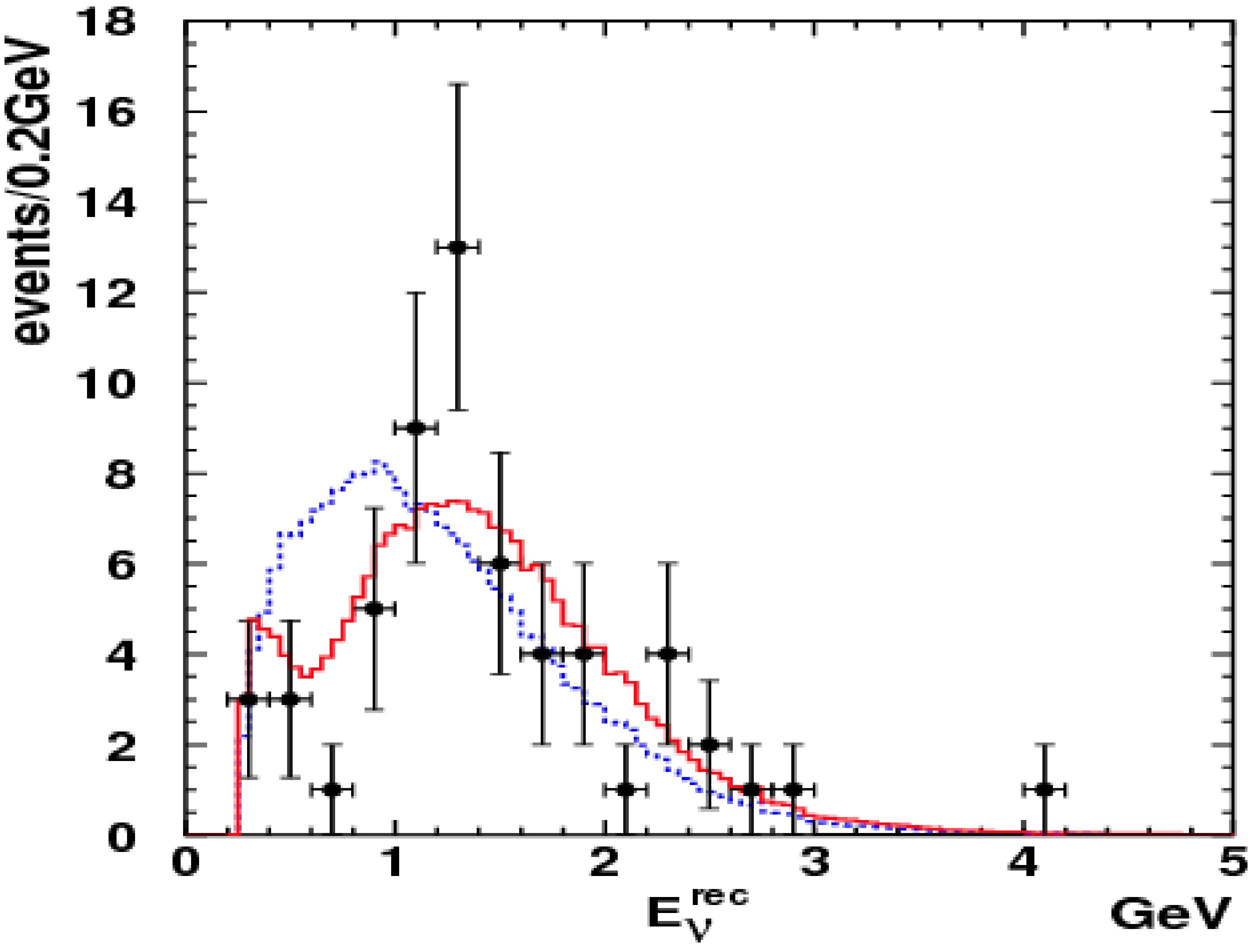
$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{\text{atm}} \equiv \sin^2 2\theta_{23} = 1.0$;

$\Delta m^2_{31} = (1.9 - 2.9) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \geq 0.92$, 99% C.L.

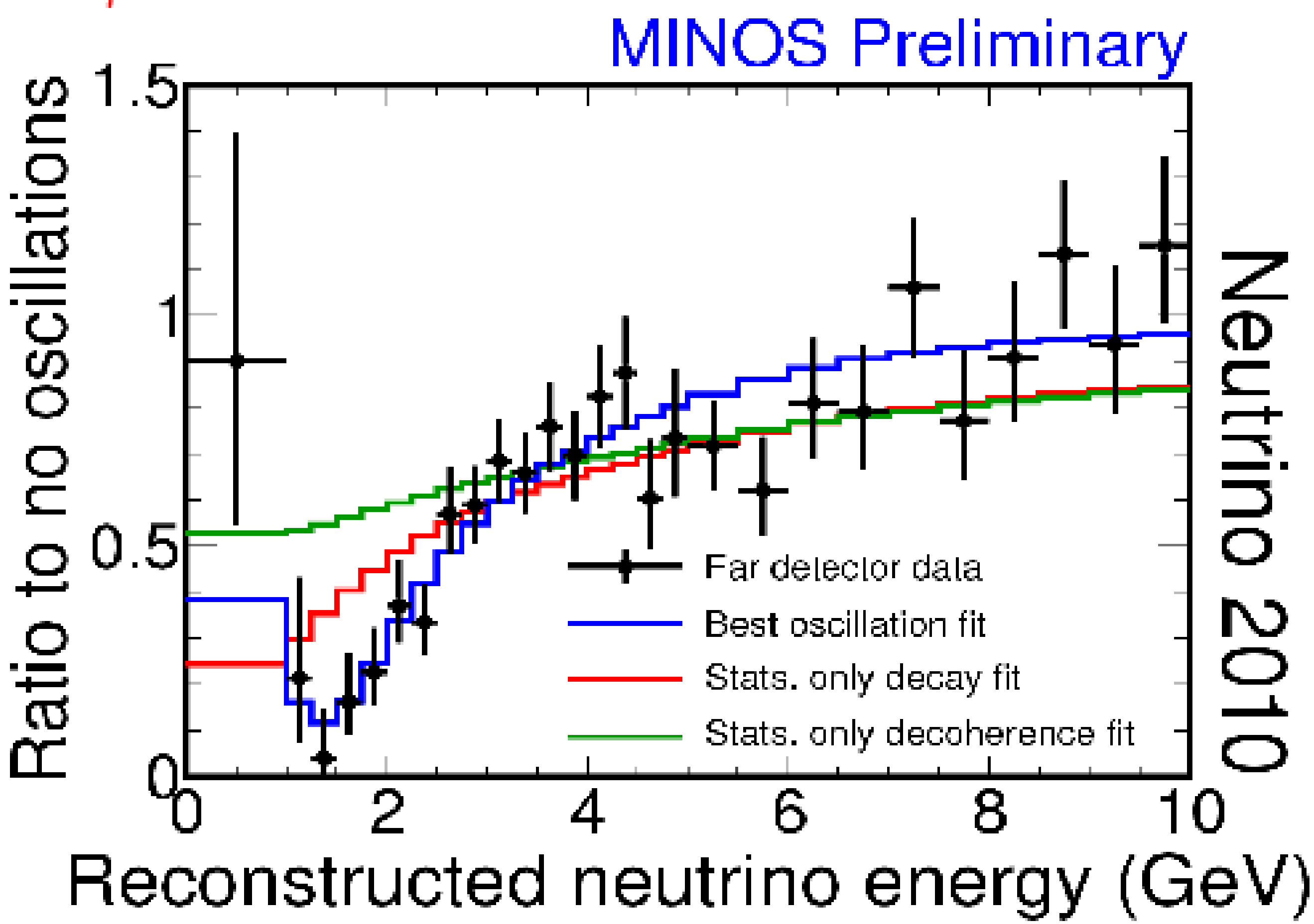
- sign of Δm^2_{atm} not determined. If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.

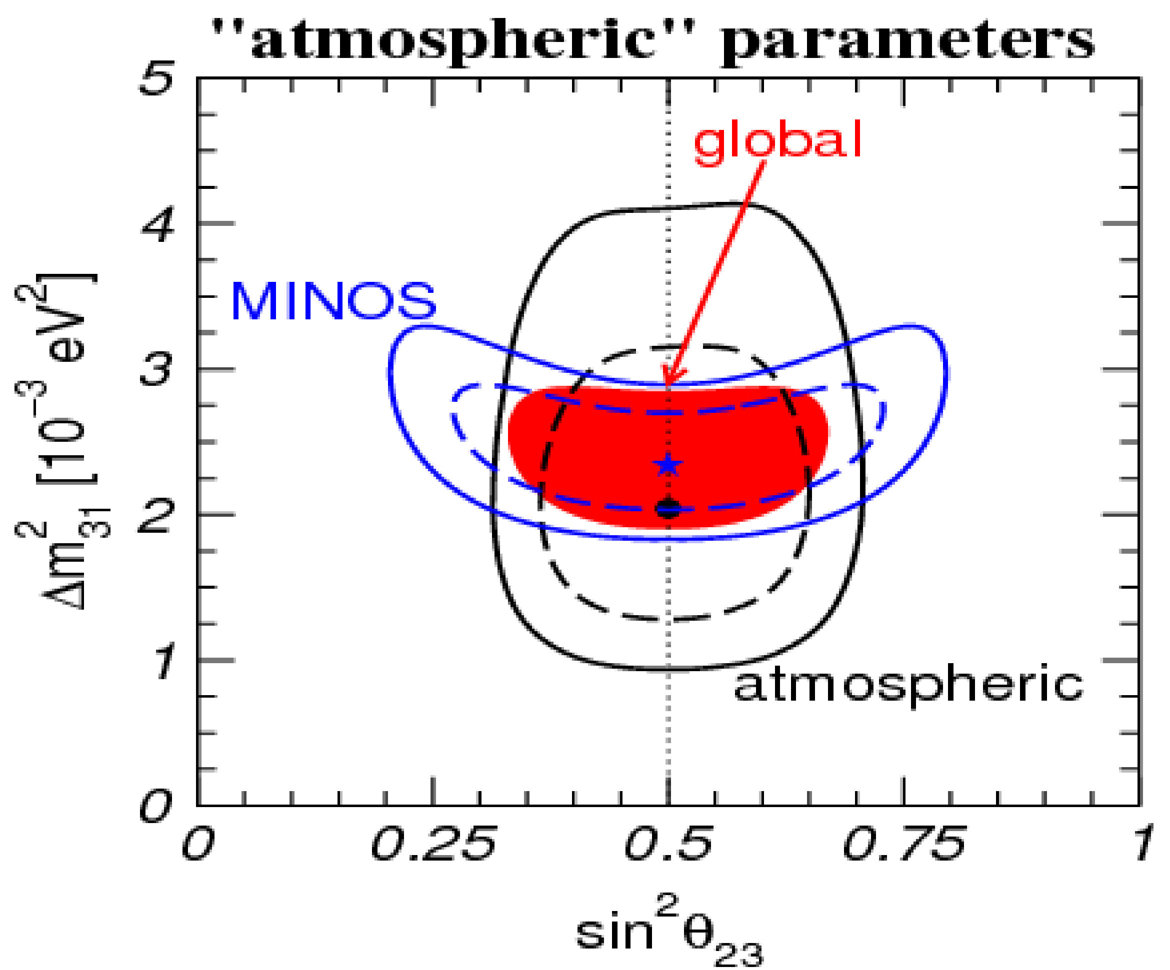
3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (NH); $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (IH).

K2K: ν_μ Spectrum



MINOS: ν_μ Spectrum





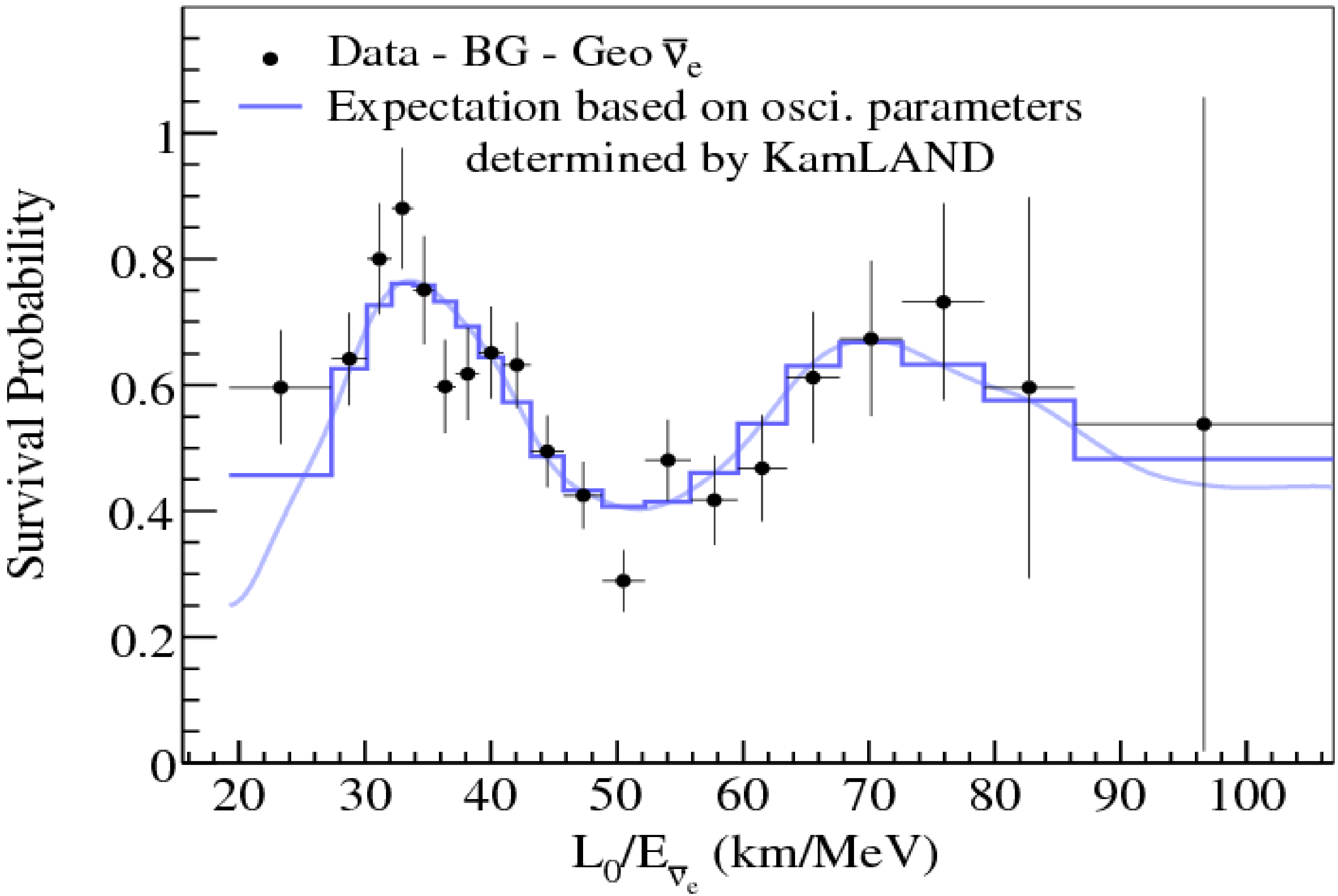
T. Schwetz, arXiv:0710.5027[hep-ph]

- sign of $\Delta m^2_{\rm atm}$ not determined;

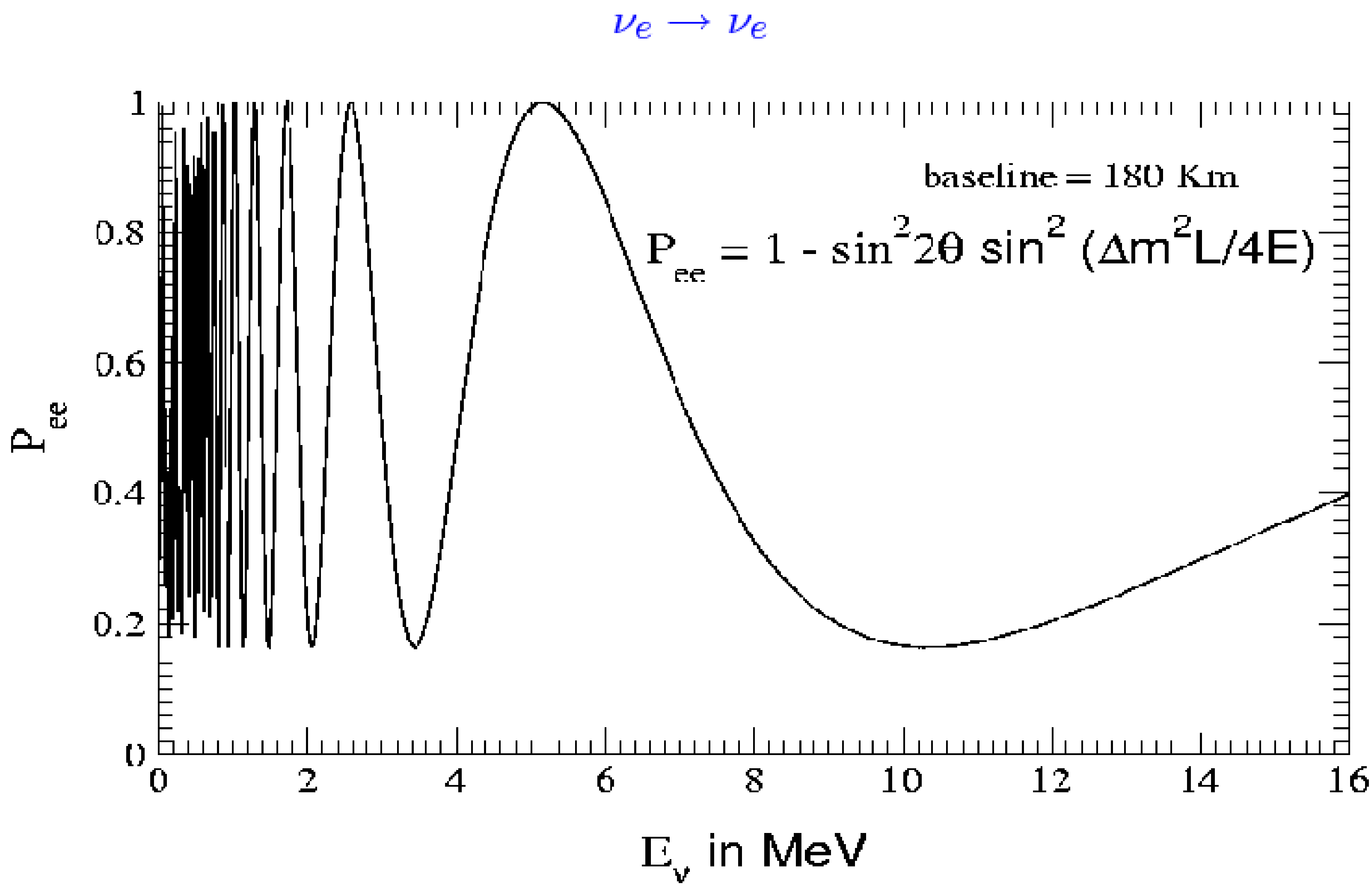
  3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

  $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

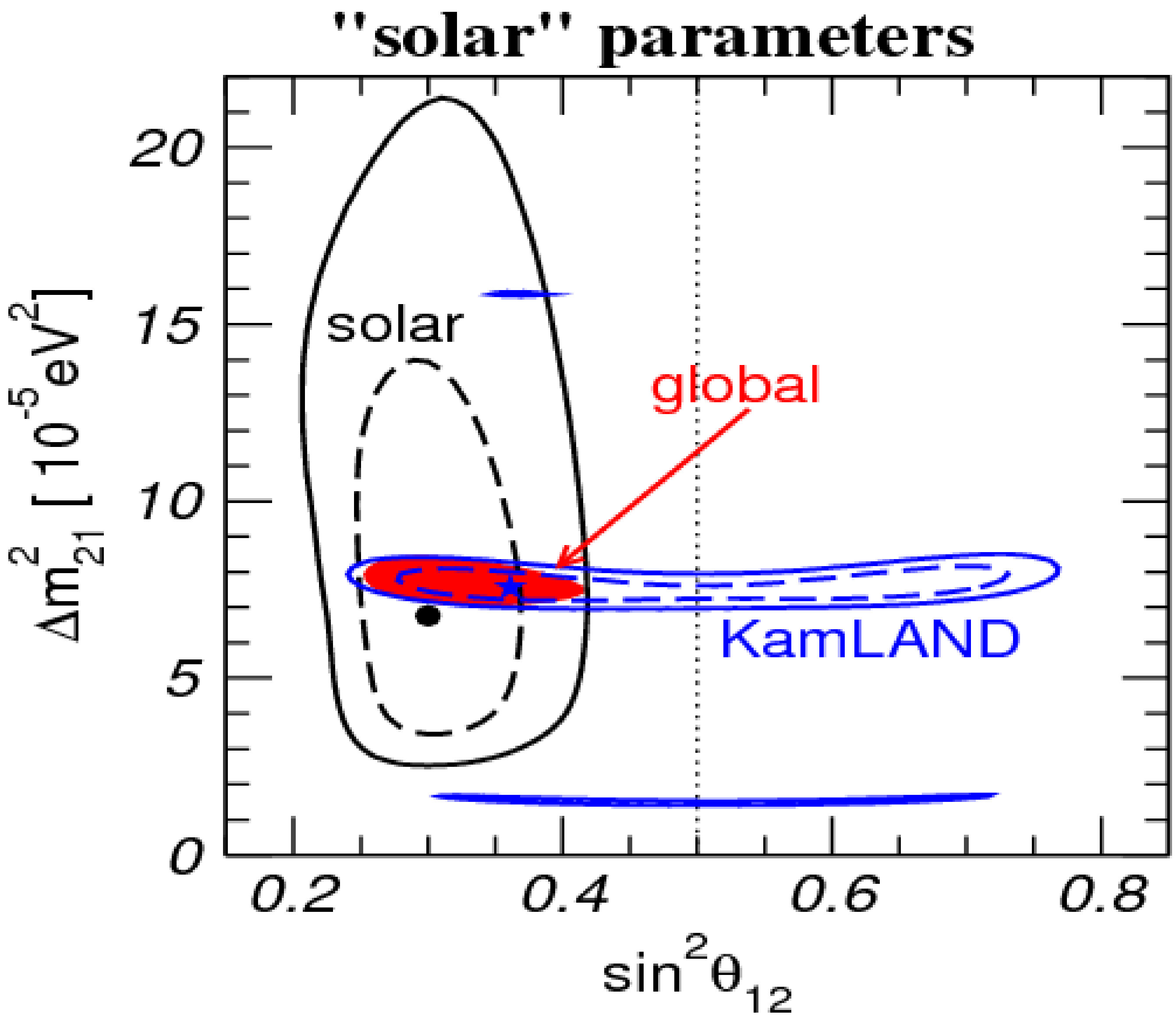
- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



Data - BG - Geo $\bar{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\bar{\nu}_e}$ (km/MeV)



$\nu_e \longrightarrow \nu_e$
baseline = 180 Km
$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L/4E)$
P_{ee}
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
E_ν in MeV



T. Schwetz, arXiv:0710.5027[hep-ph]

Matter Effects in Neutrino Oscillations

Matter can affect strongly $\nu-$oscillations:

Mean free path in matter with $\bar{\rho} = \bar{\rho}(Earth)$:

E$\sim$1 MeV, $L_f \sim 5 \times 10^{11}$ km; $R_E = 6371$ km

E$\sim$1 GeV, $L_f \sim 5 \times 10^{5}$ km

ν coherent scattering on e^-, p, n - effective potential (index of refraction)

$$V_{e\mu} = V(\nu_e) - V(\nu_\mu) = \sqrt{2} G_F N_e$$

$$\bar{V}_{e\mu} = V(\bar{\nu}_e) - V(\bar{\nu}_\mu) = - \sqrt{2} G_F N_e$$

$V_{\mu\tau} = V(\nu_\mu) - V(\nu_\tau) = 0$ (leading order)

$V_{e\mu} \neq \bar{V}_{e\mu}$: CP, CPT violated

L. Wolfenstein, 1978; V. Barger et al., 1980; P. Langacker *et al.*, 1983;
S.P. Mikheyev, A.Yu. Smirnov, 1985; etc.

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \tag{1}$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\,[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

In matter, $H_m = H_0 + H_{int}$.

$H_0|\nu_{1,2}> = E_{1,2}|\nu_{1,2}>$, not eigenstates of H_m.

Consider first $N_e = const.$

$H_m|\nu^m_{1,2}> = E^m_{1,2}|\nu^m_{1,2}>.$

Then at $t = 0$ in matter

$$|\nu_e> \; = |\nu^m_1> \cos\theta_m + |\nu^m_2> \sin\theta_m,$$

$$|\nu_{\mu(\tau)}> \; = -|\nu^m_1> \sin\theta_m + |\nu^m_2> \cos\theta_m;$$

$$\sin 2\theta_m = \frac{\epsilon'}{\sqrt{\epsilon^2+\epsilon'^2}} = \frac{\tan 2\theta}{\sqrt{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta}},$$

$$\cos 2\theta_m = \frac{1-N_e/N_e^{res}}{\sqrt{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta}},$$

$$N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F} \cong 6.56\times 10^6 \, \frac{\Delta m^2[\mathrm{eV}^2]}{E[\mathrm{MeV}]} \cos 2\theta \, \mathrm{cm}^{-3} \; \mathrm{N_A},$$

$$E^m_2 - E^m_1 = \frac{\Delta m^2}{2E}\left((1-\frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta\right)^{\frac{1}{2}}$$

$$P_m^{2\nu}(\nu_e \to \nu_\mu) = |A_\mu(t)|^2 = \frac{1}{2}\sin^2 2\theta_m \, [1 - \cos 2\pi \frac{L}{L_m}],$$

$$L_m = \frac{E_2^m - E_1^m}{2\pi} = L^v \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta \right)^{-\frac{1}{2}}.$$

The resonance condition: $N_e = N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$

At the resonance:

$\sin^2 2\theta_m = 1$, $\min(E_2^m - E_1^m)$, $L_m^{res} = L^v / \sin 2\theta$.

Limiting cases:

$N_e \ll N_e^{res}$: $\theta_m \cong \theta$, $E_{1,2}^m \cong E_{1,2}$, $L_m \cong L^v$.

$N_e \gg N_e^{res}$: $\theta_m \cong \frac{\pi}{2}$, $\nu_e \to \nu_\mu$ suppressed.

In this case: $|\nu_e\rangle \cong |\nu_2^m\rangle$, $|\nu_\mu\rangle = -|\nu_1^m\rangle$.

Antineutrinos: $N_e \to (-N_e)$

$\Delta m^2 \cos 2\theta > 0$: $\bar{\nu}_e \to \bar{\nu}_\mu$ suppressed by matter; $\nu_e \to \nu_\mu$ can be enhanced.

$\Delta m^2 \cos 2\theta < 0$: $\nu_e \to \nu_\mu$ suppressed by matter; $\bar{\nu}_e \to \bar{\nu}_\mu$ can be enhanced.

Oscillations in matter (Earth, Sun) are neither CP- nor CPT- invariant.

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}, \quad \bar{N}_e^{core} \sim 6.0\ N_A\ cm^{-3}$

$$P^m(\nu_e \to \nu_\mu; t) = \frac{1}{2}\sin^2 2\theta_m\ (1 - \cos 2\pi \frac{L}{L_{osc}^m}),\ L_{osc}^m \sim L_{osc}^{vac}$$

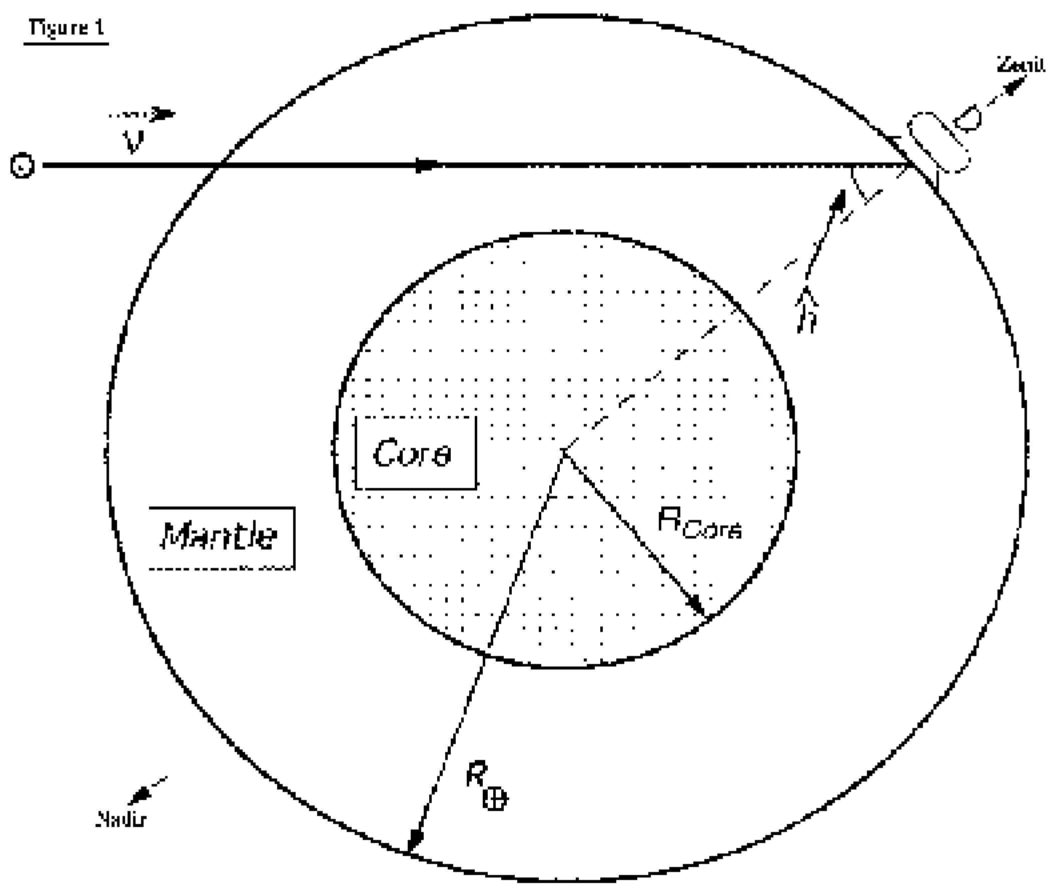
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(1-\frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta}\ , N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

$N_e = N_e^{res}$: MSW resonance

$\Delta m^2 \cos 2\theta > 0$: $\nu_e \to \nu_\mu$

$\Delta m^2 \cos 2\theta < 0$: $\bar{\nu}_e \to \bar{\nu}_\mu$

The Earth



Earth: $R_{core} = 3446$ km, $R_{mant} = 2885$ km

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

The Earth

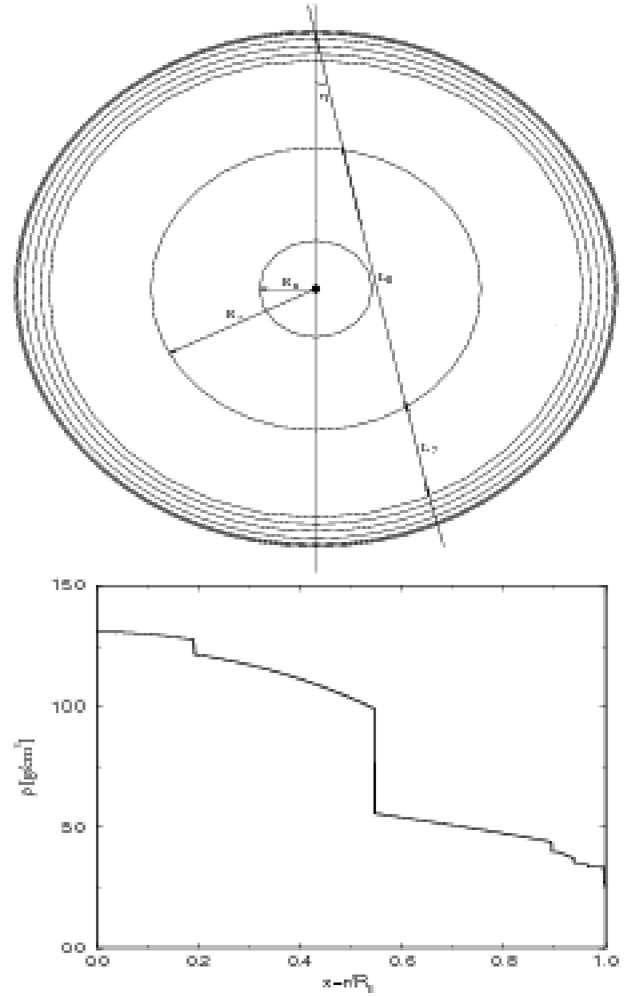
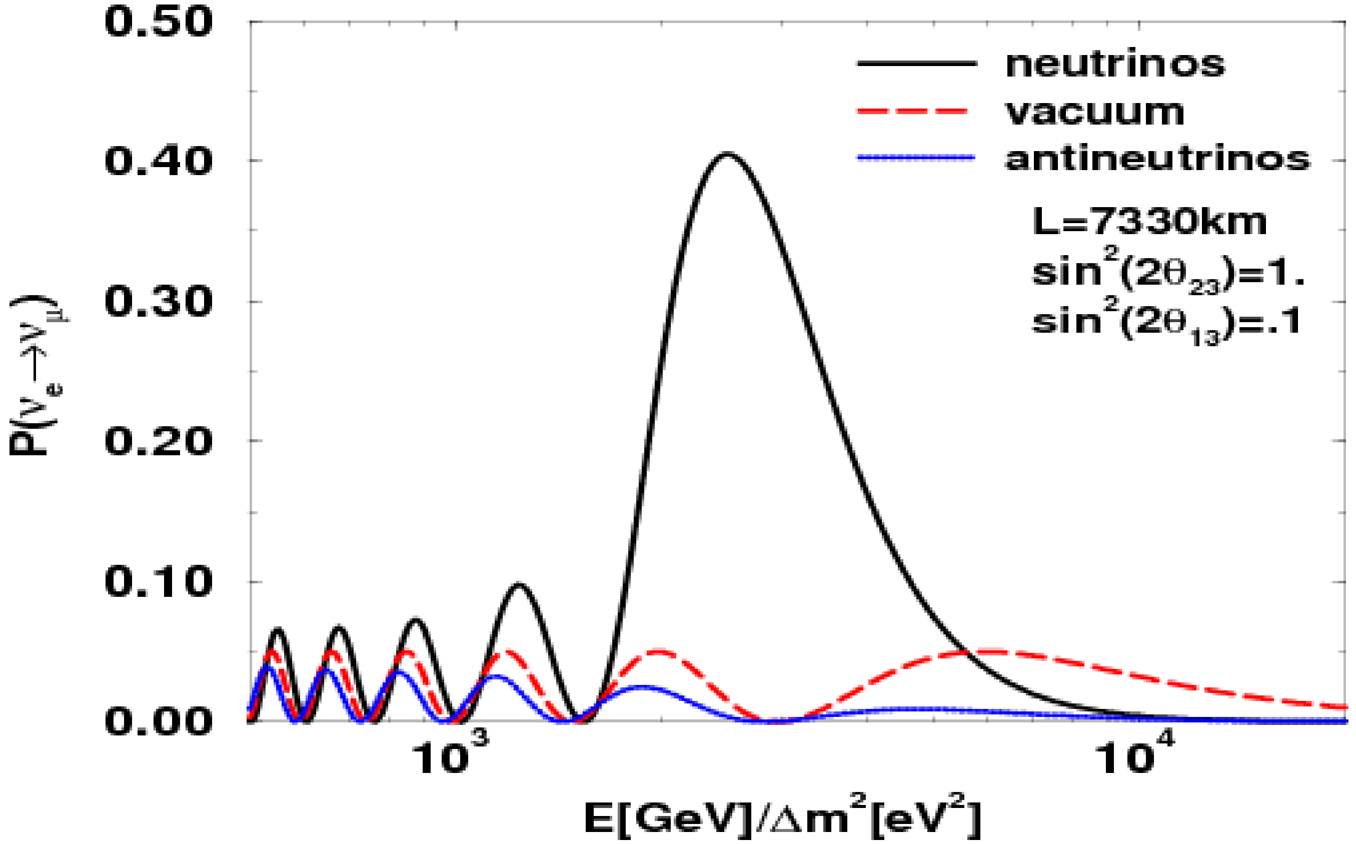


FIG. 1. Density profile of the Earth.

$R_c = 3446$ km, $R_m = 2885$ km; $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)

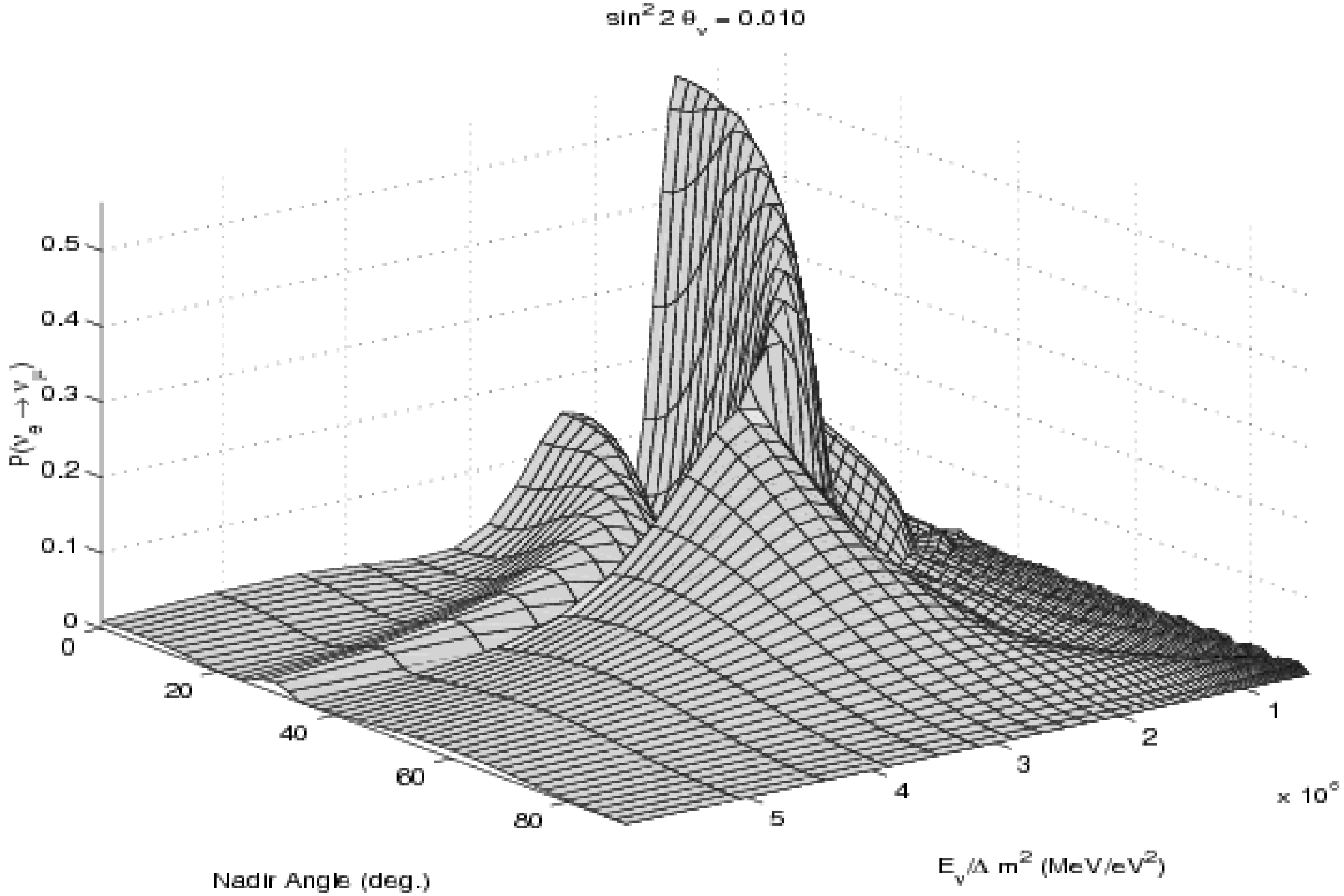


$\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 7.5$ GeV; $P^{3\nu} = \sin^2\theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$;

$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L_m^{res} = L^\nu / \sin 2\theta \cong 7500/0.33$ km; $2\pi L/L_m \cong \pi$.

I. Mocioiu, R. Shrock, 2000

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)

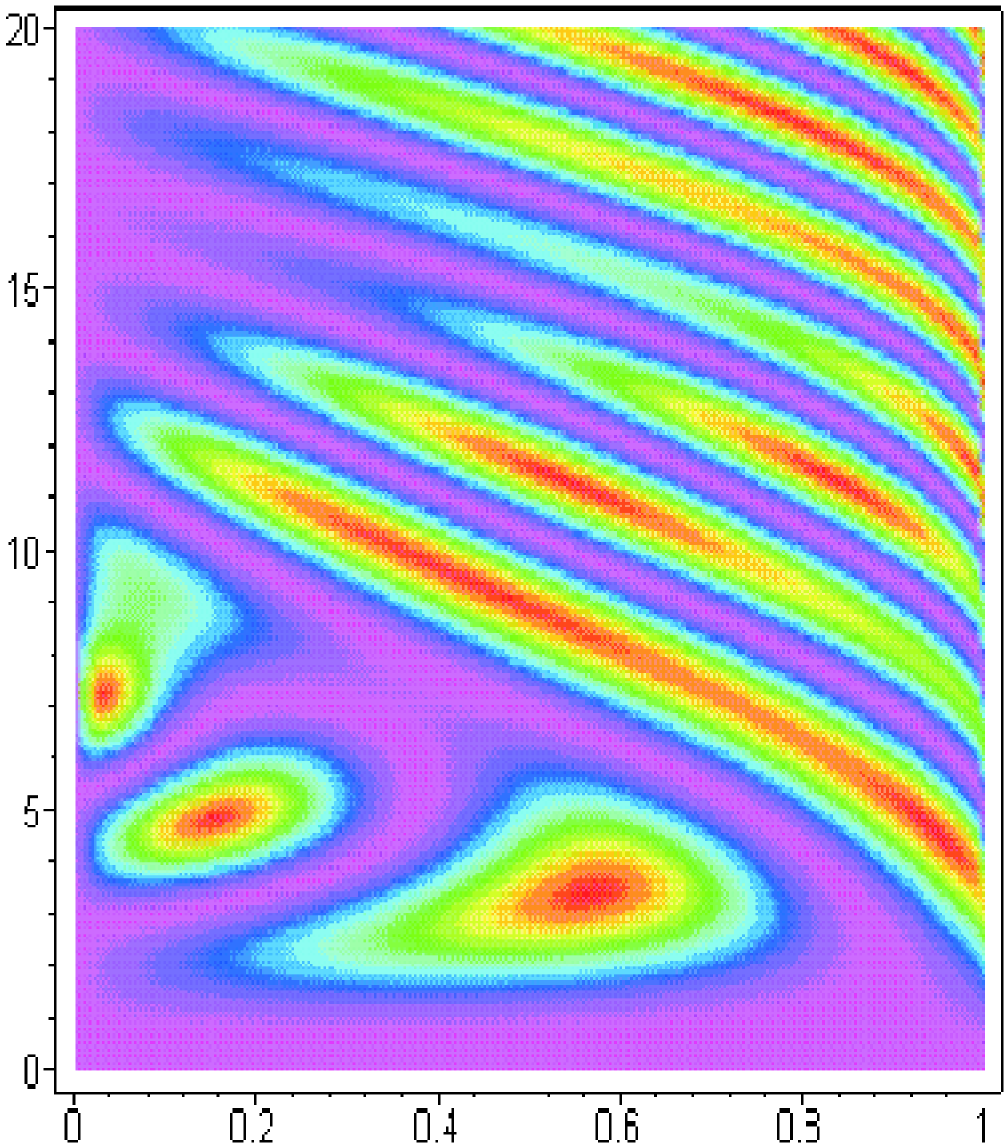


S.T.P., 1998;
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_v \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_{\rm atm}$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2}P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E$ $[10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

- **For Earth center crossing ν's ($\theta_n = 0$) and, e.g.** $\sin^2 2\theta_{13} = 0.01$, **NOLR occurs at** $E \cong 4$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

S.T.P., hep-ph/9805262

- **For the Earth core crossing ν's:** $P_{2\nu} = 1$ **due to NOLR when**

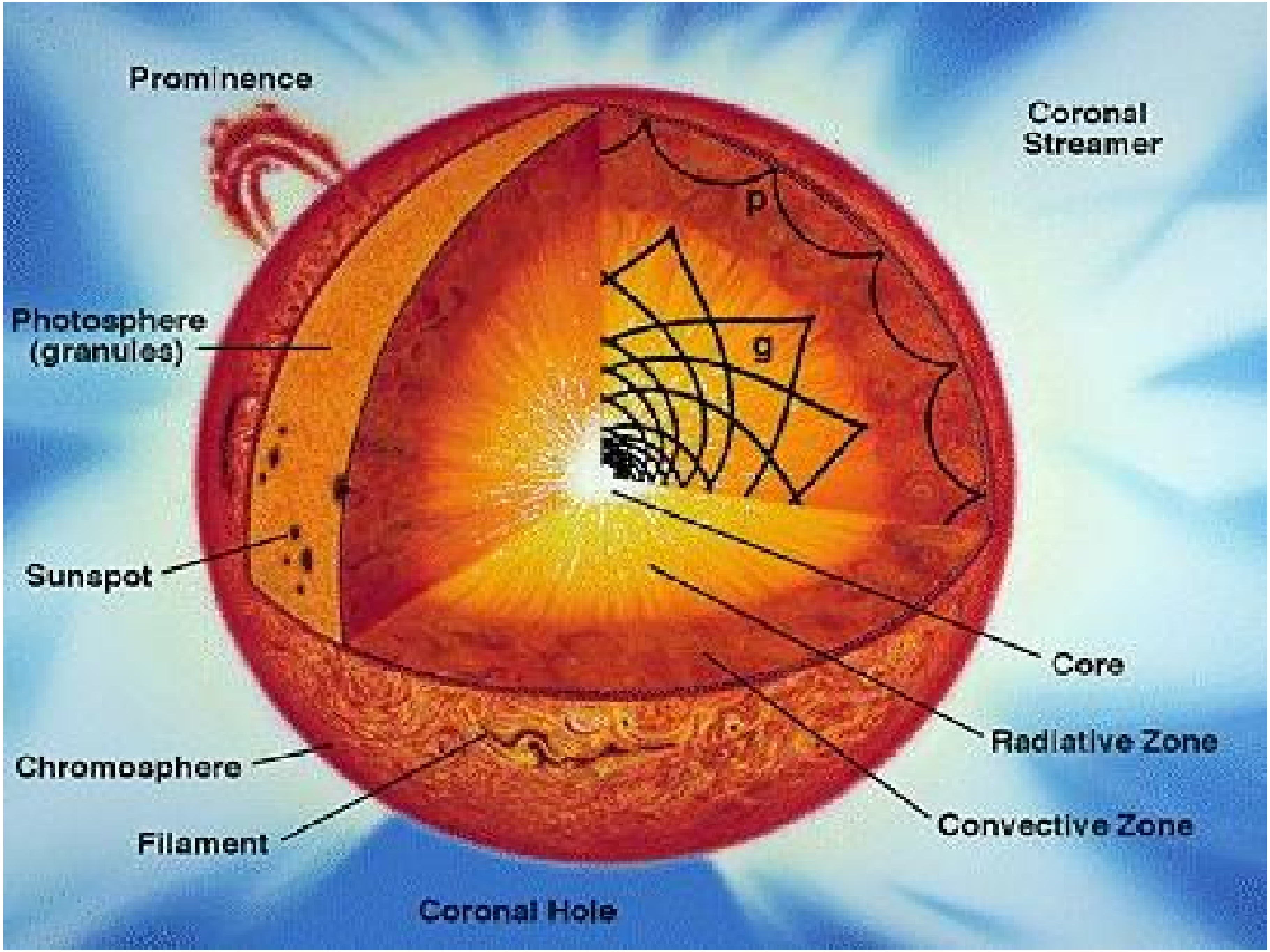
$$\tan \Phi^{\text{man}}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}},$$

$$\tan \Phi^{\text{core}}/2 \equiv \tan \phi'' = \pm \sqrt{\frac{\cos 2\theta'_m}{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}}$$

Φ^{man} (Φ^{core}) **- phase accumulated in the Earth mantle (core),**
θ'_m (θ''_m) **- the mixing angle in the Earth mantle (core).**

$P_{2\nu} = 1$ **due to NOLR for** $\theta_n = 0$ **(Earth center crossing ν's) at,**
e.g. $\sin^2 2\theta_{13} = 0.034;\ 0.154$, $E \cong 3.5;\ 5.2$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).



Prominence
Coronal
Streamer
p
g
Photosphere
(granules)
Sunspot
Core
Radiative Zone
Chromosphere
Convective Zone
Filament
Coronal Hole

Solar Neutrino Production: pp Chain

REACTION	TERM. (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2H + e^+ + \nu_e$	(99.96)	≤ 0.420
or		
$p + e^- + p \rightarrow {}^2H + \nu_e$	(0.44)	1.442
$^2H + p \rightarrow {}^3He + \gamma$	(100)	
$^3He + {}^3He \rightarrow \alpha + 2\,p$	(85)	
or		
$^3He + {}^4He \rightarrow {}^7Be + \gamma$	(15)	
$^7Be + e^- \rightarrow {}^7Li + \nu_e$	(15)	$\begin{cases} 0.861 & 90\% \\ 0.383 & 10\% \end{cases}$
$^7Li + p \rightarrow 2\,\alpha$		
or		
$^7Be + p \rightarrow {}^8B + \gamma$	(0.02)	
$^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		$\lesssim 15$
$^8Be^* \rightarrow 2\,\alpha$		
or		
$^3He + p \rightarrow {}^4He + e^+ + \nu_e$	(0.000004)	$\lesssim 18.8$

$$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e.$$

- pp neutrinos, $E \leq 0.420$ MeV, $\bar{E} = 0.265$ MeV,
- ^{7}Be neutrinos, E=0.862 MeV (89.7% of the flux), 0.384 MeV (10.3%) ,
- ^{8}B neutrinos, $E \leq 14.40$ MeV, $\bar{E} = 6.71$ MeV,
- pep neutrinos, E=1.442 MeV,
- of ^{13}N, $E \leq 1.199$ MeV, $\bar{E} = 0.707$ MeV,
- of ^{15}O, $E \leq 1.732$ MeV, $\bar{E} = 0.997$ MeV.

Flux	BP'00	Cl–Ar	Ga–Ge
$\Phi_{pp} \times 10^{-10}$	$5.95(1\ ^{+0.01}_{-0.01})$	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	$1.40(1\ ^{+0.01}_{-0.01})$	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	$4.77(1\ ^{+0.09}_{-0.09})$	1.15	34.2
$\Phi_{B} \times 10^{-6}$	$5.93(1\ ^{+0.14}_{-0.15})$	6.76	14.2
$\Phi_{N} \times 10^{-8}$	$5.48(1\ ^{+0.19}_{-0.13})$	0.09	3.4
$\Phi_{O} \times 10^{-8}$	$4.80(1\ ^{+0.22}_{-0.15})$	0.33	5.5
Total		$8.55\ ^{+1.1}_{-1.2}$	$129.8\ ^{+9}_{-7}$

Solar Neutrinos ν_e, $E\sim$1 MeV: B. Pontecorvo 1946

$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

R. Davis et al., 1967 - 1996: 615 t C_2Cl_4; 0.5 Ar atoms/day, exposure 60 days.

$$\nu + e^- \rightarrow \nu + e^-$$

Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);

$$\nu_e + D \rightarrow e^- + p + p \ , \qquad \text{SNO}$$

$$\nu_l + D \rightarrow \nu_l + n + p\,,\ l = e, \mu, \tau \qquad \text{SNO}$$

Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

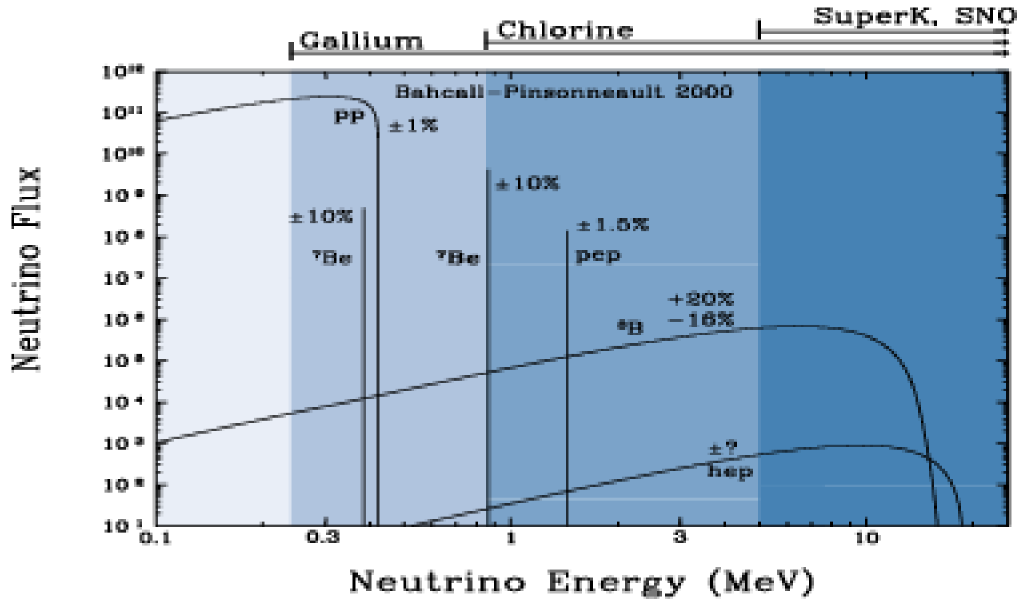


Figure 2: Differential Standard Solar Model neutrino fluxes [14].

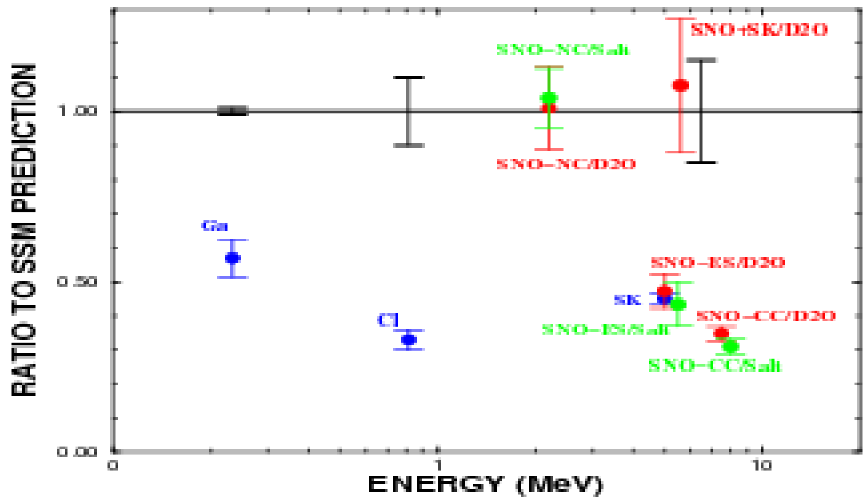
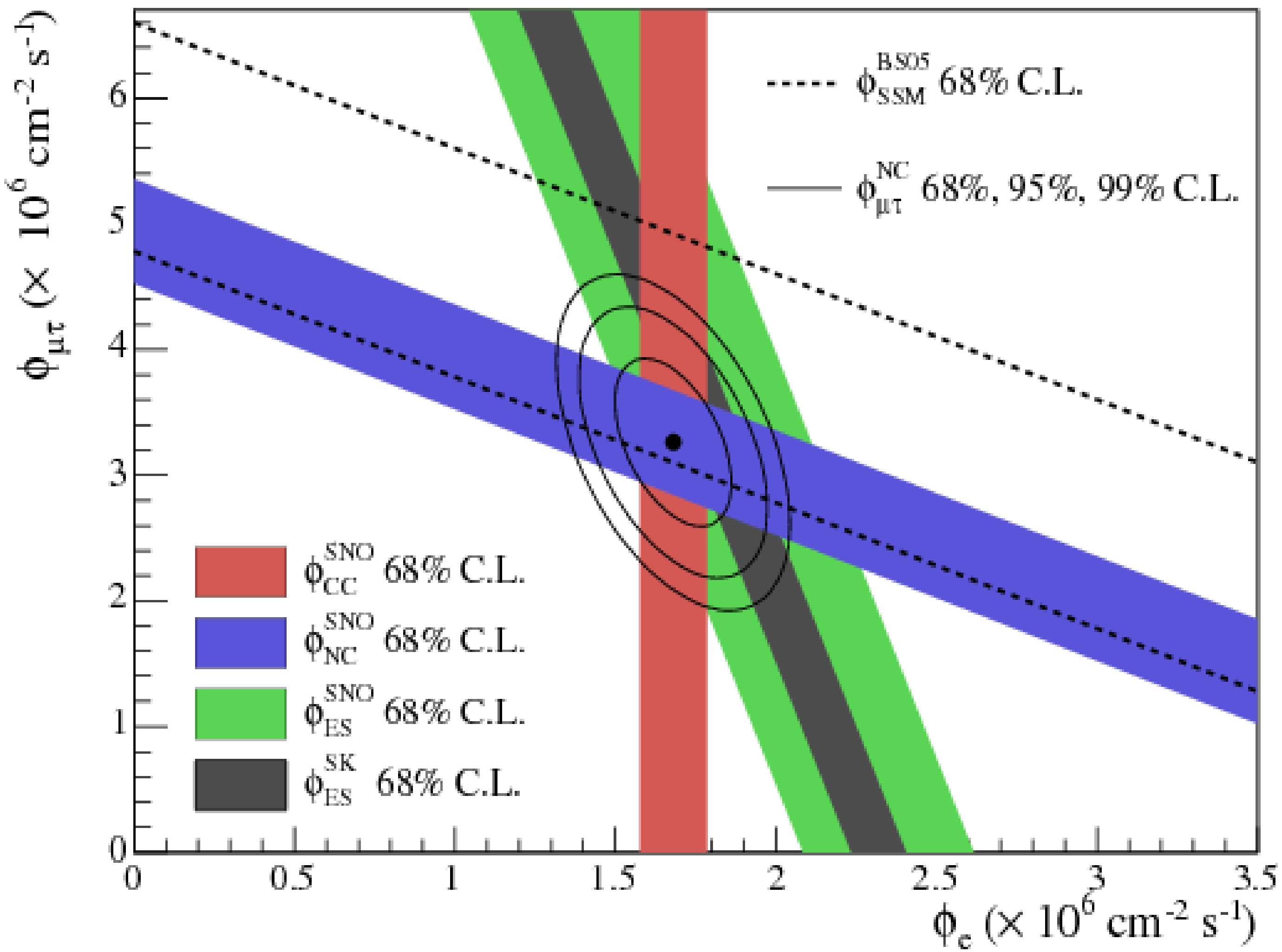


Figure 3: Comparison of measurements to Standard Solar Model predictions.

Flux	BP'00	Cl–Ar	Ga–Ge
$\Phi_{pp} \times 10^{-10}$	$5.95(1\ ^{+0.01}_{-0.01})$	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	$1.40(1\ ^{+0.01}_{-0.01})$	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	$4.77(1\ ^{+0.09}_{-0.09})$	1.15	34.2
$\Phi_{B} \times 10^{-6}$	$5.93(1\ ^{+0.14}_{-0.15})$	6.76	14.2
$\Phi_{N} \times 10^{-8}$	$5.48(1\ ^{+0.19}_{-0.13})$	0.09	3.4
$\Phi_{O} \times 10^{-8}$	$4.80(1\ ^{+0.22}_{-0.15})$	0.33	5.5
Total		$8.55\ ^{+1.1}_{-1.2}$	$129.8\ ^{+9}_{-7}$

Experiment	Observed rate/BP04 prediction	Predicted Rate at global best-fit	Predicted Rate at solar best-fit
Ga	0.52 ± 0.029	0.555	0.540
Cl	0.301 ± 0.027	0.356	0.345
SK(ES)	0.406 ± 0.014	0.394	0.395
SNO(CC)	0.274 ± 0.019	0.289	0.289
SNO(ES)	0.38 ± 0.052	0.386	0.386
SNO(NC)	0.895 ± 0.08	0.889	0.908

The observed rates w.r.t predictions from the latest Standard Solar Model BP04. Shown are also the predicted rates for the best fit values of Δm^2_{21} and $\sin^2\theta_{12}$, obtained in the analysis of the i) global solar neutrino data, and ii) global solar neutrino +KamLAND data.



ϕ_{SSM}^{BS05} 68% C.L.
$\phi_{\mu\tau}^{NC}$ 68%, 95%, 99% C.L.
ϕ_{CC}^{SNO} 68% C.L.
ϕ_{NC}^{SNO} 68% C.L.
ϕ_{ES}^{SNO} 68% C.L.
ϕ_{ES}^{SK} 68% C.L.
$\phi_{\mu\tau}$ ($\times$ 10^6 cm^{-2} s^{-1})
ϕ_e ($\times$ 10^6 cm^{-2} s^{-1})

MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \qquad (2)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\,[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

- Standard Solar Models

$N_e(t) = N_e(t_0)\exp\left\{-\frac{t-t_0}{r_0}\right\}$, $r_0 \sim 0.1 R_\odot$, $R_\odot = 6.96 \times 10^5$km

The region of $\nu_\odot$ production: $r \lesssim 0.2 R_\odot$

$20\ N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$

Suppose $N_e(x_0) \gg N_e^{res}$: $|\nu_e\rangle \cong |\nu_2^m\rangle$.

Possible evolution:

The system stays at this level; at the surface: $|\nu_2^m\rangle = |\nu_2\rangle$

$P(\nu_e \to \nu_e) \cong |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta\,,$ Adiabatic

At $N_e = N_e^{res}$, where $E_2^m - E_1^m$ is minimal, the system jumps to lower level $|\nu_1^m\rangle$; at the surface: $|\nu_1^m\rangle = |\nu_1\rangle$

$P(\nu_e \to \nu_e) \cong |\langle\nu_e|\nu_1\rangle|^2 = \cos^2\theta\,,$ Nonadiabatic

Type of transition: $P' \equiv P(\nu_2^m(t_0) \to \nu_1)\,,$ jump probability

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t, t_0) = (Z/Z_0)^{c-a}\ e^{-(Z-Z_0)+i\int_{t_0}^{t}\epsilon(t')dt'}\ A'_e(t, t_0),$$

$A'_e(t, t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{\ Z\frac{d^2}{dZ^2} + (c - Z)\ \frac{d}{dZ} - a\right\} A'_e(t, t_0) = 0,$$

where

$$a = 1 + ir_0\ \frac{\Delta m^2}{2E}\sin^2\theta, \qquad c = 1 + ir_0\ \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,

$$\Psi(\vec{r}) = \frac{1}{r}\psi_{kl}(r)Y_{lm}(\theta', \phi'),$$

r, θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, ..., l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2}\ e^{Z/2}\ \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2\,\frac{r}{a_0}\sqrt{-E_{kl}/E_I},\ a \equiv a_{kl} = l+1-\sqrt{-E_I/E_{kl}},\ c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6\ eV$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$\Phi(a,c;Z)$, $Z^{1-c}\,\Phi(a-c+1,2-c;Z)$; $\Phi(a',c';Z=0)=1$, $a',c'\neq 0,-1,-2,\ldots$

$$A(\nu_e\rightarrow\nu_{\mu(\tau)})=\frac{1}{2}\,\sin 2\theta\,\left\{\Phi(a-c,2-c;Z_0)-e^{i(t-t_0)\frac{\Delta m^2}{2E}}\,\Phi(a-1,c;Z_0)\right\}.$$

Sun: $N_e(x)\cong N_e(x_0)e^{-\frac{x}{r_0}}$, $r_0\cong 0.1R_\odot$, $R_\odot\cong 7\times 10^5$ km

The region of $\nu_\odot$ production:

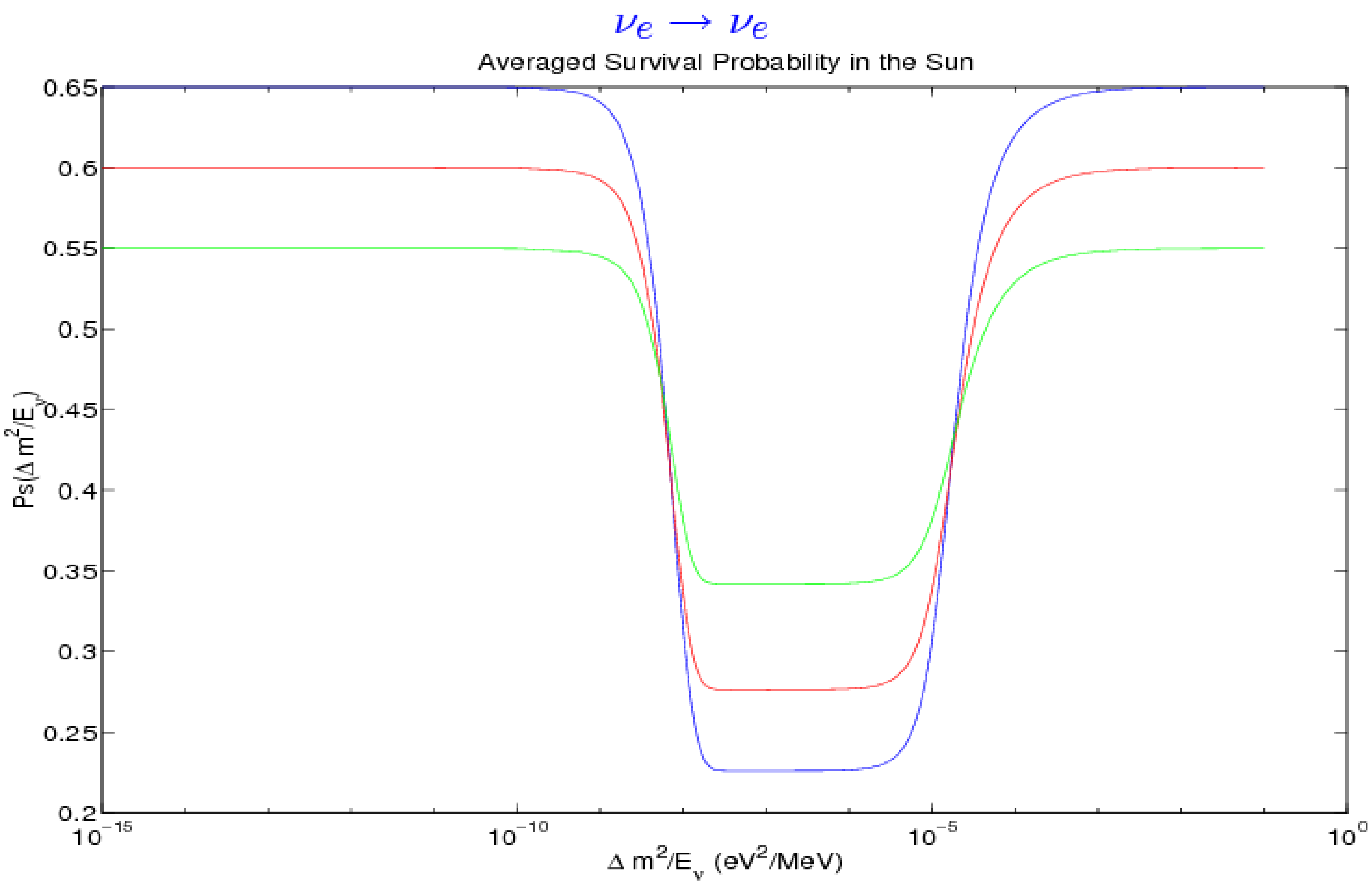
$20\ N_A\ cm^{-3}\lesssim N_e(x_0)\lesssim 100\ N_A\ cm^{-3}$: $|Z_0|>500$ (!)

The solar ν_e survival probability:

$\bar{P}(\nu_e\rightarrow\nu_e)=\frac{1}{2}+(\frac{1}{2}-P')\cos 2\theta_m^0\cos 2\theta,$

$$P' = \frac{e^{-2\pi r_0\frac{\Delta m^2}{2E}\sin^2\theta}-e^{-2\pi r_0\frac{\Delta m^2}{2E}}}{1-e^{-2\pi r_0\frac{\Delta m^2}{2E}}}$$

S.T.P., 1988


$\nu_e \longrightarrow \nu_e$
Averaged Survival Probability in the Sun
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
$Ps(\Delta m^2/E_\nu)$
10^{-15}
10^{-10}
10^{-5}
10^{0}
$\Delta m^2/E_\nu$ (eV^2/MeV)

The solar ν_e survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E} \sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

Case 1: $\cos 2\theta_m^0 = -1$, $P' = 0$, $\bar{P} = \frac{1}{2}(1 - \cos 2\theta)$.

Case 2: $\theta_m^0 = \theta$, $P' = 0$, $\bar{P}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$

Case 1: SNO, Super Kamiokande; $\bar{P} \cong 0.3$: $\cos 2\theta > 0$!

Case 2: pp neutrinos.

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.59 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.318$, $\cos 2\theta_{12} \gtrsim 0.26$ (3σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.031$ (0.047) 2σ (3σ) (see further).

Mezzetto, Schwetz *et al.*, arXiv:1003.5800

$3-\nu$ Mixing Analysis: $\Delta m^2_{\odot} \ll |\Delta m^2_{\rm atm}|$

$P^{3\nu}_{\odot} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\, P^{2\nu}_{\odot}$,

$P^{2\nu}_{\odot} = \bar{P}^{2\nu}_{\odot} + P^{2\nu}_{\odot\ \rm osc}$,

$\bar{P}^{2\nu}_{\odot} = \frac{1}{2} + (\frac{1}{2} - P')\cos 2\theta^m_{12}(t_0)\cos 2\theta_{12} \quad (\theta_{12} \equiv \theta_{\odot})$,

$P' = 0$: L. Wolfenstein, 1978; S. Mikheyev, A. Smirnov, 1985;

$P' \neq 0$ (general or LZ): S. Parke, W. Haxton, 1986;

P'-double exponential, $P^{2\nu}_{\odot\ \rm osc}$: S.T.P., 1988

$N_e \rightarrow N_e \cos^2\theta_{13}$,

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}\sin^2\theta_{12}} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}, \qquad r_0 \sim 0.1 R_{\odot}$$

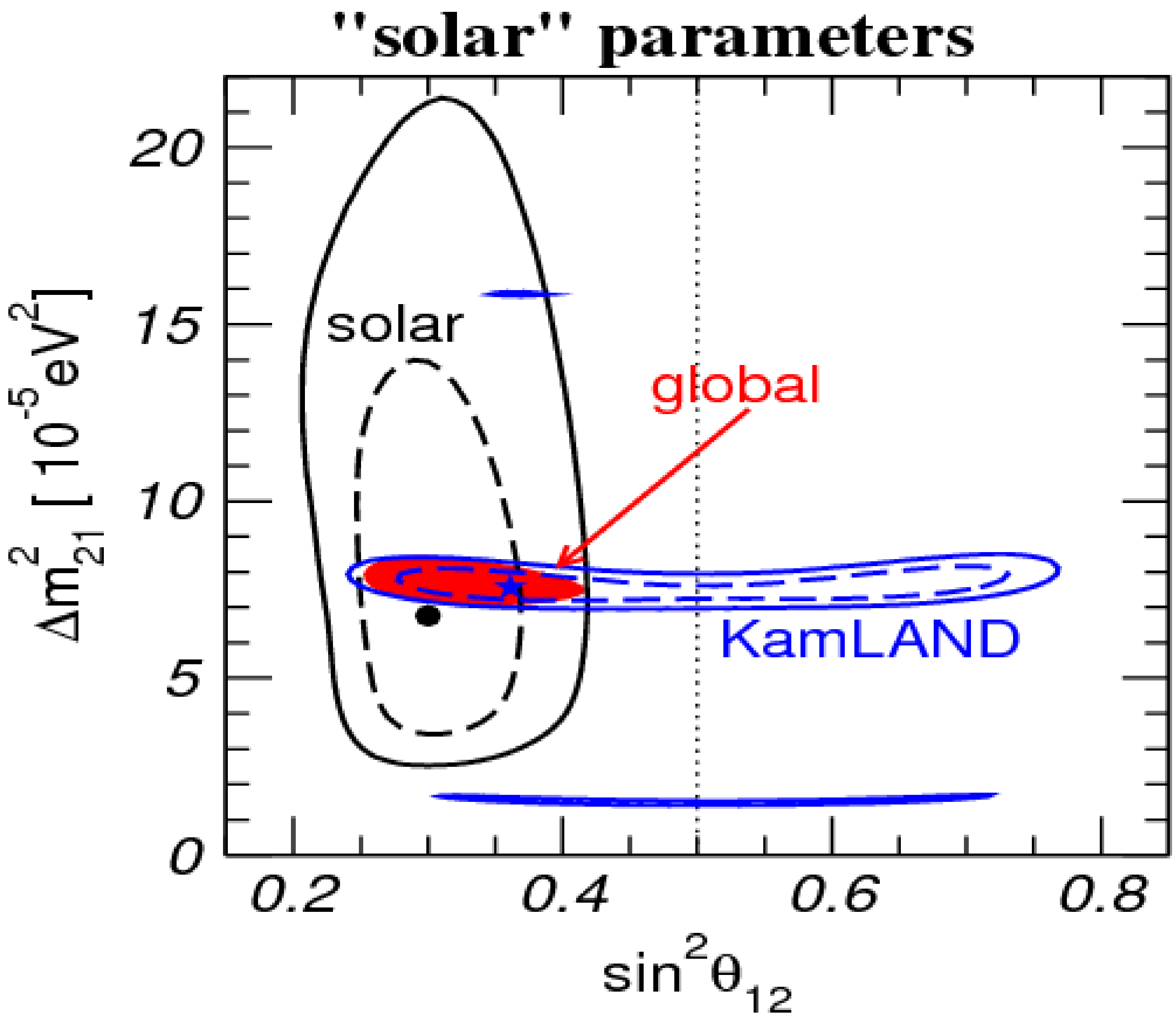
S.T.P., 1988

LMA: $P' \ll 1, \quad < P^{2\nu}_{\odot\ \rm osc} > \cong 0$

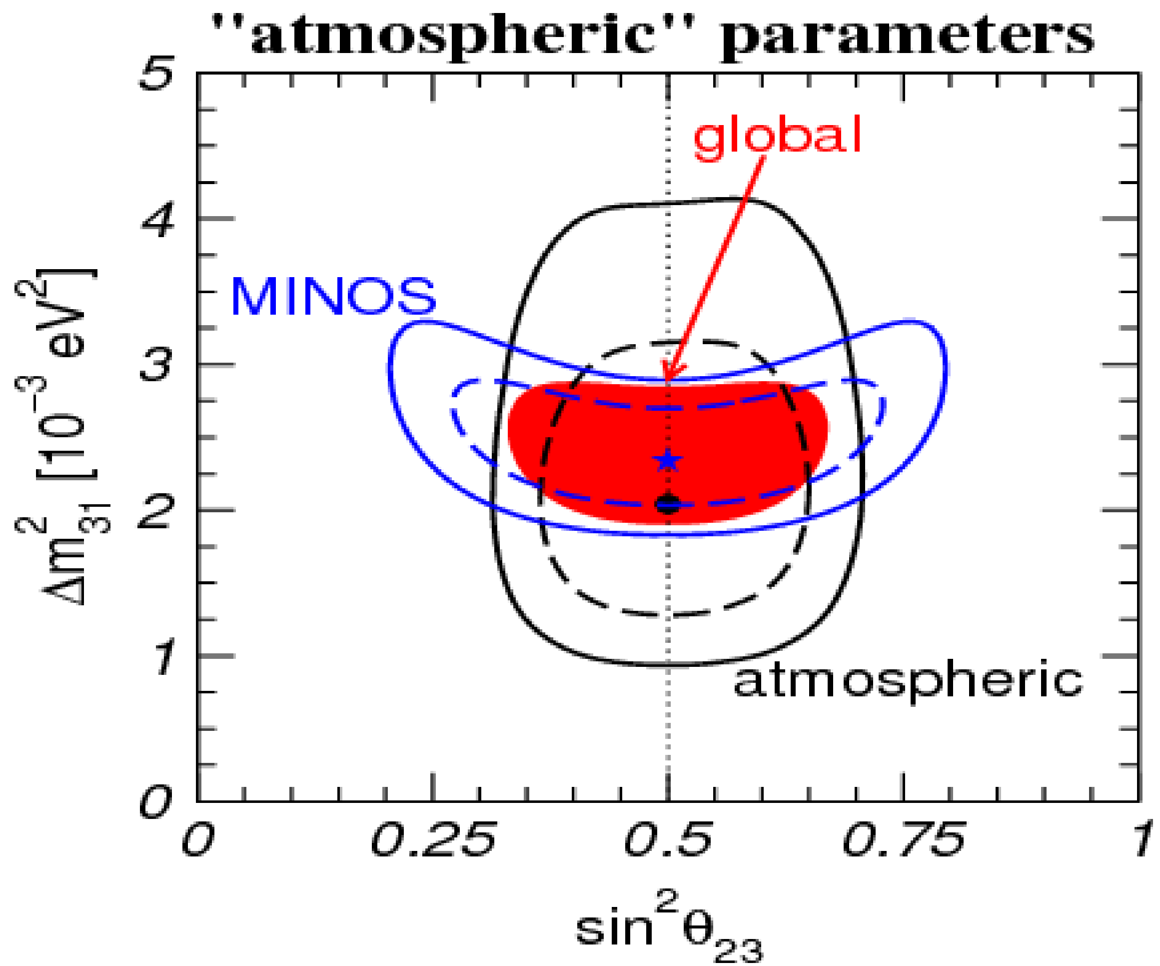
J. Rich, S.T.P., 1988

$$P^{3\nu}_{\rm KL} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2(\frac{\Delta m^2_{\odot}}{4E}L)\right]$$

$$P^{3\nu}_{\rm CHOOZ} \cong 1 - \sin^2 2\theta_{13}\sin^2(\frac{\Delta m^2_{\rm atm}}{4E}L)$$



T. Schwetz, arXiv:0710.5027[hep-ph]



T. Schwetz, arXiv:0710.5027[hep-ph]

- sign of Δm^2_{atm} not determined;

  3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

  $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.

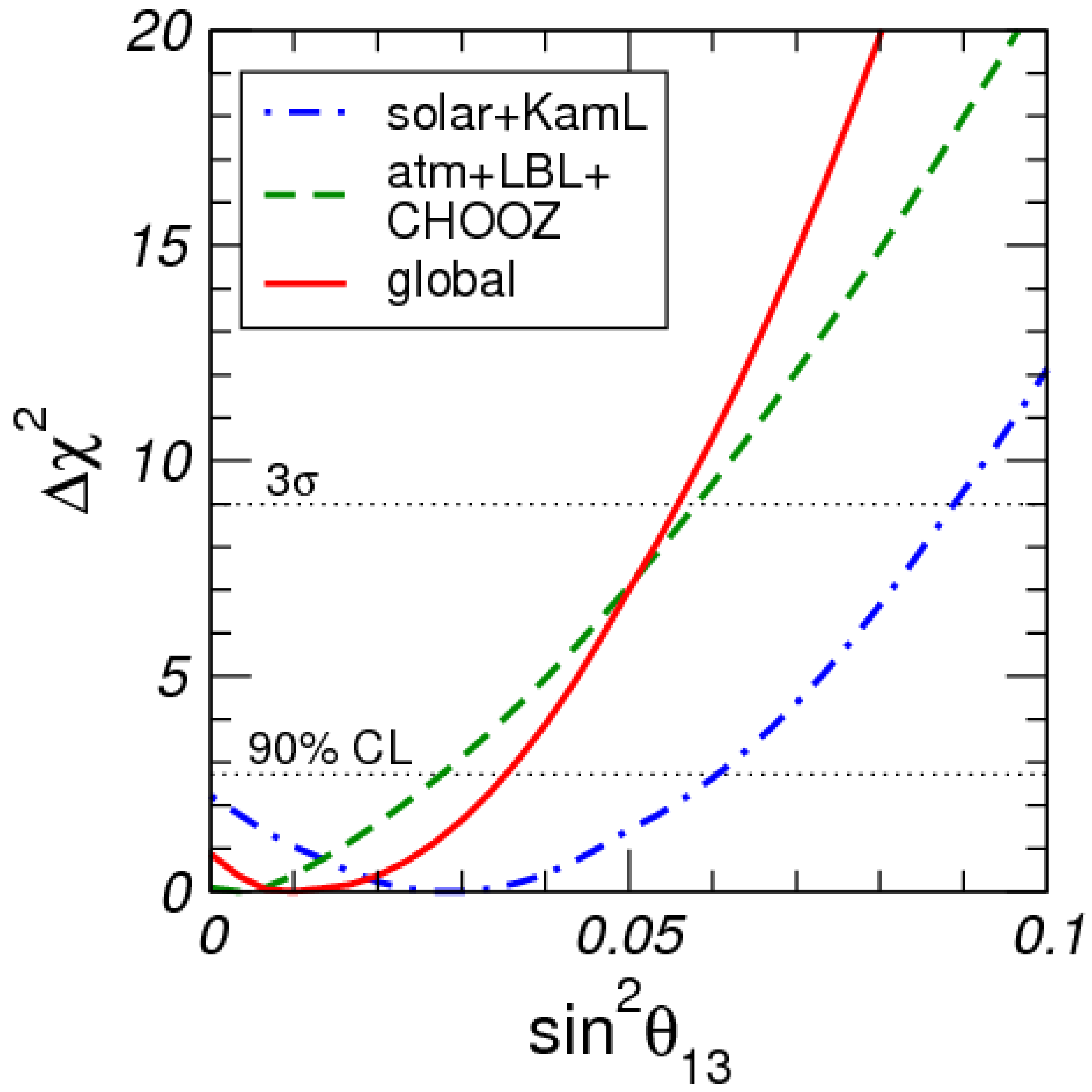
$\sin^2\theta_{13} = 0.016 \pm 0.010$, $\sin\theta_{13} = (0.077 - 0.161)$, 1σ

E. Lisi *et al.*, arXiv:0806.2649

Atmospheric ν data: $\cos\delta = -1$ favored over $\cos\delta = +1$

J. Escamilla *et al.*, arXiv:0805.2924

New data from T2K and MINOS (June 2011), to be discussed further, strengthen these indications.



T. Schwetz *et al.*, arXiv:0808.2016[hep-ph]

Neutrino Oscillation Parameters

parameter	bf$\pm 1\sigma$	1σ acc.	2σ range	3σ range
Δm^2_{21} $[10^{-5}]$	7.65 ± 0.23	3%	$7.25 - 8.11$	$7.05 - 8.34$
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}]$	$2.4^{+0.12}_{-0.11}$	5%	$2.18 - 2.64$	$2.07 - 2.75$
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	7%	$0.27 - 0.35$	$0.25 - 0.37$
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	14%	$0.38 - 0.64$	$0.36 - 0.67$
$\sin^2\theta_{13}$	0.016 ± 0.010	$0.01 - 0.026$	≤ 0.04	≤ 0.056

Best fit values (bf), 1σ errors, relative accuracies at 1σ, and 2σ and 3σ allowed ranges of three-flavor neutrino oscillation parameters from a combined analysis of global data.

M. Maltoni, T. Schwetz *et al.*, arXiv:0812.3161; E. Lisi *et al.*, arXiv:0806.2649 (θ_{13})

- sgn($\Delta m^2_{\rm atm}$) = sgn(Δm^2_{31}) not determined

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

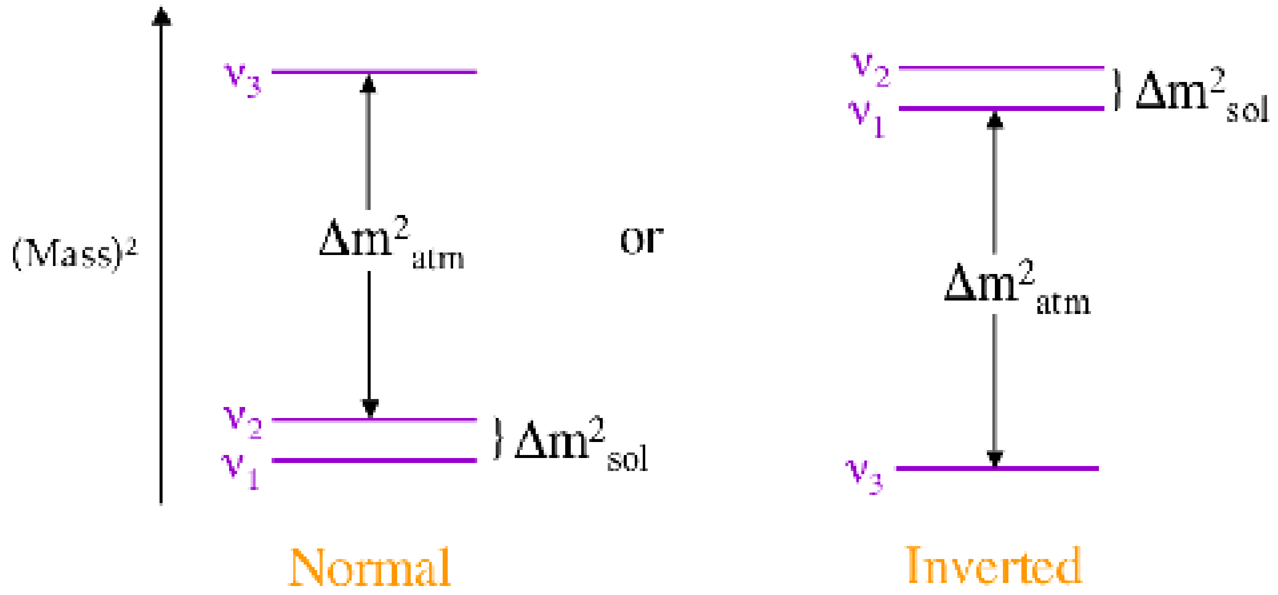
$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

The $(\text{Mass})^2$ Spectrum

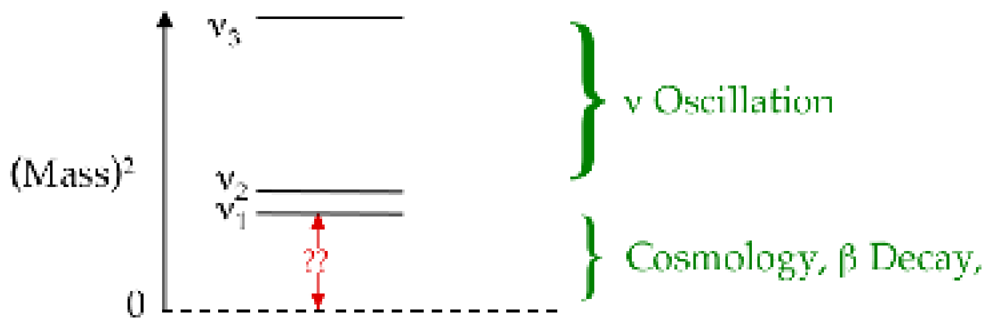


$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

Due to B. Kayser

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data ⇒ $\sqrt{\Delta m^2_{atm}}$ < Mass[Heaviest ν_i]

Due to B. Kayser

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn($\Delta m^2_{\rm atm}$), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, $\Delta m^2_{\rm atm}$, θ_{atm}.

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

  – the origin of the observed patterns of ν-mixing and ν-masses ;

  – the physical origin of CPV phases in $U_{\rm PMNS}$;

  – Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

  – Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

  – Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

  – Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

- Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

  – Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

HOW?

- $\nu_{\odot}-$, $\nu_{\rm atm}-$ experiments

  SK ($\nu_{\rm atm}$);

  INO ($\nu_{\rm atm}$); MEMPHYS

  MINOS ($\nu_{\mu}^{\rm atm}$); ATLAS, CMS ($\nu_{\mu}^{\rm atm}$) (?)

  SNO (2006)

  SAGE

  BOREXINO

  LowNu (XMASS, LENS,...)

- Reactor Experiments $\sim (1 - 180)$ km (SKGd)
- Accelerator Experiments

  MINOS 732 km

  CNGS (OPERA) 732 km

- Super Beams

  T2K, SK (HK) 295 km

  NOνA $\sim$800 km

  LBNE (Fermilab-DUSEL) $\sim$1200 km

  SPL+$\beta-$beams, MEMPHYS (0.5 megaton):
  CERN-Frejus $\sim$140 km

  $\nu-$Factories $\sim$ 3000, 7000 km

- $(\beta\beta)_{0\nu}-$Decay, ^{3}H $\beta-$Decay
- Astrophysics, Cosmology


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

T2K: First results March 2011

Observed 1 e^--event passing all cuts and having all the characteristics of being due to $\nu_\mu \to \nu_e$ oscillations. The estimated background is: 0.30 ± 0.07**(syst.) events;** 29% **probability to observe 1 event when the expected average is 0.3 events.**

NH: $\sin^2 2\theta_{13} < 0.44$; IH: $\sin^2 2\theta_{13} < 0.53$ (90% C.L.)

Assumed: $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$.

T2K: Latest results June 14, 2011 (1.43×10^{20} **pot, full data analysis)**

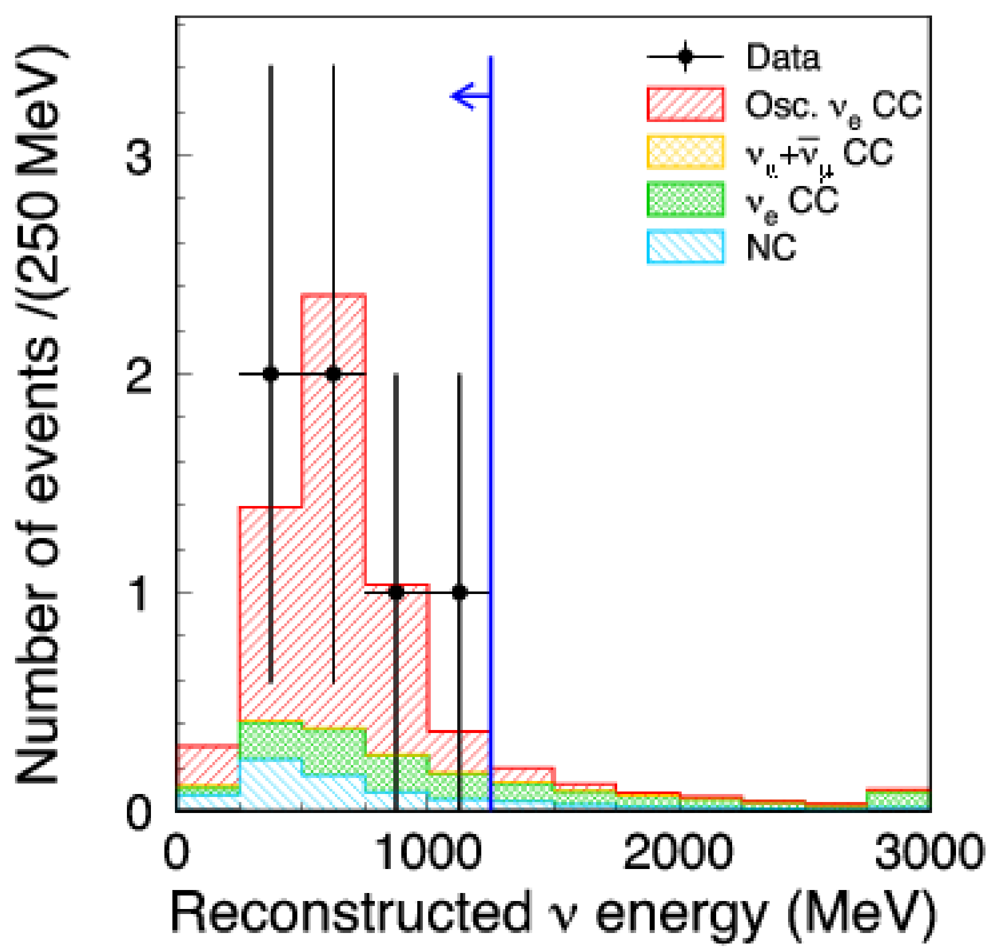
Observed 6 e^--events passing all criteria for being due to $\nu_\mu \to \nu_e$ oscillations. The estimated background ($\theta_{13} = 0$): 1.5 ± 0.03**(syst.) events; the probability to observe ≥ 6 events given 1.5 are expected:** 7×10^{-3}.

Evidence for $\theta_{13} \neq 0$ at 2.5σ.

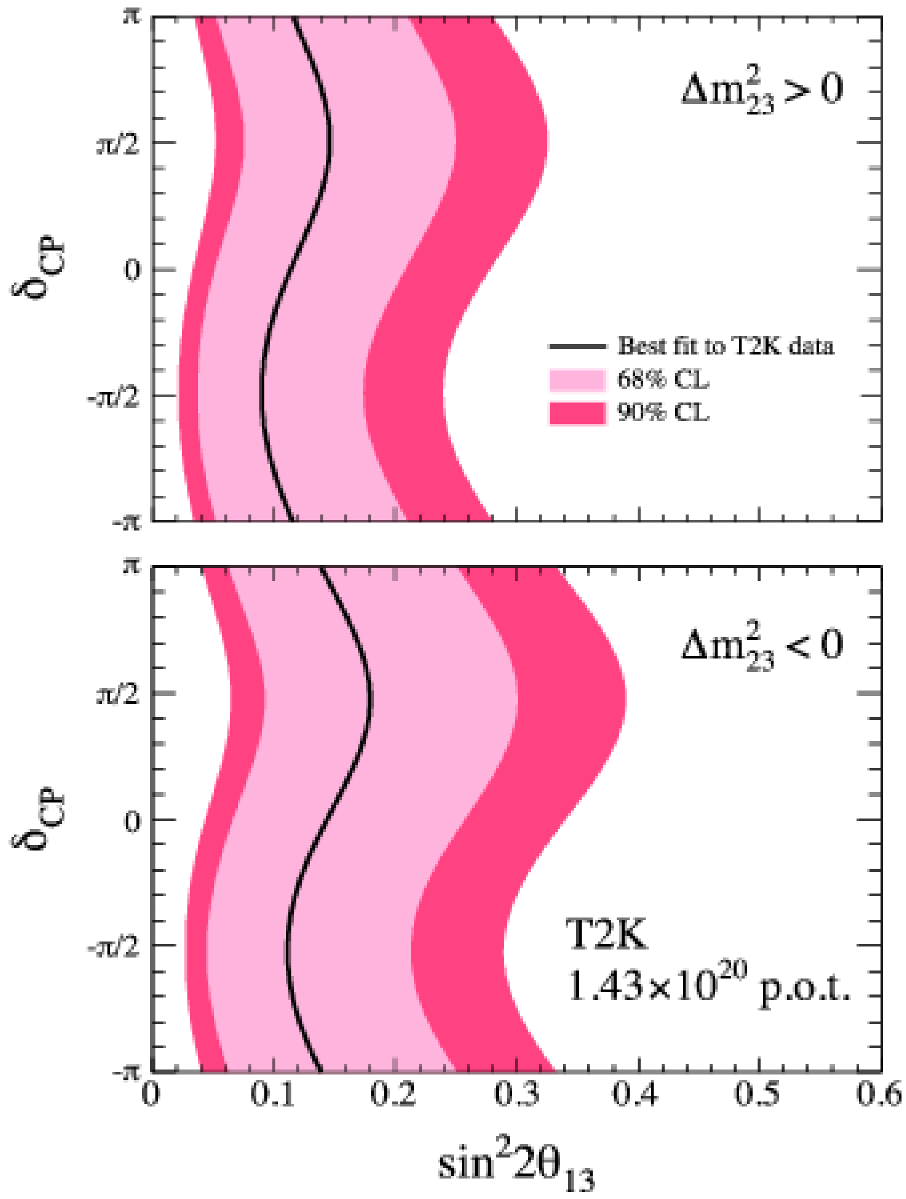
For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, **NO (IO) spectrum:**

$(2\sin^2\theta_{23})\sin^2 2\theta_{13} = 0.11\ (0.14)$, best fit;

$0.03(0.04) < (2\sin^2\theta_{23})\sin^2 2\theta_{13} < 0.28(0.34)$, 90%C.L.



Data
Osc. ν_e CC
$\nu_\mu+\bar{\nu}_\mu$ CC
ν_e CC
NC
Number of events /(250 MeV)
Reconstructed ν energy (MeV)
0
1
2
3
0
1000
2000
3000



Δm²₂₃ > 0
Best fit to T2K data
68% CL
90% CL
δCP
Δm²₂₃ < 0
T2K
1.43×10²⁰ p.o.t.
sin²2θ13

Japan experienced very severe earthquake on March 11th 2011 at 14:46 JST. The J-PARC facility suffered some damages. Fortunately, the Tsunami did not hit the J-PARC.

"Our present priority is to restore life-supporting infrastructure such as electricity, water supply and gas at J-PARC. It may take some time, but we promise the full recovery of the J-PARC accelerator and T2K experiment in the near future. I thank you for the messages of solidarity and sympathy".
Director of the Institute of Particle and Nuclear Studies, KEK
Koichiro Nishikawa
Spokesperson of the T2K experiment
Takashi Kobayashi

We wish success to our colleagues from T2K, J-PARC and KEK in their reconstruction efforts.

MINOS: Latest results June 24, 2011 (8.2×10^{20} pot, prelim.)

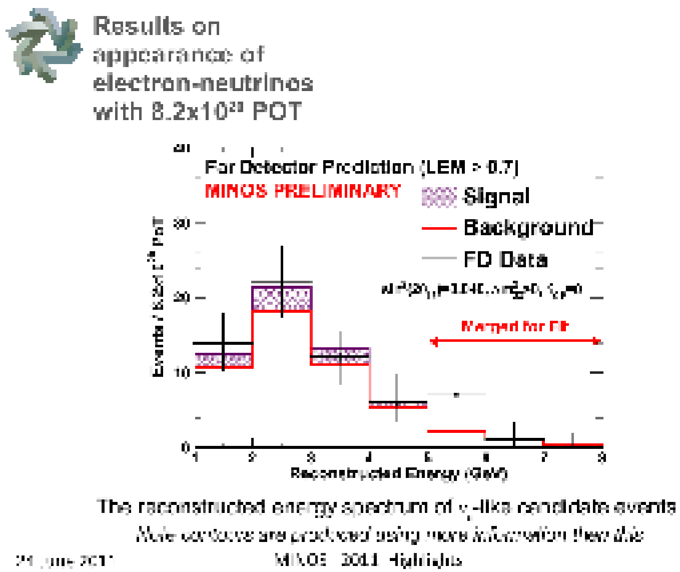
Observed 62 e^--events passing all criteria for being due to $\nu_\mu \to \nu_e$ oscillations. Estimated background: $49.5 \pm 2.8 \pm 7.0$(stat.) events.

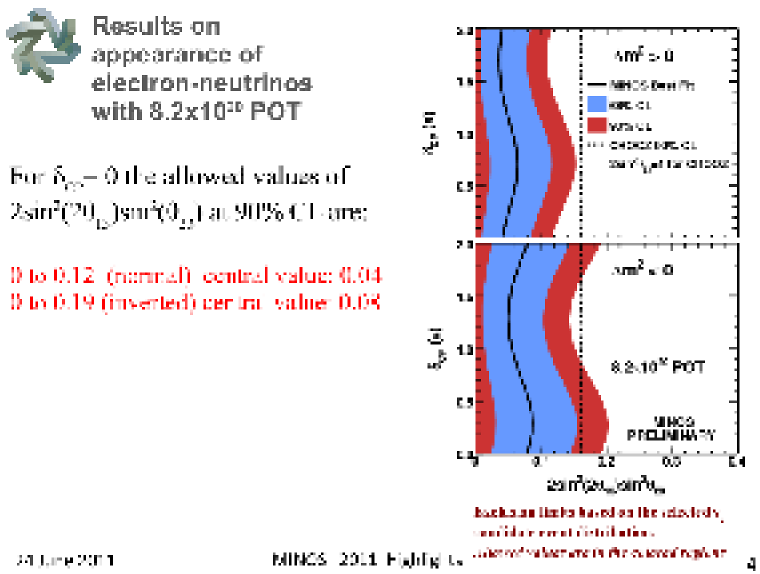
1.7σ excess above background.

For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$$(2\sin^2\theta_{23})\sin^2 2\theta_{13} = 0.04\ (0.08),\ \text{best fit;}$$

$$0 \le (2\sin^2\theta_{23})\sin^2 2\theta_{13} \le 0.12\ (0.19),\ 90\%\text{C.L.}$$





G.L. Fogli *et al.*, June 29, 2011: global data analysis.

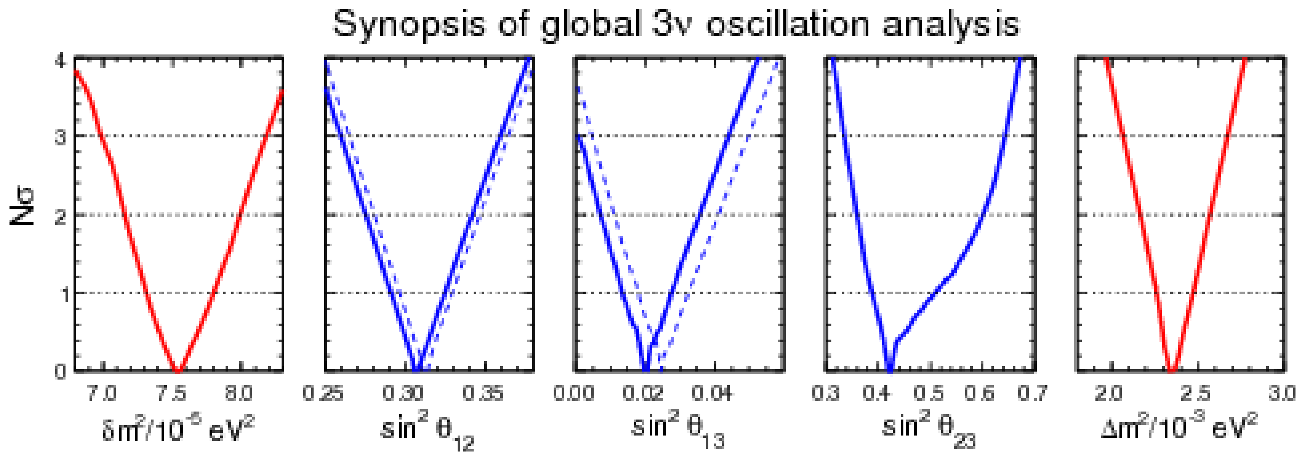
Evidence for $\theta_{13} \neq 0$ at $\geq 3\sigma$.

$\sin^2\theta_{13} = 0.021\ (0.025) \pm 0.007$, old (new) reactor fluxes;

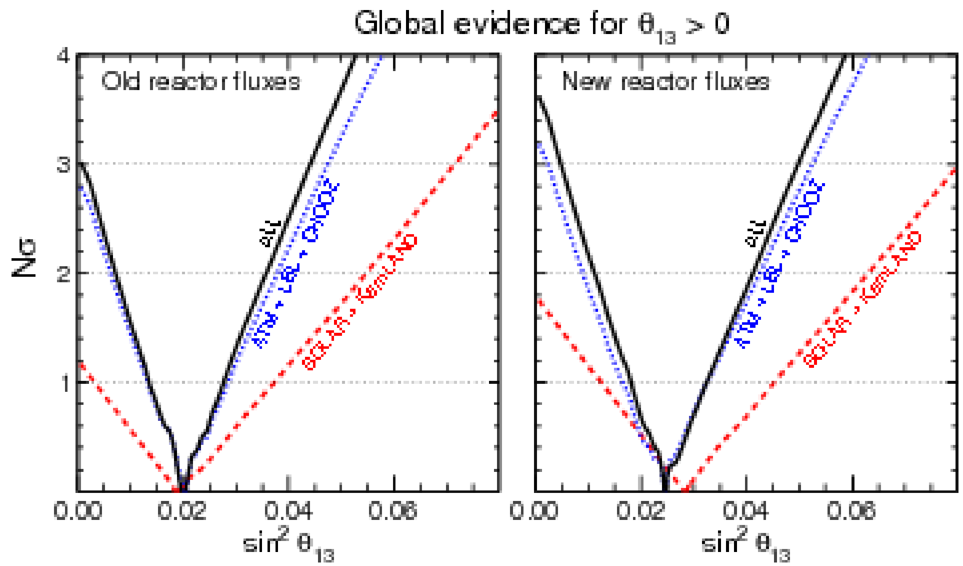
$0.001(0.005) < \sin^2\theta_{13} < 0.044(0.050)$, 3σ.

Global data: $\cos\delta = -1$ clearly favored over $\cos\delta = +1$;

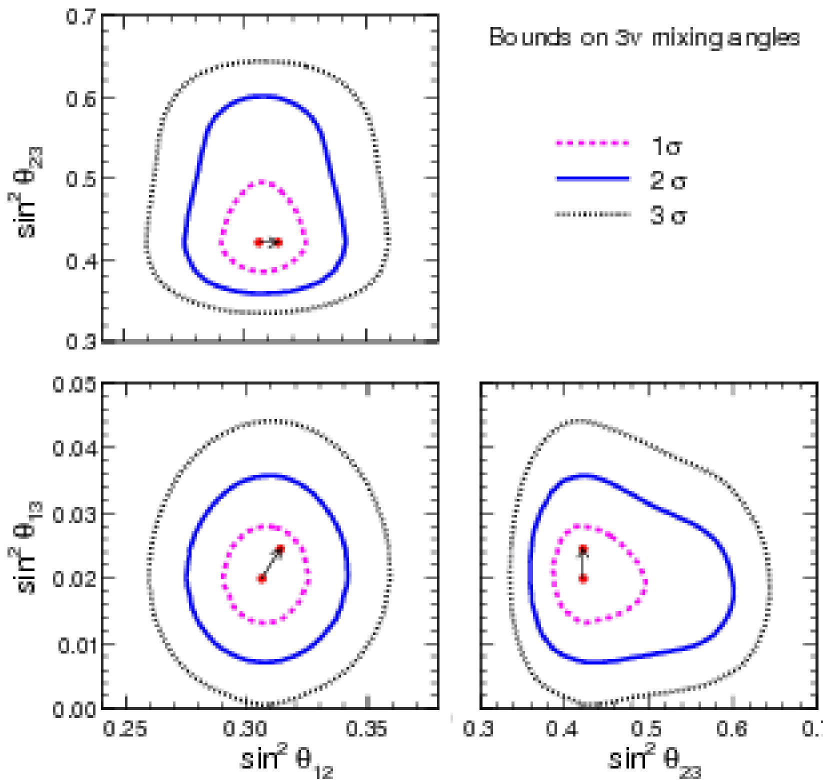
best fit: $\sin\theta_{13}\cos\delta = -0.14$.

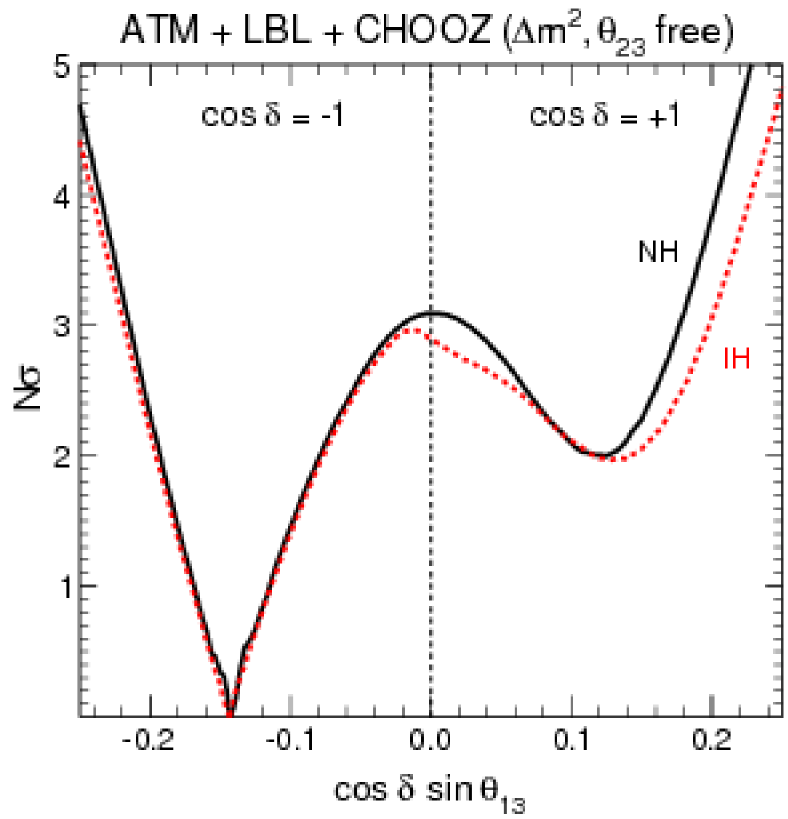


Synopsis of global 3ν oscillation analysis
Nσ
δm²/10⁻⁵ eV²
sin² θ12
sin² θ13
sin² θ23
Δm²/10⁻³ eV²



Global evidence for θ13 > 0
Old reactor fluxes
New reactor fluxes
ALL
ATM + LBL + CHOOZ
SOLAR + KamLAND
Nσ
sin² θ13


Bounds on 3ν mixing angles
1σ
2 σ
3 σ
sin² θ23
sin² θ13
sin² θ12
sin² θ23
0.7
0.6
0.5
0.4
0.3
0.05
0.04
0.03
0.02
0.01
0.00
0.25
0.30
0.35
0.3
0.4
0.5
0.6
0.7



Global data: $\cos\delta = -1$ clearly favored over $\cos\delta = +1$;

best fit: $\sin\theta_{13}\cos\delta = -0.14$.

Neutrino oscillation parameters summary.

Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the mass-mixing parameters, assuming old reactor neutrino fluxes. By using new reactor fluxes, the corresponding best fits and ranges for $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ (in parentheses) are basically shifted by about $+0.006$ and $+0.004$, respectively, while the other parameters are essentially unchanged.

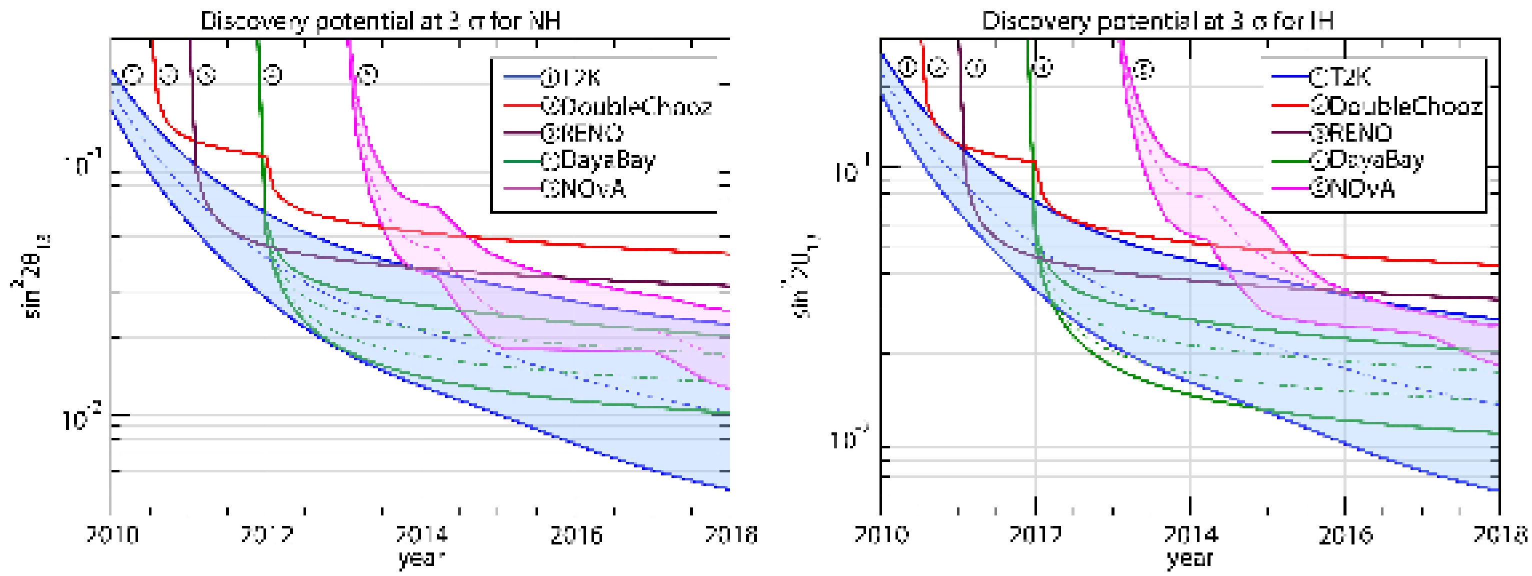
Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.58	0.306 (0.312)	0.021 (0.025)	0.42	2.35
1σ range	7.32 – 7.80	0.291 – 0.324 (0.296 – 0.329)	0.013 – 0.028 (0.018 – 0.032)	0.39 – 0.50	2.26 – 2.47
2σ range	7.16 – 7.99	0.275 – 0.342 (0.280 – 0.347)	0.008 – 0.036 (0.012 – 0.041)	0.36 – 0.60	2.17 – 2.57
3σ range	6.99 – 8.18	0.259 – 0.359 (0.265 – 0.364)	0.001 – 0.044 (0.005 – 0.050)	0.34 – 0.64	2.06 – 2.67

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

Large $\sin^2\theta_{13} \cong 0.021$ - far-reaching implications for the program of reserach in neutrino physics:

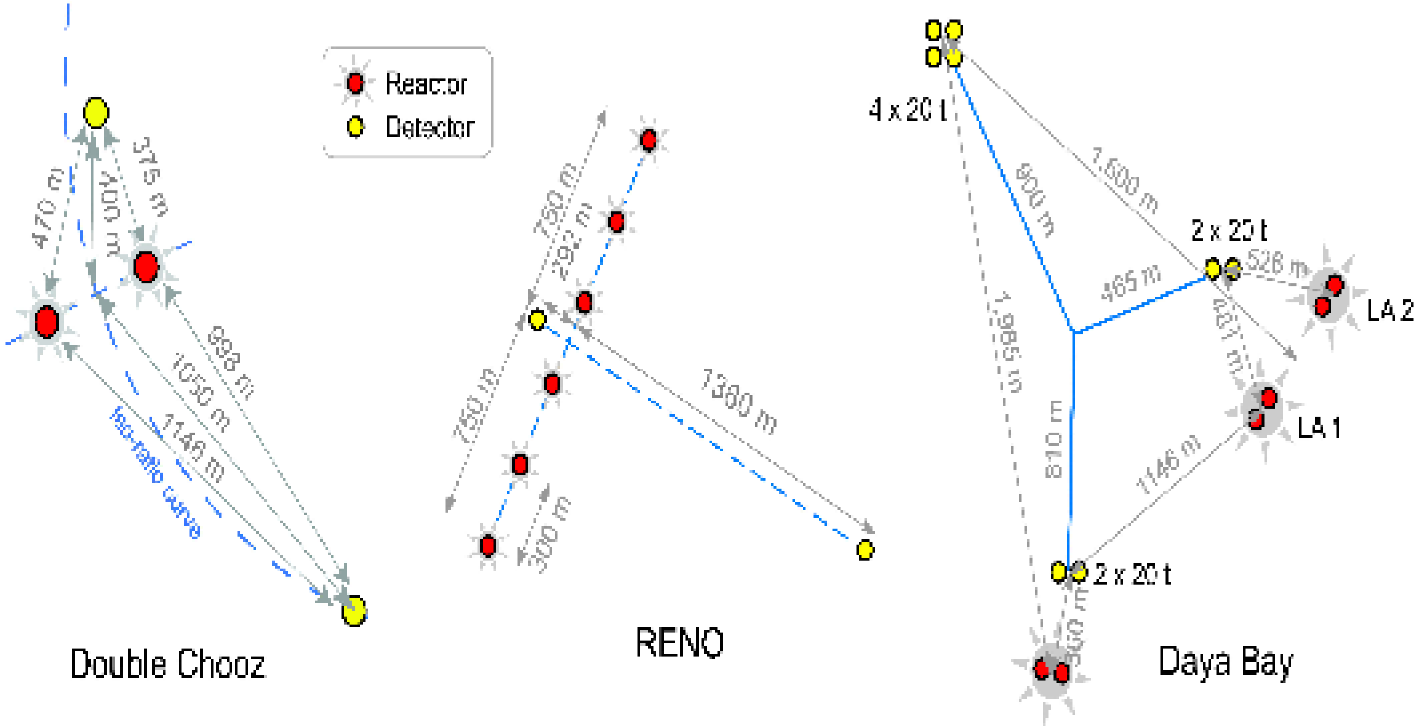
- For the searches for CP violation in ν-oscillations;
- For the determination of the type of $\nu-$ mass spectrum (or of sgn(Δm^2_{atm})) in neutrino oscillation experiments.
- For understanding the patern of the neutrino mixing and its origins (symmetry, etc.?).
- For the observation of the Earth mantle-core enhancement (Neutrino Oscillation Length Resonance) in the $\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}$ (or $\bar{\nu}_{e(\mu)} \rightarrow \bar{\nu}_{\mu(e)}$) oscillations of the atmospheric neutrinos crossing the Earth core.
- For the predictions for the $(\beta\beta)_{0\nu}$-decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).
- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe.

Prospects for θ_{13}



Width: experiments with ν beams - dependence on the CP phase; DayaBay - syst. uncert. $0.18\% - 0.6\%$

Mezzetto, Schwetz, 1003.5800



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

The 3-neutrino mixing - still the reference framework.

3-ν oscillation parameters summary.

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.58^{+0.22}_{-0.26}$	7.16–7.99	6.99–8.18
$\|\Delta m^2_{31}\|$ $[10^{-3}\ \text{eV}^2]$	$2.35^{+0.12}_{-0.09}$	2.26 – 2.47	2.06 – 2.67
$\sin^2\theta_{12}$	$0.306^{+0.018}_{-0.015}$	0.275–0.342	0.259–0.359
$\sin^2\theta_{23}$	$0.42^{+0.08}_{-0.03}$	0.36–0.60	0.34–0.64
$\sin^2\theta_{13}$	$0.021^{+0.007}_{-0.008}$ $(0.025^{+0.007}_{-0.007})$	0.08 – 0.036 (0.012 – 0.041)	0.001 – 0.044 (0.005 – 0.050)

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

$\sin^2\theta_{13}$: upper (lower) row - old (new) reactor fluxes.

Neutrino Mixing:

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}$, $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}$, $\theta_{13} < \frac{\pi}{13}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix};$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1+\lambda)$, $\sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1-\lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\theta_{\odot} + \theta_c = (47 \pm 1.2)^{\text{deg}} = \pi/4$?
- U_{PMNS} **due to new approximate symmetry?**

A Natural Possibility:

$$U = U^{\dagger}_{\rm lep}(\lambda)\, U_{\rm bim(tri)}\, P(\alpha_{21}, \alpha_{31})\,,$$

with

$$U_{\rm tri} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm bim} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\rm lep}(\lambda)$ - from diagonalization of the l^- mass matrix,
- $U_{\rm bim(tri)}P$ - from diagonalization of the $\nu-$mass matrix.
- $U_{\rm (tri)}$: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$, in agreement with the current data.

Harrison, Perkins, Scott, 2002

$U_{\rm (tri)}$: Groups A4, S4,... (vast literature)

(Reviews: Altarelli, Feruglio, 1002.0211, Tanimoto et al., 1003.3552)

Typically, $\theta_{13} \sim (\lambda_c)^2$ is predicted.

- U_{bim}: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{12} = 1/2$ must be corrected.

U_{bim}: Groups S4,...; typically $\theta_{13} \sim \lambda_c$.

- U_{bim}: alternatively $U(1)$,

$$L' = L_e - L_\mu - L_\tau \qquad (\Delta m^2_\odot \ll |\Delta m^2_{\text{atm}}|)$$

S.T.P., 1982

$U_{\text{bim(tri)}}$ requires a $\mu - \tau$ symmetry of M_ν

- Additional possibility: $\sin^2\theta_{12} = \frac{2}{5+\sqrt{5}} \cong 0.28$, "Golden ratio"; A_5.

Kajiyama et al., 2007; Everett, Stuart, 2008, 2011

$$U_{\text{GR}} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{2} & \frac{\cos\theta_{12}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{2} & \frac{\cos\theta_{12}}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

Feruglio, 2011

Which is the correct approximate form of U_{PMNS}? Perhaps none of the above(?).

For $\sin^{\ell}\theta_{ij} \equiv \lambda_{ij}$ "small", $\lambda_{12} \gg \lambda_{13}$ (natural),

$\sin^2\theta_{12} = \frac{1}{2} - \sin 2\theta^{\nu}_{12} \sin\theta_{13} \cos\delta$, $U_{\rm bim}$,

δ is the Dirac CPV phase;

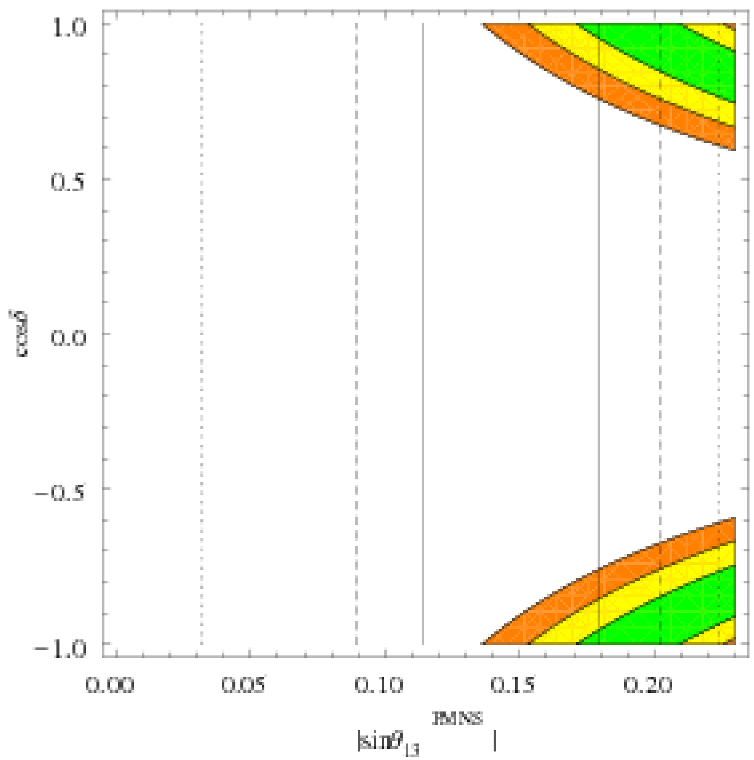
$U_{\rm bim}$: $\sin 2\theta^{\nu}_{12} = \pm 1$, only $\sin 2\theta^{\nu}_{12} = -1$ is possible
if $\sin\theta_{13}\cos\delta < 0$.

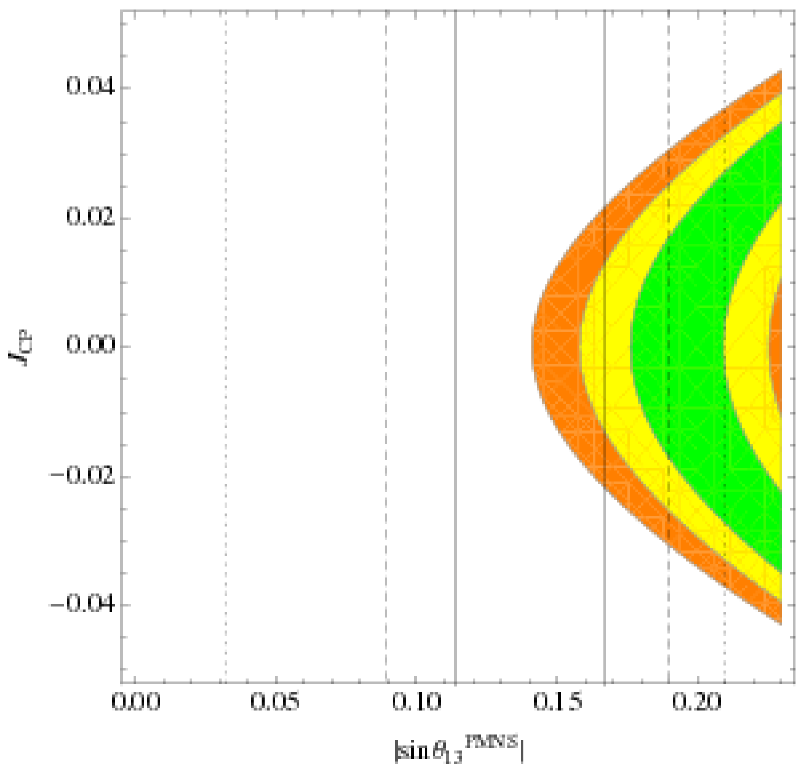
$\sin^2\theta_{12} = \frac{1}{3} \mp 2\frac{\sqrt{2}}{3} \sin\theta_{13}\cos\delta$, $U_{\rm tri}$.

P. Frampton, S.T.P., W. Rodejohann, 2004;
S. King, 2005; S. Antusch, S. King, 2005; I. Masina, 2006;
K. Hochmuth, S.T.P., W. Rodejohann, 2007

Can be tested experimentally.

The case of U_{bim}

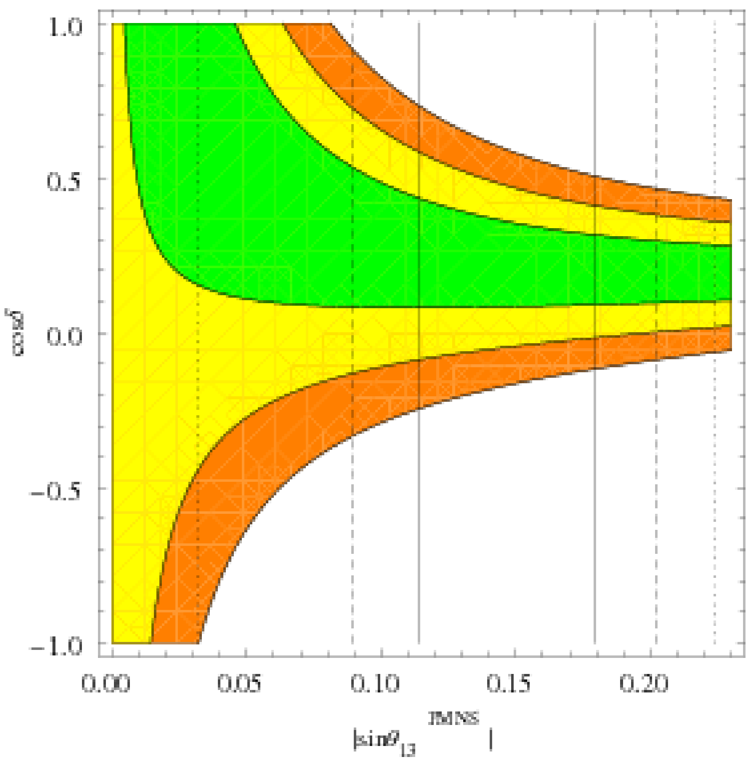


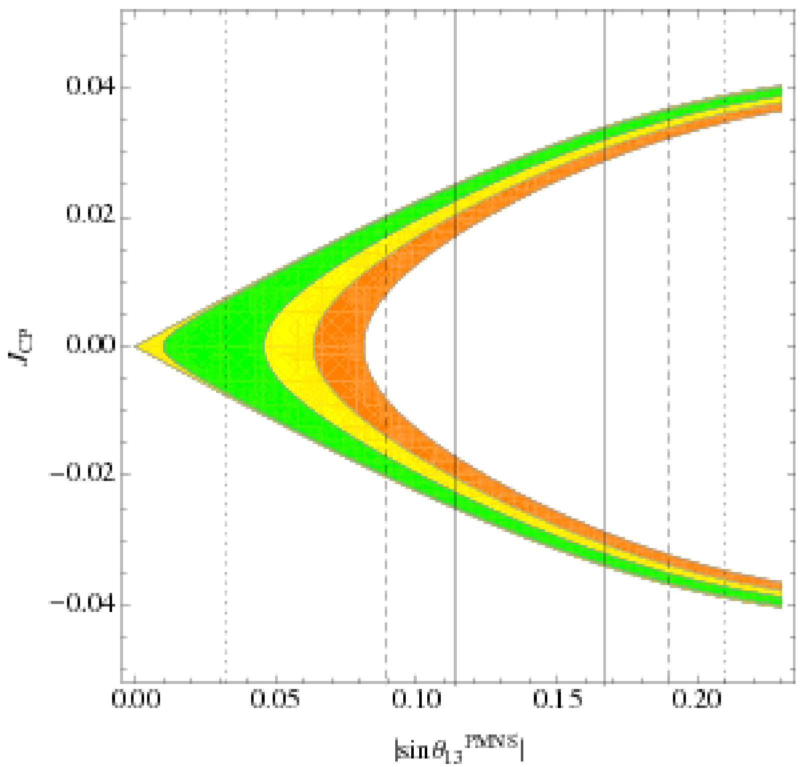


$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

D. Marzocca, S.T.P., A. Romanino, M. Spinrath, 2011

The case of U_{tri}





D. Marzocca, S.T.P., A. Romanino, M. Spinrath, 2011

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} \ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\{U_{e1}U^*_{e3}\}\,, \quad S_2 = \text{Im}\{U_{e2}U^*_{e3}\} \quad \text{(not unique);} \quad \text{or}$$
$$S'_1 = \text{Im}\{U_{\tau 1}U^*_{\tau 2}\}\,, \quad S'_2 = \text{Im}\{U_{\tau 2}U^*_{\tau 3}\}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im}\{U_{e1}U^*_{e3}\} \neq 0$ and $\text{Re}\{U_{e1}U^*_{e3}\} \neq 0$.

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} \ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} \ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21}, \ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger et al.,1980.

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'}) \ , \quad l \neq l' = e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l), \ l \neq l'$$

3ν — mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'}) \ , \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l), \ l \neq l'$$

$$A_{\text{T}}^{(e,\mu)} = A_{\text{T}}^{(\mu,\tau)} = -A_{\text{T}}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988

In vacuum: $A^{(e,\mu)}_{CP(T)} = J_{CP} F^{vac}_{osc}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin(\frac{\Delta m^2_{21}}{2E}L) + \sin(\frac{\Delta m^2_{32}}{2E}L) + \sin(\frac{\Delta m^2_{13}}{2E}L)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'})$

CPT : $P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_{l'} \to \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance **(Earth)**

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

In matter with constant density: $A^{(e,\mu)}_{T} = J^{mat}_{CP} F^{mat}_{osc}$

$$J^{mat}_{CP} = J^{vac}_{CP}\, R_{CP}$$

R_{CP} does not depend on θ_{23} and δ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P^{3\nu\ man}_m(\nu_e \rightarrow \nu_\mu) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = \alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

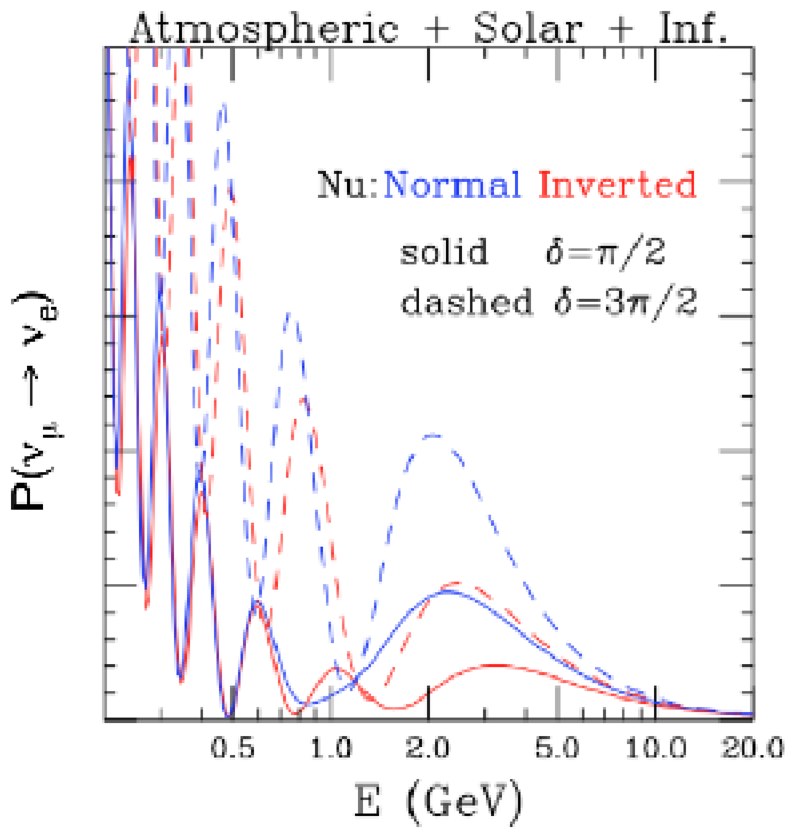
$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E},\;\; A = \sqrt{2}G_F N_e^{man}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_e \rightarrow \bar{\nu}_\mu$: $\delta,\;\; A \rightarrow (-\delta),\;\; (-A)$

For $L = 1200$ km (~ Fermilab to DUSEL),
and $\sin^2 2\theta_{13} = 0.04$

(Parke)



Note how the 2nd maximum can help.

HOW?

- Reactor Experiments at $L \sim 1$ km: D-CHOOZ, Daya Bay, RENO (start: 2010; 2012; 2011)

MINOS, CNGS (OPERA), $L \sim 730$ km:

$\sin^2\theta_{13}$

- Super Beams: θ_{13}, δ, ...

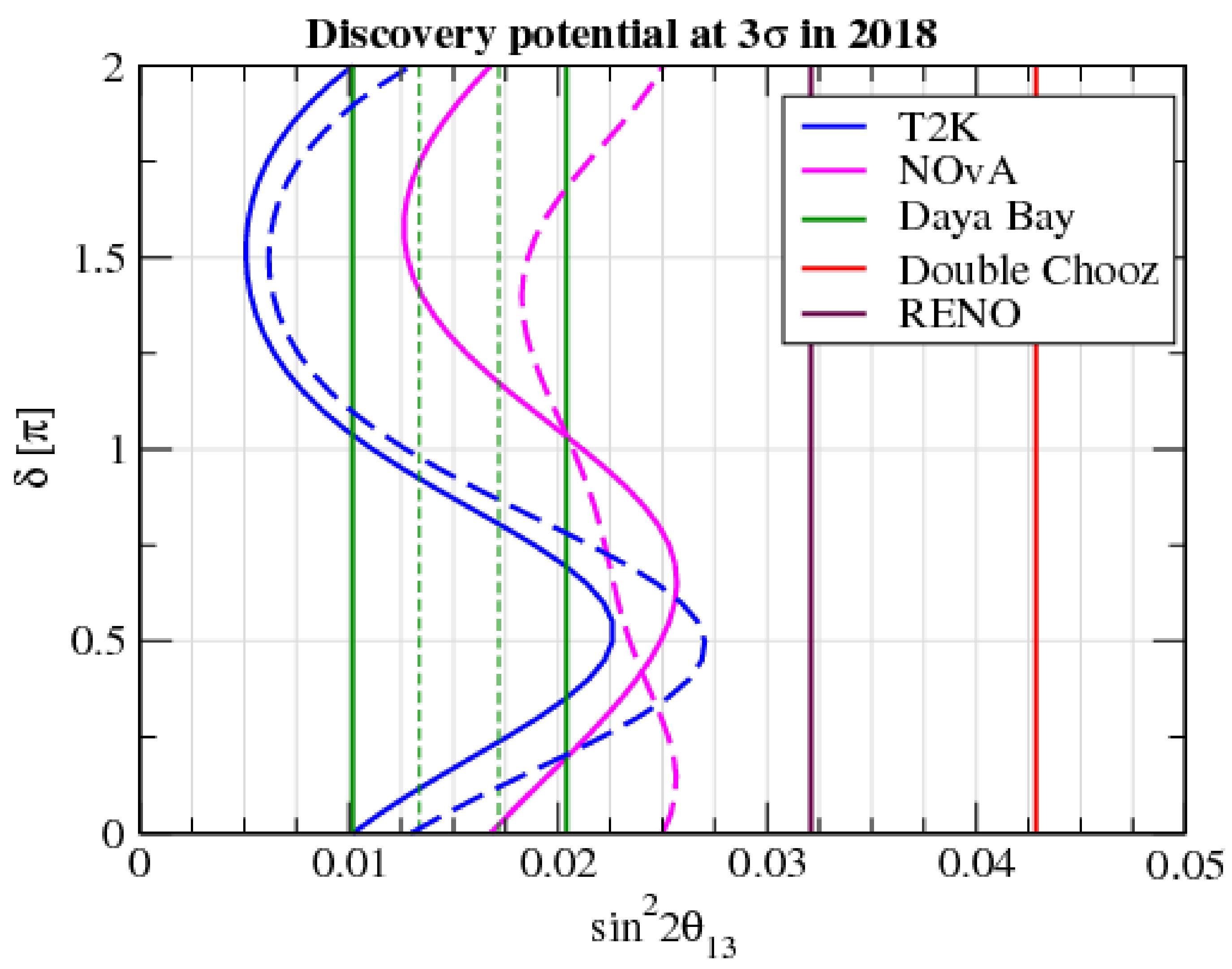
JHF (T2K), SK (HK) 295 km (started)

NuMI (NOνA) ~800 km (2013)

LBNE (Fermilab-DUSEL) ~1200 km (2020)

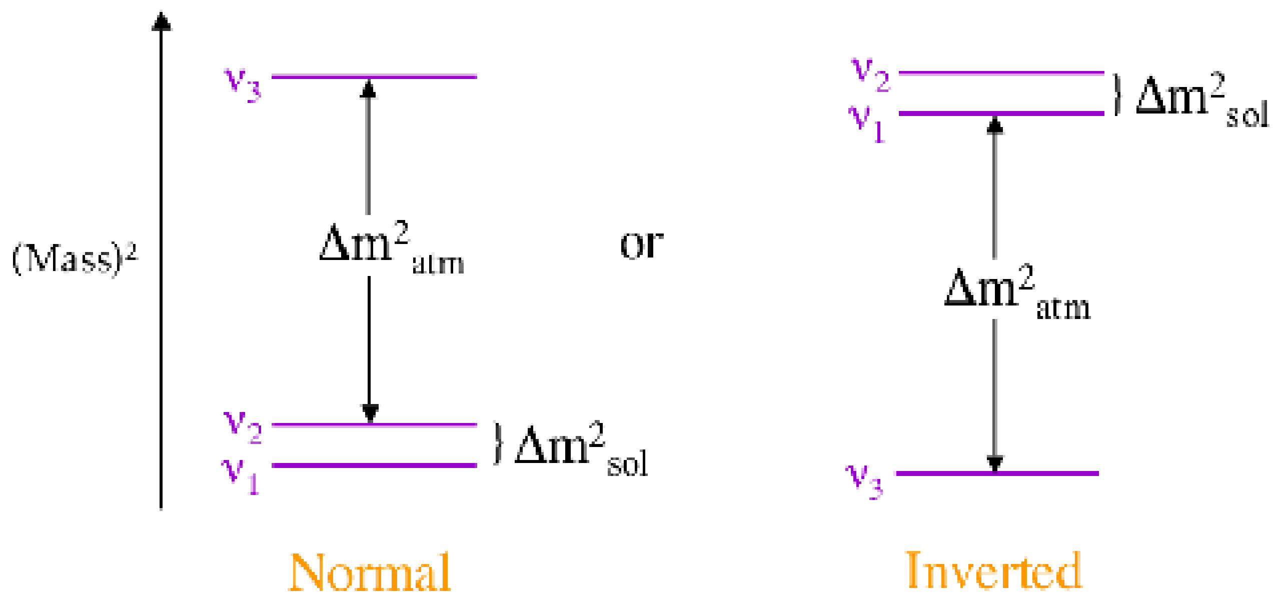
SPL+$\beta-$beams, UNO (1 megaton):
CERN-Frejus ~140 km

$\nu-$Factories $\sim$ 3000, 7000 km



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

Due to B. Kayser

If ν_j- Majorana particles, $U_{\rm PMNS}$ contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana physical CPV phases

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l,l' = e,\mu,\tau$,

- are not sensitive to the nature of ν_j,

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + (\mathrm{A,Z}) \rightarrow \mu^+ + (\mathrm{A,Z}-2)$$

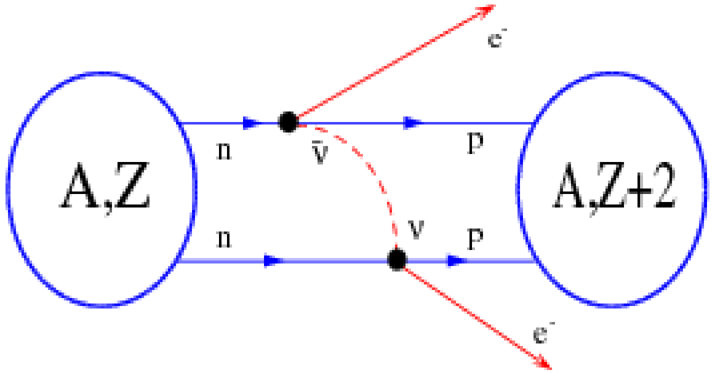
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\mathrm{A,Z}) \rightarrow (\mathrm{A,Z}+2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

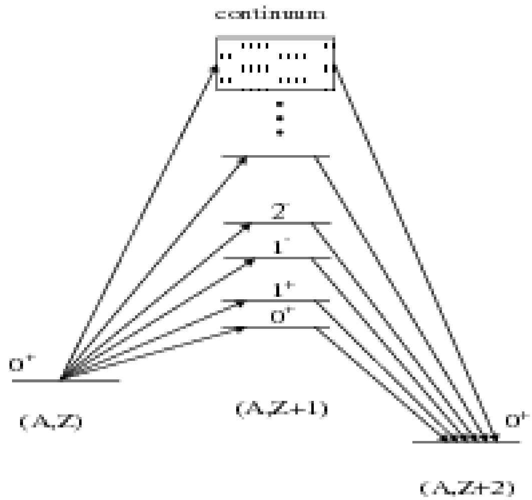
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \to uue^- e^- (\bar{\nu}_e \bar{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}$−Decay Experiments:

- Majorana nature of ν_j
- Type of ν−mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^3H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j− Dirac or Majorana particles, **fundamental problem**

ν_j−Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j−Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of ν−mixing and of Δm^2_{atm} and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j− Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of ν− oscillations.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z),   M(A,Z) - NME,

$$|<m>| = \left|m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 \; e^{i\alpha_{21}} + m_3 |U_{e3}|^2 \; e^{i\alpha'_{31}}\right|$$

$$= \left|m_1 \; c_{12}^2 \; c_{13}^2 + m_2 \; s_{12}^2 \; c_{13}^2 \; e^{i\alpha_{21}} + m_3 \; s_{13}^2 \; e^{i\alpha'_{31}}\right|, \;\; \theta_{12} \equiv \theta_{\odot}, \; \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix; $\alpha'_{31} \equiv \alpha_{31} - 2\delta$

CP-invariance: $\alpha_{21} = 0, \pm\pi, \; \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$|<m>|$: m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of $\min(m_j)$, $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering,} \quad \text{or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) if $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|<m>| = |<m>| \,(m_{\rm min}, \alpha_{21}, \alpha_{31}; S), \; S = NO(NH), IO(IH).$$

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$\left|<m>\right| \cong \left|\sqrt{\Delta m^2_{\odot}} \sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} e^{i\beta}\right|$, $m_1 \ll m_2 \ll m_3$ (NH),

$\left|<m>\right| \cong \sqrt{m_3^2 + \Delta m^2_{13}} \left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|$, $m_3 < (\ll) m_1 < m_2$ (IH),

$\left|<m>\right| \cong m \left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|$, $m_{1,2,3} \cong m \gtrsim 0.10$ eV (QD),

$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta + 2\delta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$\left|<m>\right| \lesssim 5 \times 10^{-3}$ eV, NH;

$\sqrt{\Delta m^2_{13}} \cos 2\theta_{12} \cong 0.013$ eV $\lesssim \left|<m>\right| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055$ eV, IH;

$m \cos 2\theta_{12} \lesssim \left|<m>\right| \lesssim m$, $m \gtrsim 0.10$ eV, QD .

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|<m>| = (0.1 - 0.9)$ eV (99.73% C.L.).

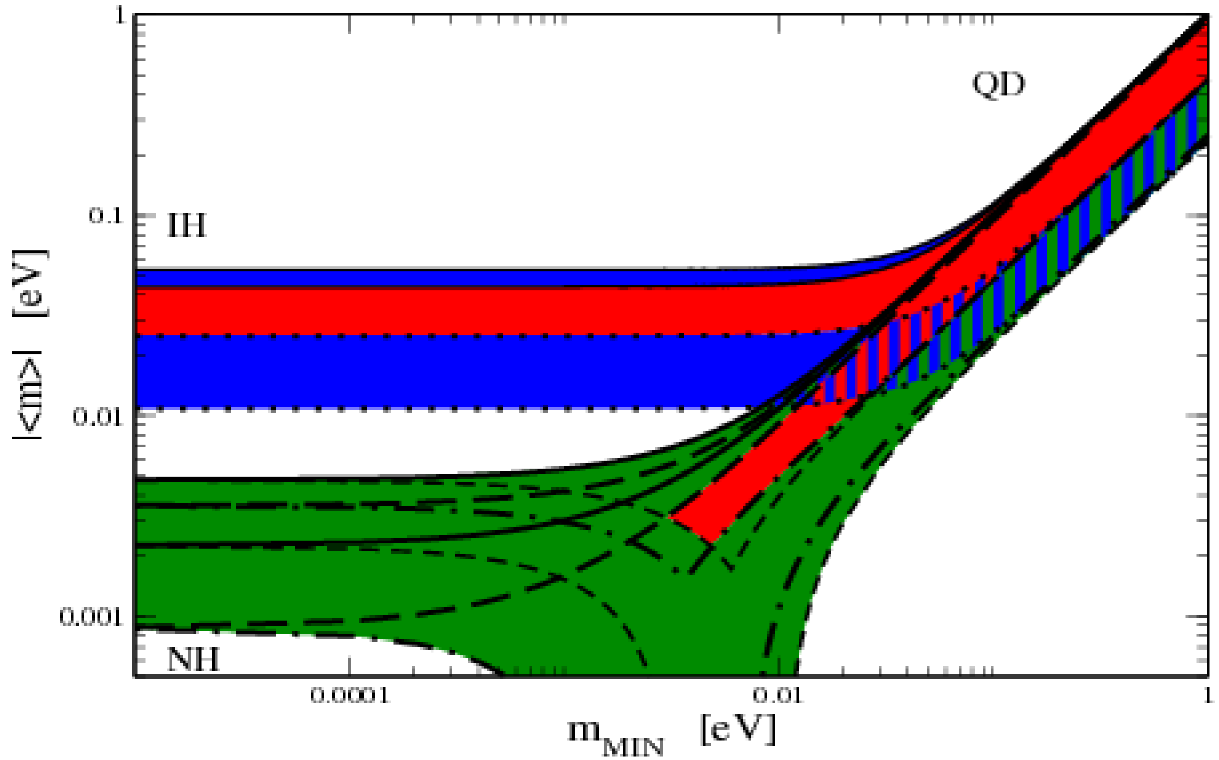
IGEX ^{76}Ge: $|<m>| < (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|<m>|$ <(0.7–1.2) eV, $|<m>|$ <(0.18–0.90) eV (90% C.L.).

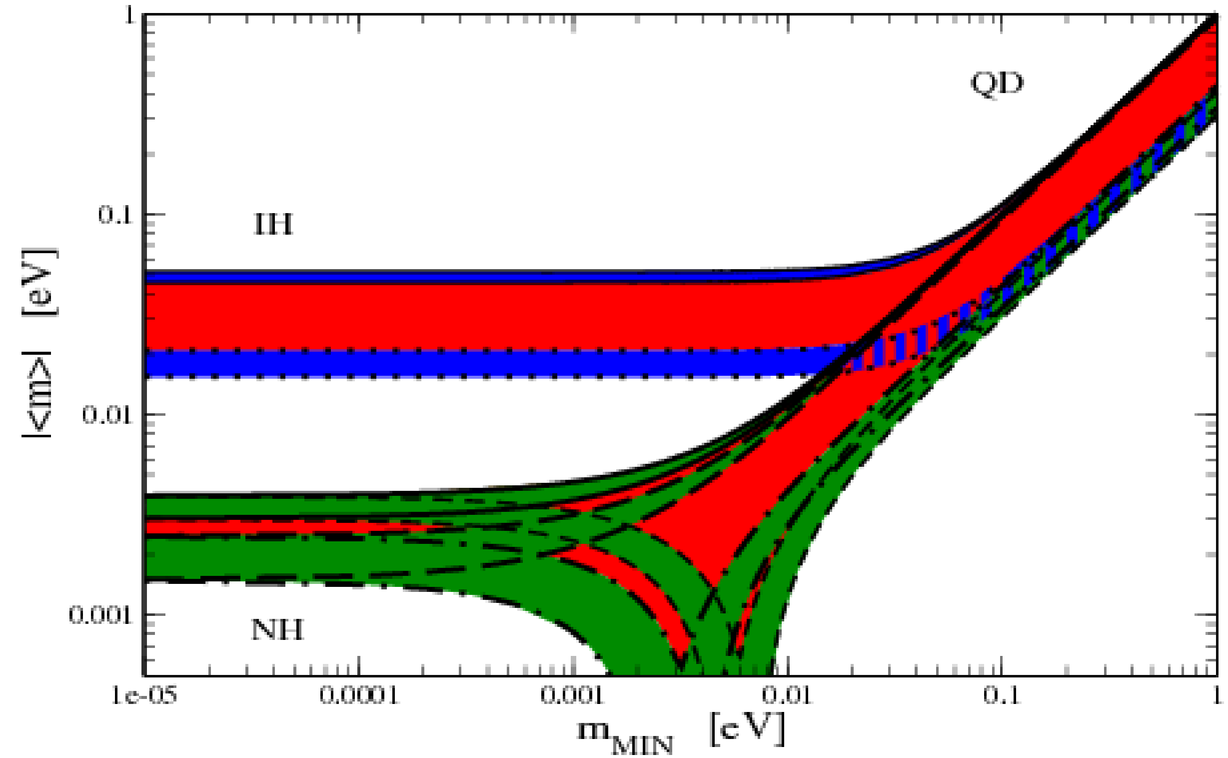
Large number of projects: $|<m>| \sim (0.01 - 0.05)$ eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
SuperNEMO,
COBRA - ^{116}Cd,
EXO - ^{136}Xe,
MAJORANA - ^{76}Ge,
MOON - ^{100}Mo,
CANDLES - ^{48}Ca,
XMASS - ^{136}Xe.



S. Pascoli, S.T.P., 2006

The current 2σ ranges of values of the parameters used.



$\sin^2\theta_{13} = 0.015 \pm 0.006$; $1\sigma(\Delta m^2_\odot) = 4\%$, $1\sigma(\sin^2\theta_\odot) = 4\%$, $1\sigma(|\Delta m^2_{\rm atm}|) = 6\%$;

$2\sigma(|<m>|)$ used.

Majorana CPV Phases and $|<m>|$

CPV can be established provided

- $|<m>|$ measured with $\Delta \lesssim 15\%$;
- $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi \lesssim 1.5$;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
UNITED KINGDOM
IRELAND
Dublin
London
Birmingham
Manchester
Liverpool
Cardiff
Celtic Sea
Bay of Biscay
FRANCE
Paris
Nantes
Bordeaux
Toulouse
Lyon
Marseille
SPAIN
Madrid
Barcelona
Valencia
Sevilla
Malaga
Bilbao
PORTUGAL
Lisbon
Porto
MOROCCO
Rabat
Casablanca
ALGERIA
Algiers
Oran
TUNISIA
Tunis
Mediterranean Sea
BALEARIC ISLANDS
Sardinia
Cagliari
Tyrrhenian Sea
Ionian Sea
Palermo
Sicily
Valletta
MALTA
GERMANY
Berlin
Hamburg
Bremen
Essen
Cologne
Frankfurt am Main
Stuttgart
Munich
Leipzig
Luxembourg
Brussels
Amsterdam
Rotterdam
POLAND
Warsaw
Gdańsk
Poznań
Wrocław
Kraków
CZECH REPUBLIC
Prague
Brno
SLOVAKIA
Bratislava
AUSTRIA
Vienna
HUNGARY
Budapest
Zagreb
Ljubljana
Sarajevo
Belgrade
Serbia
Tirana
Skopje
BULGARIA
Sofia
Varna
ROMANIA
Bucharest
Constanta
Black Sea
UKRAINE
Kiev
BELARUS
Minsk
Vilnius
GREECE
Athens
TURKEY
Istanbul
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of $\nu-$masses.

- Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986.

- In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

Instead of Conclusions

We are at the beginning of the Road...

The future of neutrino physics is bright.

Supporting Slides

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\, \Theta[(E_0 - m_i) - E]$$

Maximum β energy when there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

Presently: $\langle m_\beta \rangle < 2$ eV

Mainz & Troitzk

Due to B. Kayser