Neutrino Yukawa Textures and Consequences

Probir Roy

Saha Institute of Nuclear Physics, Kolkata,India

B.Adhikary, A. Ghosal, P. R.: JHEP **10** (2009) 040, JCAP **1101** (2011) 025

2011 BCSPINV Advanced Study Institute in Particle and Cosmology
Hue, Vietnam, 25-30 July

TALK PLAN

- Four zero textures and Motivation
- Neutrino factfile
- $\mu\tau$ symmetry their effects
- Radiative lepton Decay : $l_\alpha \to l_\beta \gamma$
- Phenomenology
- Deviations from RG running
- Conclusions from four zero Yukawa textures within type-I seesaw

Four zero textures and Motivation

Magic of four zero Yukawa textures :
Great phenomenological success in quark sector: u-type, d-type mass matrices (Fritzsch and Xing, Prog. Nucl. Part. Phys. **45** 2000, 1).
Type-I See-Saw

Lagrangian mass term : SM leptons + 3 RH N's

$$-\mathscr{L}^m = \bar{l}_L (M_\ell)_{ll'} l'_R + \bar{\nu}_{lL} (m_D)_{ll'} N_{l'R} + \frac{1}{2} \bar{N}^c_{lL} (M_R)_{ll'} N_{l'R} + h.c.$$

Neutrino mass matrix

$$-\mathscr{L}^m_\nu = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{N}^c_L \end{pmatrix}_l \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}_{ll'} \begin{pmatrix} \nu^c_R \\ N_R \end{pmatrix}_{l'} + h.c.$$

for $O(M_R) \gg O(m_D) \Longrightarrow$

$$-\mathscr{L}^m_{\nu_L} = \frac{1}{2}\bar{\nu}_{lL}(m_\nu)_{ll'}\nu^c_{l_R} + h.c.$$

$m_\nu \simeq -m_D M_R^{-1} m_D^T \quad (\mathrm{See-Saw\ formula})$.

Without loss of generality choose basis: Diagonal M_ℓ and M_R with real positive entries.
m_ν Diagonalisation:

$$U^\dagger m_\nu U^* = m_d^\nu = \mathrm{diag}(m_1,\ m_2,\ m_3)$$

m_i's Real Positive.
Now

$$\begin{aligned} m_\nu &= \tfrac{1}{2} v_u^2 Y_\nu \mathrm{diag}(M_1^{-1}, M_2^{-1}, M_3^{-1}) Y_\nu^T \\ &= U \mathrm{diag}(m_1, m_2, m_3) U^T \\ U &= U_{\mathrm{PMNS}} = \tilde{U}_{\mathrm{CKM}} U_{\mathrm{Maj}} \end{aligned}$$

Relation of flavor basis to mass basis: $(\nu_L)_l = (U)_{li}\,\nu_{iL}$

Charged current interaction in mass basis:

$$\mathcal{L}_l^{cc} = (-g/\sqrt{2})\bar{l}_{iL}\gamma^\mu U_{ij}\nu_{Lj}W_\mu^-$$

U Parametrization:

$$\begin{aligned} U &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & -s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta_D} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\frac{\alpha_{M_1}}{2}} & 0 & 0 \\ 0 & e^{i\frac{\alpha_{M_2}}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \text{PMNS} \times \text{Majorana Phase Matrix} \end{aligned}$$

$c_{ij} \equiv \cos\theta_{ij}, \quad s_{ij} \equiv \sin\theta_{ij}$

$\delta_D =$ Dirac phase

$\alpha_{M_1}, \alpha_{M_2} =$ Majorana phases

In our chosen basis :

$$M_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

and in this basis

$$m_D = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

with complex elements.

m_ν from see-saw

$$m_\nu = -m_D M_R^{-1} m_D^T = -\begin{pmatrix} \frac{a_1^2}{M_1} + \frac{a_2^2}{M_2} + \frac{a_3^2}{M_3} & \frac{a_1 b_1}{M_1} + \frac{a_2 b_2}{M_2} + \frac{a_3 b_3}{M_3} & \frac{a_1 c_1}{M_1} + \frac{a_2 c_2}{M_2} + \frac{a_3 c_3}{M_3} \\ \frac{a_1 b_1}{M_1} + \frac{a_2 b_2}{M_2} + \frac{a_3 b3}{M_3} & \frac{b_1^2}{M_1} + \frac{b_2^2}{M_2} + \frac{b_3^2}{M_3} & \frac{b_1 c_1}{M_1} + \frac{b_2 c_2}{M_2} + \frac{b_3 c_3}{M_3} \\ \frac{a_1 c_1}{M_1} + \frac{a_2 c_2}{M_2} + \frac{a_3 c_3}{M_3} & \frac{b_1 c_1}{M_1} + \frac{b_2 c_2}{M_2} + \frac{b_3 c_3}{M_3} & \frac{c_1^2}{M_1} + \frac{c_2^2}{M_2} + \frac{c_3^2}{M_3} \end{pmatrix}$$

Additional inputs:

No family of neutrino is decoupled

$m_i \neq 0$ and all M_i's are large (like 10^9 Gev or more)

$$\det m_\nu \neq 0 \neq \det m_D$$

No artificial cancellations!

Maximum number of zeros allowed in m_D with above constraints $= 4$

(G.C. Branco, D.E.-Costa, M.N. Rebelo and P. Roy, PRD 08)

All four zero textures have been classified. 72 allowed textures One important property of these:

High Scale CP Violation completely determined by the Low Energy CP Violation

72 textures categorised into two

Category **A:** 54 Textures:

- 18 Textures, First row orthogonal to second row, $(m_\nu)_{12} = (m_\nu)_{21} = 0$
- 18 Textures, First row orthogonal to third row, $(m_\nu)_{13} = (m_\nu)_{31} = 0$
- 18 Textures, Second row orthogonal to third row, $(m_\nu)_{23} = (m_\nu)_{32} = 0$

Category B: Textures with two zeros in one row and one each in the rest

- 6 Textures, First row with two zeros
- 6 Textures, Second row with two zeros
- 6 Textures, Third row with two zeros

If the rows with one zero each are k, l then det cofactor $[(m_\nu)_{kl}] = 0$

In addition for each textures in B: $(m_\nu)_{ll'} \neq 0$

Neutrino Factfile

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j$$

- $|\Delta m^2_{32}| \sim 2.4 \times 10^{-3}\mathrm{eV}^2$, $|\Delta m^2_{21}| \sim 7.6 \times 10^{-5}\mathrm{eV}^2$
- $|R| = \frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \simeq 3.2 \times 10^{-2}$
- $0.05\mathrm{eV} \leq \Sigma_i m_i \leq 0.5\mathrm{eV} \leftarrow$ cosmological bound
- $\theta_{23} \simeq \frac{\pi}{4}$, $\theta_{12} \simeq \sin^{-1}\frac{1}{\sqrt{3}}$, θ_{13} small

$\mu\tau$ symmetry and its consequences

Under $\mu\tau$ symmetry, $\mathcal{L}$ invariant under $\nu_\mu \longleftrightarrow \nu_\tau$, $N_\mu \longleftrightarrow N_\tau$.
$\Longrightarrow$

$$m_D = \begin{pmatrix} a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \\ b_1 & b_3 & b_2 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}.$$

Custodial $\mu\tau$ symmetry in m_ν:

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} = m_\nu^T$$

$$\Longrightarrow \quad h = m_\nu m_\nu^\dagger = \begin{pmatrix} P & Q & Q \\ Q^* & R & S \\ Q^* & S & R \end{pmatrix} \quad \text{is} \quad \mu\tau \quad \text{symmetric,}$$

where

$$P = |A|^2 + 2|B|^2, \qquad Q = A^*B + B^*(C + D)$$
$$R = |B|^2 + |C|^2 + |D|^2, \qquad S = |B|^2 + CD^* + DC^*.$$

Apply to four zero textures.

Category A:

•

$\mu\tau$ symmetry in Lagrangian is incompatible with only four zeros in m_D for 52 textures here

In the remaining 2 textures $\mu\tau$ symmetry can be fitted

$$m_D^{(1)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_2 & 0 \\ 0 & 0 & b_2 \end{pmatrix}, \qquad m_D^{(2)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

See-Saw formula$\Longrightarrow$

$$m_\nu = -\begin{pmatrix} a_1^2/M_1 + 2a_2^2/M_2 & a_2b_2/M_2 & a_2b_2/M_2 \\ a_2b_2/M_2 & b_2^2/M_2 & 0 \\ a_2b_2/M_2 & 0 & b_2^2/M_2 \end{pmatrix},$$

for both.

Under parametrization

$$m_3 = -\frac{b_2^2}{M_2}, \qquad \sqrt{\frac{M_2}{M_1}} \times \frac{a_1}{b_2} = k_1 e^{i(\bar{\alpha}+\bar{\alpha}')} \qquad \frac{a_2}{b_2} = k_2 e^{i\bar{\alpha}'}$$

m_ν after phase factor absorption $e^{i\bar{\alpha}'}$ by ν_e :

$$m_\nu = m_3 \begin{pmatrix} k_1^2 e^{2i\bar{\alpha}} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix}.$$

3 real parameters $k_1, \; k_2, \; \bar{\alpha}$

Further, tribimaximal mixing, i.e. $\theta_{12} = \sin^{-1}(1/\sqrt{3}) \Longrightarrow$, assumption

$$A + B = C + D$$

$$\Longrightarrow \qquad \cos\bar{\alpha} = 0 \qquad k_1 = (2k_2^2 + k_2 - 1)^{1/2}.$$

One real parameter.

Category B:

Again, 16 textures here with only four zeros in m_D are incompatible with $\mu\tau$ symmetry. Hence ruled out In remaining two textures $\mu\tau$ Symmetry imposed without ambiguity

$$m_D = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \qquad \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

See-Saw formula$\Longrightarrow$

$$m_\nu = -\begin{pmatrix} a_1^2/M_1 & a_1b_1/M_1 & a_1b_1/M_1 \\ a_1b_1/M_1 & b_1^2/M_1 + b_2^2/M_2 & b_1^2/M_1 \\ a_1b_1/M_1 & b_1^2/M_1 & b_1^2/M_1 + b_2^2/M_2 \end{pmatrix},$$

for both.

Under parametrization

$$m_3 = -\frac{b_2^2}{M_2}, \qquad \sqrt{\frac{M_2}{M_1}} \times \frac{a_1}{b_2} = l_1 e^{i\bar{\beta}'} \qquad \sqrt{\frac{M_2}{M_1}} \times \frac{b_1}{b_2} = l_2 e^{i\bar{\beta}}$$

Absorbing phase factor $e^{i\bar{\beta}'}$ in ν_e:

$$m_\nu = m_3 \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\bar{\beta}} & l_1 l_2 e^{i\bar{\beta}} \\ l_1 l_2 e^{i\bar{\beta}} & l_2^2 e^{2i\bar{\beta}} + 1 & l_2^2 e^{2i\bar{\beta}} \\ l_1 l_2 e^{i\bar{\beta}} & l_2^2 e^{2i\bar{\beta}} & l_2^2 e^{2i\bar{\beta}} + 1 \end{pmatrix}.$$

Again, 3 real parameters $l_1,\ l_2,\ \bar{\beta}$.

Now, additional TBM, $\theta_{12} \ = \ \sin^{-1}(1/\sqrt{3})$, assumption

$$A + B = C + D$$

$\longrightarrow$

$$\bar{\beta} = \cos^{-1}(l_1/4l_2),$$
$$l_2 = \frac{1}{2}(1 - l_1^2)^{1/2}.$$

One real parameter.

Parametric Functions for Category A

$$X_{1A} = 2\sqrt{2}k_2\sqrt{(2k_2^2+1)^2 + 2k_1^2(2k_2^2+1)\cos 2\bar{\alpha} + k_1^4}$$
$$X_{2A} = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\bar{\alpha}$$
$$X_{3A} = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\bar{\alpha} - 4k_2^2$$
$$X_{4A} = k_1^4 + 4k_2^4 + 4k_1^2k_2^2\cos 2\bar{\alpha}$$
$$X_{5A} = -8k_1^2k_2^4\sin 2\bar{\alpha}$$

Parametric functions for Category B

$$X_{1B} = 2\sqrt{2}k_2\sqrt{(2l_2^2+l_1^2)^2 + 2(2l_2^2+l_1^2)\cos 2\bar{\beta} + 1}$$
$$X_{2B} = 4l_2^4 - l_1^4 + 4l_2^2\cos 2\bar{\beta} + 1$$
$$X_{3B} = 1 - (2l_2^2+l_1^2)^2 - 4l_2^2\cos 2\bar{\beta}$$
$$X_{4B} = l_1^4$$
$$X_{5B} = -8l_1^2l_2^4\sin 2\bar{\beta}$$

Phenomenology

Physical Quantities

$$\tan 2\theta_{12} = \frac{X_1}{X_2}, \qquad R = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{2X}{X_3 - X}, \qquad X = \sqrt{X_1^2 + X_2^2}$$

$$m_3 = |\Delta m^2_{21} X^{-1}|^{1/2}, \quad m_{2,1} = |\Delta m^2_{21}(X^{-1} - \frac{1}{2}X_3 X^{-1} \pm \frac{1}{2})|^{1/2},$$

$$m_{\beta\beta} = |(m_\nu)_{ee}| = |\Delta m^2_{21} X_4 X^{-1}|^{1/2}$$

and Majorana Phases

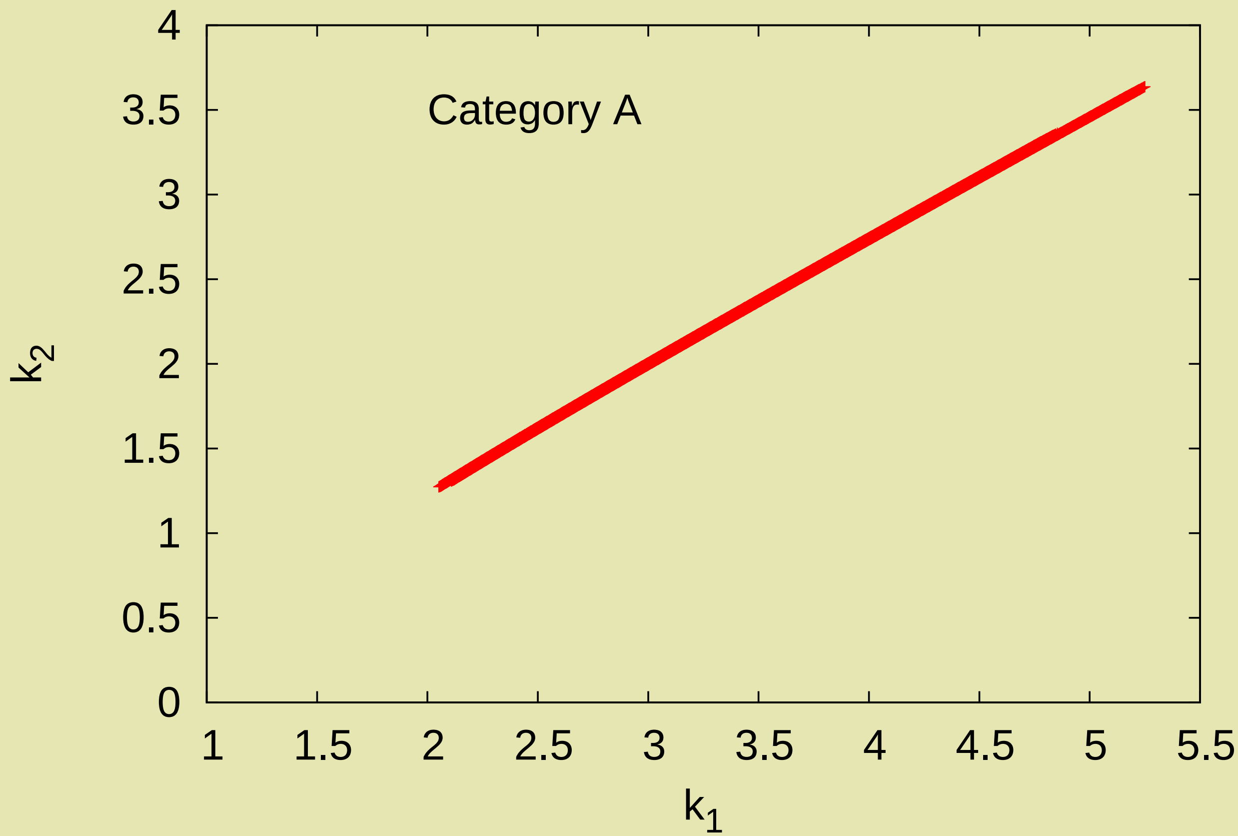
$$\cos(\alpha_{M_1} - \arg Z) = \frac{|Z|^2 m_3^2 + m_1^2 s_{12}^4 - m_2^2 c_{12}^4}{2m_1 m_3 s_{12}^2 |Z|}$$

$$\cos(\alpha_{M_2} - \arg Z) = \frac{|Z|^2 m_3^2 + m_2^2 c_{12}^4 - m_1^2 s_{12}^4}{2m_2 m_3 c_{12}^2 |Z|}$$

$$Z = \frac{(m_\nu)_{22} + (m_\nu)_{23}}{(m_\nu)_{22} - (m_\nu)_{23}}$$

Input data (M.C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP 1008:117, 2010)

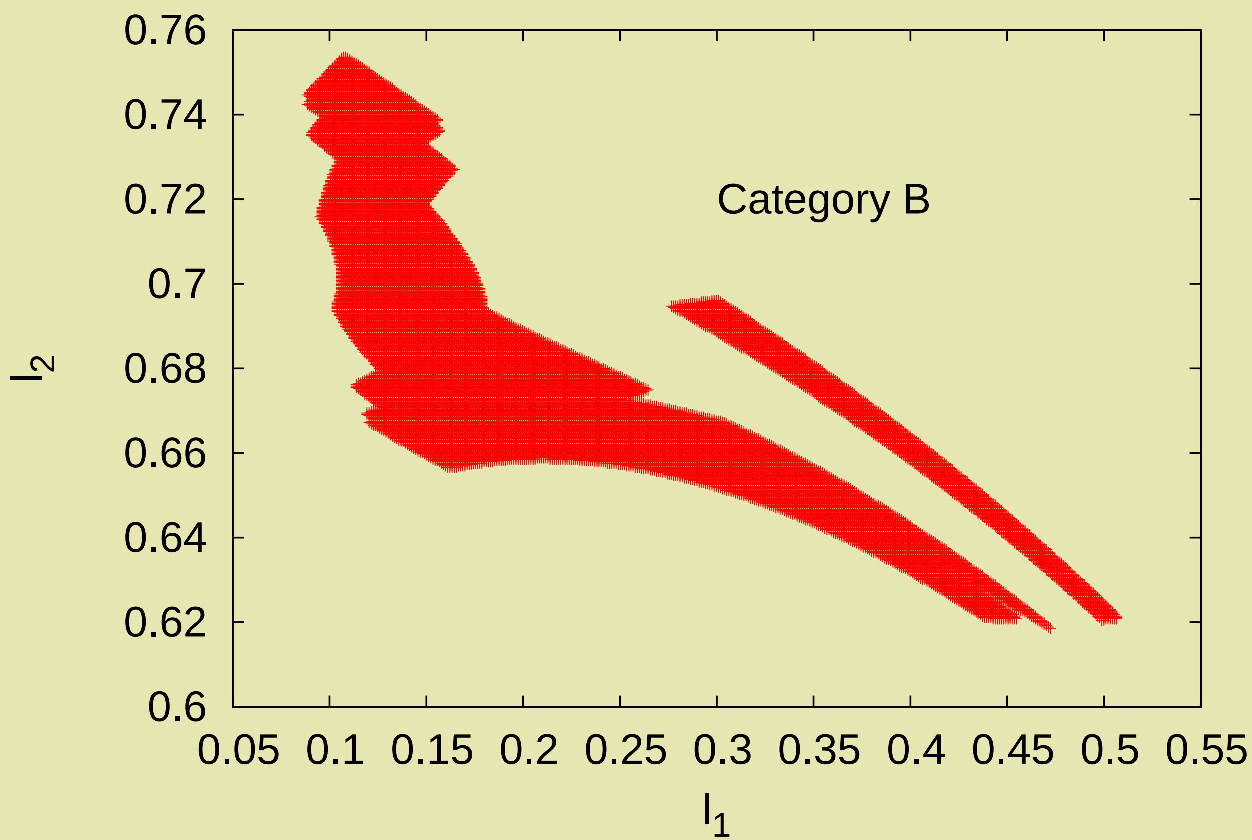
Quantity	Experimental 3σ range
θ_{12}	$31.5°$ to $37.6°$
$R_{inverted}$	-4.13×10^{-2} to -2.53×10^{-2}
R_{normal}	2.46×10^{-2} to 3.92×10^{-2}
Δm^2_{21}	$6.90 \times 10^{-5}\ eV^2$ to $8.20 \times 10^{-5}\ eV^2$
Δm^2_{32} (inverted)	$-2.73 \times 10^{-3}\ eV^2$ to $-1.99 \times 10^{-3}\ eV^2$
Δm^2_{32} (normal)	$2.09 \times 10^{-3}\ eV^2$ to $2.83 \times 10^{-3}\ eV^2$


Category A
4
3.5
3
2.5
2
1.5
1
0.5
0
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
k_2
k_1

Restrictions on parameter space

- $89^\circ \leq \bar{\alpha} \leq 90^\circ$
- $2.0 < k_1 < 5.3$
- $1.2 < k_2 < 3.7$
- Only inverted mass ordering allowed

TBM $\Longrightarrow$ $1.95 \leq k_2 \leq 1.97$ at the 3σ level


0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0.6
l_2
Category B
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
l_1

Restrictions on parameter space

- $87^\circ \leq \bar{\beta} \leq 90^\circ$
- $0.1 < l_1 < 0.55$
- $0.6 < l_2 < 0.76$
- Only normal ordered mass spectrum allowed

TBM $\Longrightarrow$ $0.11 \leq l_1 \leq 0.15$ at the 3σ level

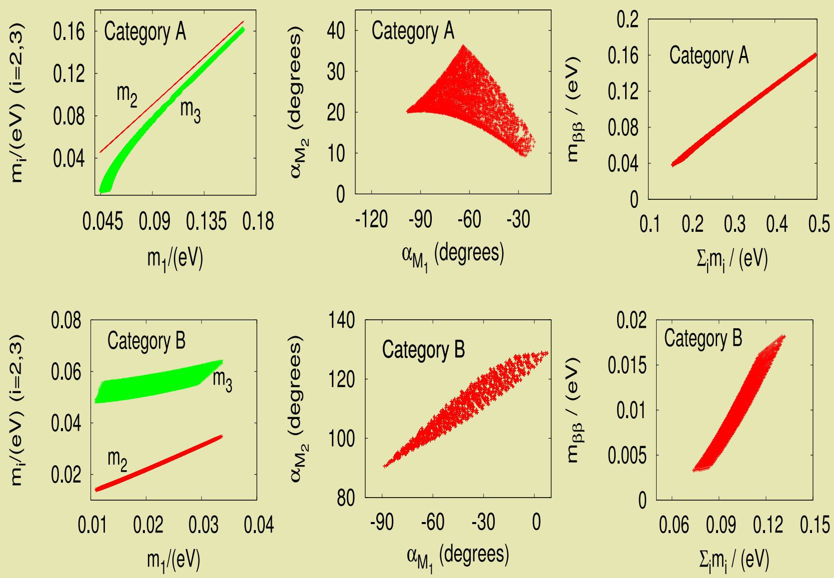
Mass ranges in our scheme

Category A	Category B
$\Sigma\, m_i = 0.092$ to 1.35 eV	$\Sigma\, m_i = 0.052$ to 0.123 eV
$m_{\beta\beta} = 0.005$ to 0.450 eV	$m_{\beta\beta} = 0.0003$ to 0.0156 eV

Latest cosmological bound $\Sigma\, m_i < 0.28$ eV removes a large part of the allowed parameter space of Category A.
S. A. Thomas, F. B. Abdalla, O. Lahav, PRL, 105, 031301, (2010)

Majorana Phases

$$-98.0^\circ \leq \alpha_{M_1} \leq 20.0^\circ \qquad 9.2^\circ \leq \alpha_{M_2} \leq 36.4^\circ \quad \text{(Category A)}$$
$$-88.6^\circ \leq \alpha_{M_1} \leq 7.97^\circ \qquad 90.7^\circ \leq \alpha_{M_2} \leq 128.8^\circ \quad \text{(Category B)}$$



Category A
m2
m3
mi/(eV) (i=2,3)
m1/(eV)
0.16
0.12
0.08
0.04
0.045
0.09
0.135
0.18
Category A
αM2 (degrees)
αM1 (degrees)
40
30
20
10
0
-120
-90
-60
-30
Category A
mββ / (eV)
Σimi / (eV)
0.2
0.16
0.12
0.08
0.04
0
0.1
0.2
0.3
0.4
0.5
Category B
m3
m2
mi/(eV) (i=2,3)
m1/(eV)
0.08
0.06
0.04
0.02
0.01
0.02
0.03
0.04
Category B
αM2 (degrees)
αM1 (degrees)
140
120
100
80
-90
-60
-30
0
Category B
mββ / (eV)
Σimi / (eV)
0.02
0.015
0.01
0.005
0
0.06
0.09
0.12
0.15

Radiative lepton decay: $l_\alpha \to l_\beta \gamma$

$$l_\alpha \to l_\beta \gamma \qquad \alpha > \beta : l_1 = e, \quad l_2 = \mu, \quad l_3 = \tau$$

mSUGRA scenarios with universal scalar masses at high scale M_X**(**$\sim 2 \times 10^{16}$**GeV)**$\Rightarrow$

$$\mathrm{BR}(l_\alpha \to l_\beta \gamma) \propto \mathrm{BR}(l_\alpha \to l_\beta \nu \bar{\nu}) \left| (m_D)_{\alpha i} (m_D)^*_{\beta i} \ln \frac{M_X}{M_i} \right|$$

M_i = mass of i-th heavy right chiral neutrino Category **A**: Allowed two textures$\rightarrow$ **(**$m_\nu)_{23} = 0 \Rightarrow$ BR**(**$\tau \to \mu\gamma = 0$**)** In both Category **A** and Category **B**: $(m_\nu)_{13} = (m_\nu)_{12} \neq 0 \Rightarrow$ BR**(**$\tau \to e\gamma$**)**$\neq 0 \neq$BR**(**$\mu \to e\gamma$**)**

$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\mathrm{BR}(\tau \to e\nu_e\bar{\nu}_e)}{\mathrm{BR}(\mu \to e\nu_\mu\bar{\nu}_\mu)} \simeq 0.178.$$

Baryogenesis via SUSY leptogenesis

Baryon asymmetry

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -\left(\frac{24 + 4n_H}{66 + 13n_H}\right)\frac{n_l - n_{\bar{l}}}{n_\gamma}.$$

For MSSM n_H = 2 and

$$\eta = -2.45\frac{n_l - n_{\bar{l}}}{n_\gamma} \simeq (5.5 - 7.0) \times 10^{-10}\ (3\sigma).$$

Sakharov conditions (1967):

- Baryon no. violation
- CP and C violation
- Out of equilibrium condition

Baryogenesis via leptogenesis (Fukugita and Yanagida 1986): Lepton no. and CP violation from Majorana nature of N_i via $N_i \to \phi_u \bar{l}_\alpha, \phi_u^\dagger l_\alpha$ interference between tree and one loop diagrams.
Sphaleronic conversion at $T \sim M_W$: $\Delta B = \Delta L$

$$\eta = -2.45 \frac{n_l - n_{\bar{l}}}{n_\gamma}.$$

$$\begin{aligned} \epsilon_i^\alpha &\equiv \frac{\Gamma(N_i \to \phi_u \bar{l}_\alpha) - \Gamma(N_i \to \phi_u^\dagger l_\alpha)}{\Sigma_\beta [\Gamma(N_i \to \phi_u \bar{l}_\beta) + \Gamma(N_i \to \phi_u^\dagger l_\beta)]} \\ &\propto \sum_{j \neq i} \left[\mathcal{I}_{ij}^\alpha f\left(\frac{M_j^2}{M_i^2}\right) + \mathcal{J}_{ij}^\alpha \left(1 - \frac{M_j^2}{M_i^2}\right)^{-1} \right] + O(M_W^2/M_i^2) \end{aligned}$$

$$\mathcal{I}_{ij}^\alpha = \mathrm{Im}(m_D^\dagger)_{j\alpha}(m_D)_{\alpha i}(m_D^\dagger m_D)_{ji}$$
$$\mathcal{J}_{ij}^\alpha = \mathrm{Im}(m_D^\dagger)_{j\alpha}(m_D)_{\alpha i}(m_D^\dagger m_D)_{ij} \to 0 \text{ (here)}$$
$$f(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\frac{1+x}{x}\right] \quad \text{(MSSM)}$$

Calculation of $\frac{n_l - n_{\bar{l}}}{n_\gamma}$ from ϵ_i^α involved.

Leptogenesis at scale M_{lowest}=min$(M_1,\ M_2,\ M_3)$.

3 possible mass hierarchical cases for N_i:
(1) NHN: $M_{lowest} = M_1 \ll M_2 = M_3$, (2) IHN: $M_{lowest} = M_2 = M_3 \ll M_1$, (3) QDN: $M_{lowest} \simeq M_1 \simeq M_2 = M_3$,

3 possible regimes:
(1) Unflavored leptogenesis:$M_{lowest}(1+\tan^2\beta)^{-1} > 10^{12}$ GeV, (2) Fully flavored leptogenesis:$M_{lowest}(1+\tan^2\beta)^{-1} < 10^9$ GeV, (3) τ flavored leptogenesis:10^9 GeV$<$ $M_{lowest}(1+\tan^2\beta)^{-1} < 10^{12}$ GeV,
Two Categories: **A** and **B**. Thus $3\times 3\times 2 = 18$ different possibilities
Sphaleron conversion $\longrightarrow$

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -2.45\frac{n_l - n_{\bar{l}}}{n_\gamma}.$$

Standard calculation using flavor dependent Boltzman equations.

Results: $\bar{\alpha}$, $\bar{\beta}$ in fourth quadrant (< 0) for NHN and QDN cases and in the first quadrant ($0 >$) for IHN in all regimes.

Unflavored Leptogenesis

Category A				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\alpha}$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\alpha} > 0$ $89.95^o - 89.99^o$	$\bar{\alpha} < 0$ $89.1^o - 89.9^o$	$\bar{\alpha} > 0$ $89.10^o - 89.99^o$
x	$10 - 10^3$	$0.001 - 0.1$	$2.0 - 9.1$	$0.1 - 0.9$
$\tan\beta$	$2 - 60$	$2 - 5$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	5.0×10^3 — 4.9×10^6	5.0×10^3 — 2.6×10^4	5×10^3 — 3.6×10^6	5.0×10^3 — 4.9×10^6

Unflavored Leptogenesis

Category B				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\beta}$	$\bar{\beta} < 0$ $88.8^o - 89.9^o$	$\bar{\beta} > 0$ $89.48^o - 89.99^o$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $89.84^o - 89.99^o$
x	$10 - 10^3$	$0.001 - 0.1$	**8.3 - 9.5**	$0.1 - 0.9$
$\tan\beta$	$2 - 8$	$2 - 12$	$2 - 60$	$2 - 10$
$\frac{M_{lowest}}{10^9 GeV}$	8.4×10^3 — 8.5×10^4	5.0×10^3 — 1.6×10^5	5.0×10^3 — 4.9×10^6	5.0×10^3 — 1.0×10^5

Fully flavored Leptogenesis

Category A				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\alpha}$	$\bar{\alpha} < 0$ $89.4^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.8^o$	$\bar{\alpha} < 0$ $89.1^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10.0$	$0.1 - 0.9$
$\tan\beta$	$25 - 60$	$22 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	67 — 3.6×10^3	4.9×10^2 — 3.6×10^3	23 — 3.60×10^3	10 — 3.6×10^3

Fully flavored Leptogenesis

Category B				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\beta}$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10$	$0.3 - 0.9$
$\tan\beta$	$16 - 60$	$24 - 60$	$6 - 60$	$7 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	2.4×10^2 — 3.6×10^3	5.7×10^2 — 3.6×10^3	0.35×10^2 — 3.6×10^3	0.49×10^2 — 3.6×10^3

τ-flavored Leptogenesis

Category A				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\alpha}$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10.0$	$0.1 - 0.9$
$\tan\beta$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	1.7×10^3 — 4.0×10^4	50 — 1.03×10^4	100 — 1.97×10^4	100 — 1.45×10^4

τ-flavored Leptogenesis

Category B				
Parameters	NHN	IHN	QDN	
			NON	ION
$\bar{\beta}$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$	$\bar{\beta} < 0$ $87.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$
x	$10 - 10^3$	$0.001 - 0.1$	$1.1 - 10.0$	$0.1 - 0.9$
$\tan\beta$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	3.25×10^2 — 2.3×10^4	6.25×10^2 — 5.0×10^4	0.37×10^2 — 2.1×10^4	0.37×10^2 — 1.6×10^5

Deviations from RG running

Have considered radiative breaking of $\mu\tau$ symmetry. Assume $\mu\tau$ symmetry at $\Lambda \sim 10^{12}$ GeV (highest M_i) One loop RG running from Λ to $\lambda \sim 10^3$ **GeV** will break it owing to charged leptons in the loop.

In MSSM using $m_\tau^2 \gg m_{e,\mu}^2$ deviation Δ_τ:

$$\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2}(\tan^2\beta + 1)\ln\left(\frac{\Lambda}{\lambda}\right)$$

Now

$$Y_\nu^\lambda = \mathrm{diag}\,(1, 1, 1 - \Delta_\tau) Y_\nu^\Lambda$$

and

$$m_\nu^\lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\Delta_\tau) \end{pmatrix} m_\nu^\Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\Delta_\tau) \end{pmatrix}$$

Have worked out $m_{1,2,3}$, $\Sigma\, m_i$ and $m_{\beta\beta}$ as well as η. Their changes are marginal.

Results for 3σ variations of R and θ_{12}:

Category A

- $0^\circ < \theta_{13} \leq 2.7^\circ$
- $36.3^\circ \leq \theta_{23} \leq 45^\circ$
- Inverted mass ordering retained

Category B

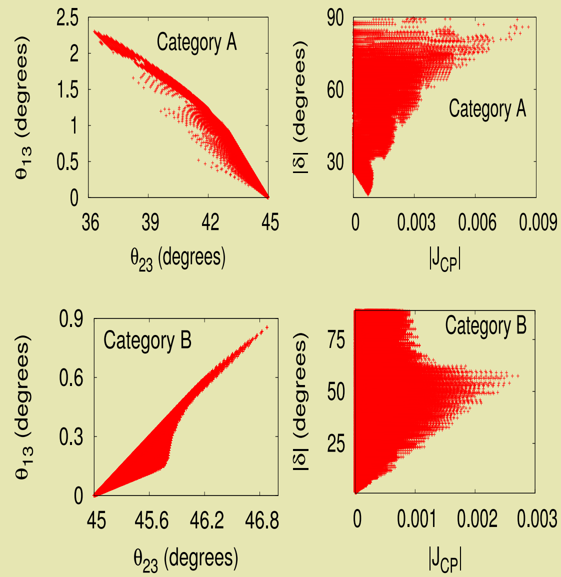
- $0^\circ < \theta_{13} \leq 0.85^\circ$
- $45^\circ \leq \theta_{23} \leq 46.8^\circ$
- Normal mass ordering retained

But measurable low energy CP violation

$$J_{CP} \simeq \frac{X_5 \Delta_\tau}{X(X_3^2 - X^2)}$$

$$|J_{CP}^A| \simeq 2.0 \times 10^{-3} \qquad |J_{CP}^B| \simeq 3.0 \times 10^{-4}$$

The same phase $\bar{\alpha}$, $\bar{\beta}$ contributes to J_{CP}, $0\nu\beta\beta$ decay and leptogenesis


Category A
2.5
2
1.5
1
0.5
0
36
39
42
45
θ13 (degrees)
θ23 (degrees)
90
60
30
0
0.003
0.006
0.009
|δ| (degrees)
|JCP|
Category B
0.9
0.6
0.3
0
45
45.6
46.2
46.8
75
50
25
0.001
0.002
0.003

More general perturbation

New result from the T2K collaboration (6 signal events with 1 estimated background event), arxiv:1106.2822.

$$4.1^o < \theta_{13}$$

for $\delta_{cp} \neq 0$ with a best fit value $\theta_{13} \sim 9^o$.

Not compatible with θ_{13} from RG evolved $\mu\tau$ symmetry breaking in our scheme.
Have to consider more general perturbation

Hypothesis : $\mu\tau$ breaking from $\nu_\tau - N_3$ contribution to Y_ν.

Now

$$Y_\nu = \mathrm{diag.}(1, 1, 1 - \epsilon)\mathrm{Y}_\nu^{\mu\tau}$$

and

$$m_\nu^\lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\epsilon) \end{pmatrix} m_\nu^{\mu\tau} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\epsilon) \end{pmatrix}$$

Proceeding as before, for $\epsilon \simeq 0.2$, we find

$$0^o < \theta_{13} < 12^o \text{ (Category A)}$$

$$0^o < \theta_{13} < 8.5^o \text{ (Category B)}$$

$$|J_{CP}^A| \simeq \mathbf{2.0} \times 10^{-2}$$

$$|J_{CP}^B| \simeq \mathbf{3.0} \times 10^{-3}$$

Conclusions on four zero Yukawa textures within type-I seesaw

- Out of 126 four zero textures $\mu\tau$ symmetry compatible with FOUR textures only leading to only two forms of m_ν: one for category A, one for category B.
- For these θ_{12} and R admit restricted regions in the parameter space and $M_\nu^{(A)}$ is in some tension with data.
- Tri-bimaximal mixing farther highly restricts the parameters.
- Small radiative deviations from $\mu\tau$ symmetry yield rather small θ_{13}, generate J_{CP} and further restrict θ_{23}.
- More general perturbative deviations can generate a larger θ_{13}.