CALIFORNIA STATE UNIVERSITY, FRESNO

Department of Physics

Introduction to Dark Energy

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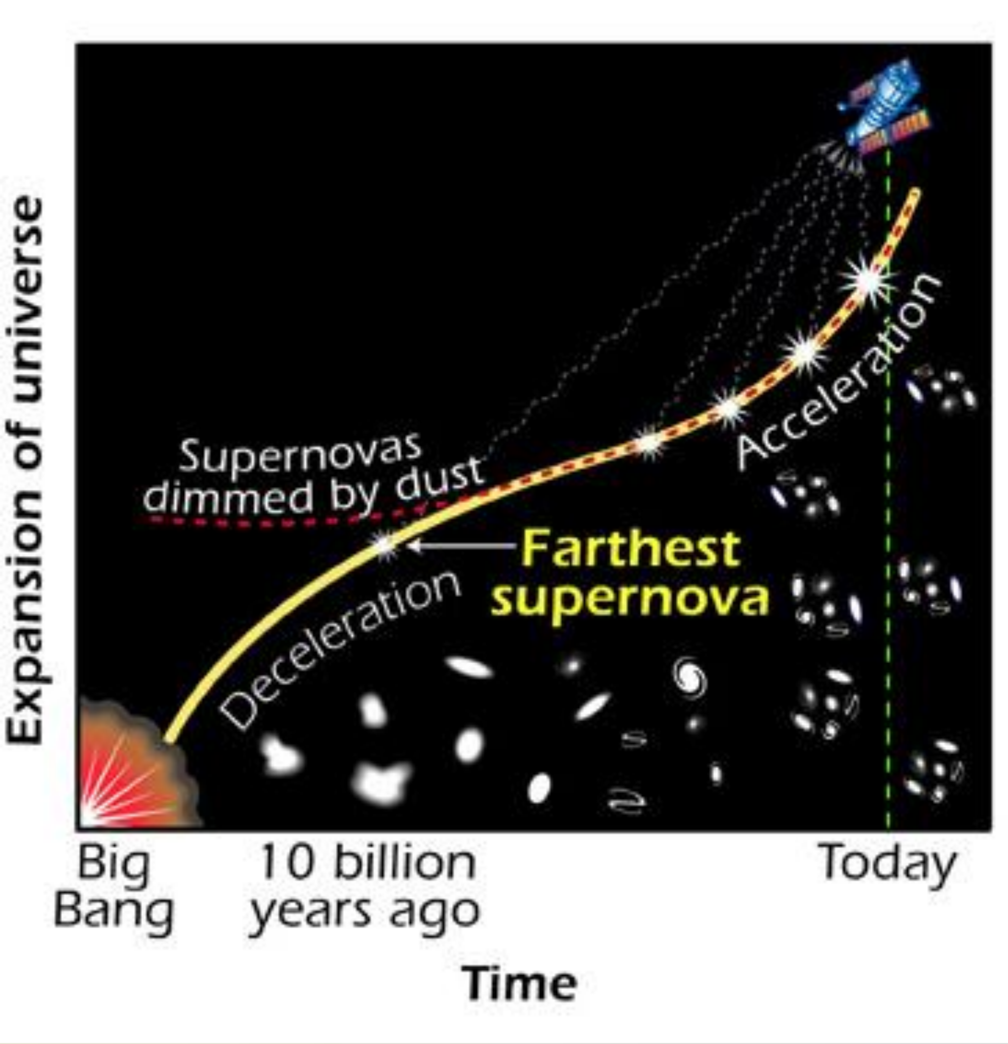
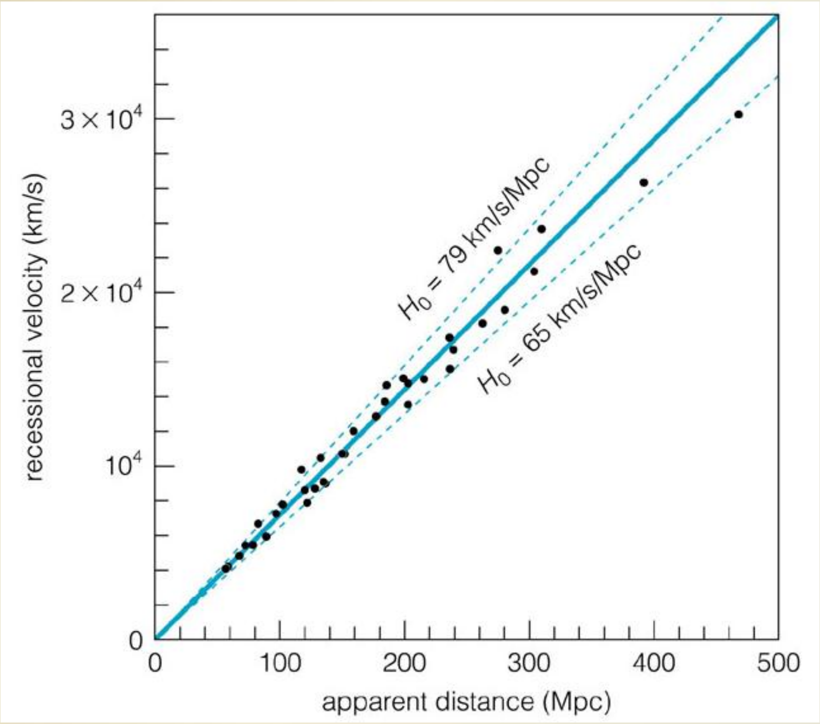
Outline of Lecture

- Description of Dark Energy
- General Relativity and Dark Energy
- Evidence for Dark Energy
- Some Models for Dark Energy
- Summary



Description of Dark Energy

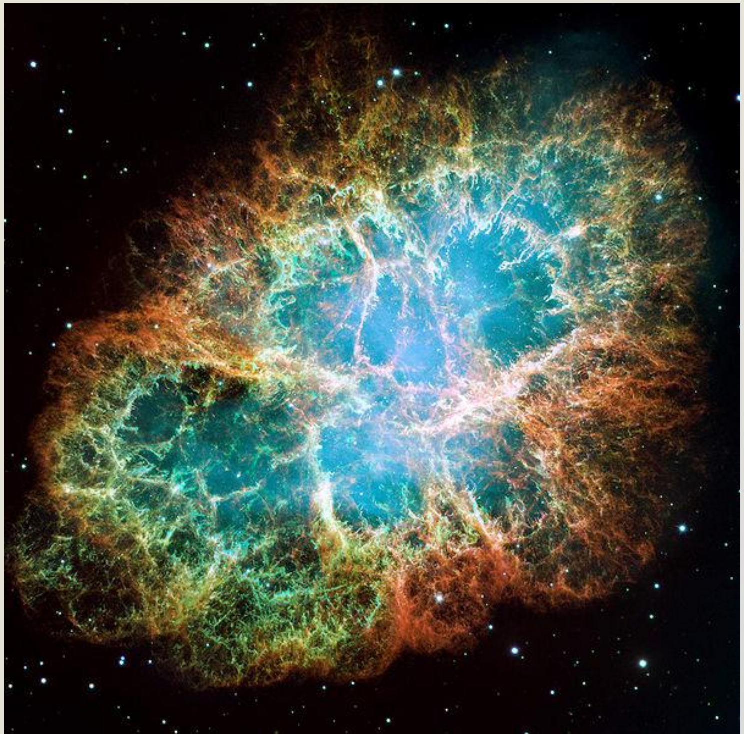
- Expanding Universe discovered by Hubble in 1929. Hubble's Law $v=H_0d$ (presently H_0 ~ 70 km/(sec-Mpc) ~2.3×10^{-18} sec^{-1}).
- Gave rise to the hot Big Bang model – expanding Universe with matter, radiation and maybe spatial curvature (k=0, ±1).
- Ordinary matter/radiation is gravitationally attractive -> expansion rate should slow down (decelerate).



Description of Dark Energy

- In 1998 two groups* set out the measure this deceleration rate using type Ia Supernova as standard candles. The expansion rate was accelerating!!
- The Universe appears to be filled with some field/fluid/stuff which gives gravitational repulsion. This “stuff” is generically called ***dark energy***.
- Dark energy appears to be very homogeneously distributed and appears to interact only via gravity.

S.J. Perlmutter et al., Astroph. J. **517, 565 (1999) and A.G. Riess et al., Astron. J. **116**, 1009 (1998).*

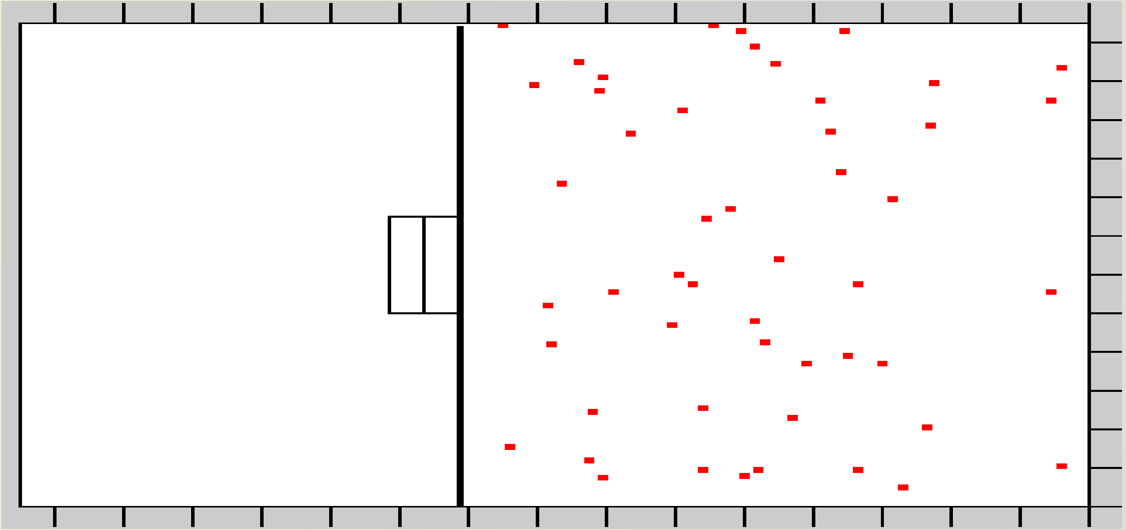


Right: Crab Nebula the remnants of the supernova in 1054

Description of Dark Energy

- To give gravitational repulsion dark energy should have negative pressure. Compressing a metal rod is positive pressure; stretching a metal rod is negative pressure/tension

- Example: a cylinder of gas with *equation of state* (EoS) $P=w\rho$, and ρ assumed fixed. Energy increase due to volume increase is $\Delta E=\rho(\Delta V)=\text{Work}=-P(\Delta V)=-w\rho(\Delta V)$. Thus need $w=-1$or if $\rho>0$ need $P<0$.



Description of Dark Energy

- In an expanding Universe most densities decrease with the size/scale factor of the Universe *a(t).*
- For matter $\rho_{matter} \sim a^{-3}$ (decrease in volume).
- For radiation $\rho_{radiation} \sim a^{-4}$ (decrease in volume + redshift).
- Vacuum energy (i.e. a cosmological constant) has $\rho_{vacuum} \sim$ *const.*
- Data give $\rho_{vacuum} \sim (10^{-12}\ \text{GeV})^4 \sim 10^{-48}\ (\text{GeV})^4 \sim 10^9\ \text{eV/m}^3 \sim 10^{-8}\ \text{erg/cm}^3$

Description of Dark Energy

- The above ρ_{vacuum} poses two problems: (i) The small size compared to (naïve) QFT predictions (ii) The coincidence problems.

- Summing over the zero modes (i.e. ½ ħω for QM SHO) in QFT fields expansion

$$\rho_{vacuum} \propto \int_0^M \sqrt{k^2+m^2}\,k^2 dk$$

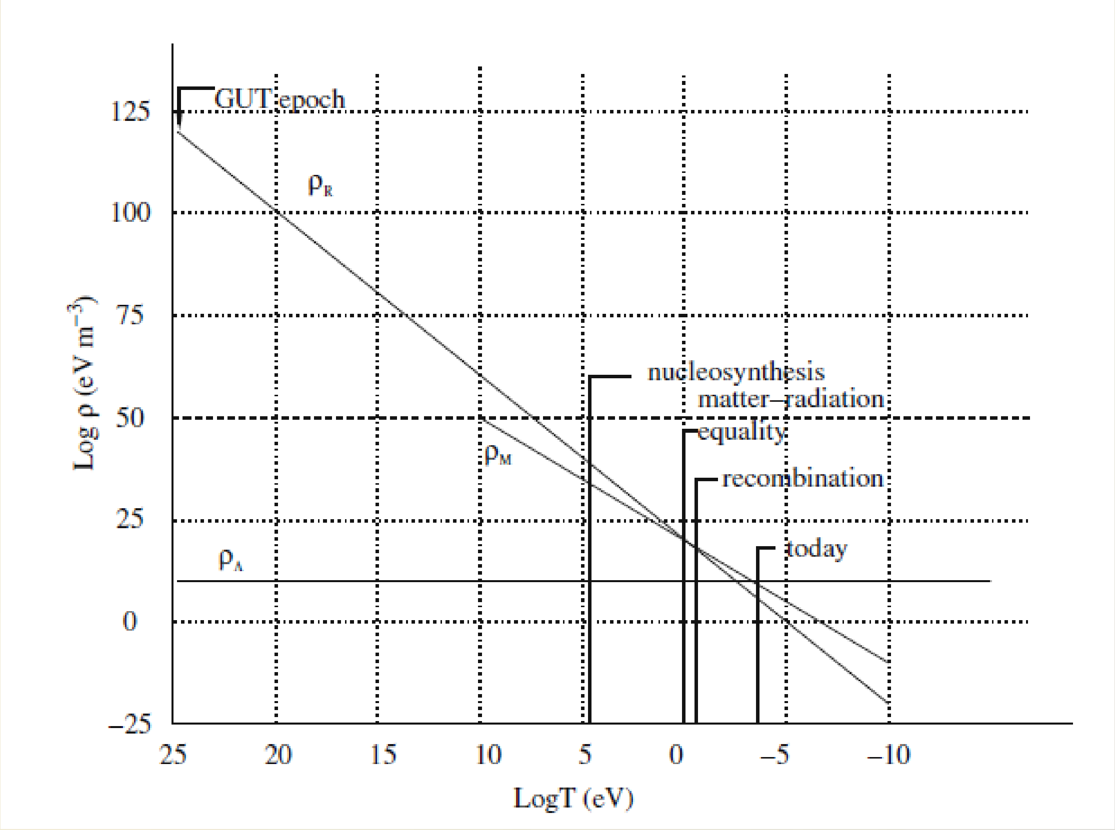
- One must cut-off at some mass scale M since the integral diverges as k^4

- $M=10^{19}$ GeV (Planck scale) → $\rho_{vacuum} \sim (10^{19}\ \text{GeV})^4 \sim 10^{76}\ (\text{GeV})^4 \sim 10^{112}\ \text{erg/cm}^3$
M=200-300 MeV (QCD scale) one finds $\rho_{vacuum} \sim 10^{-3}\ (\text{GeV})^4$

- "Only" off by 123 orders of magnitude (or 44 for QCD scale).

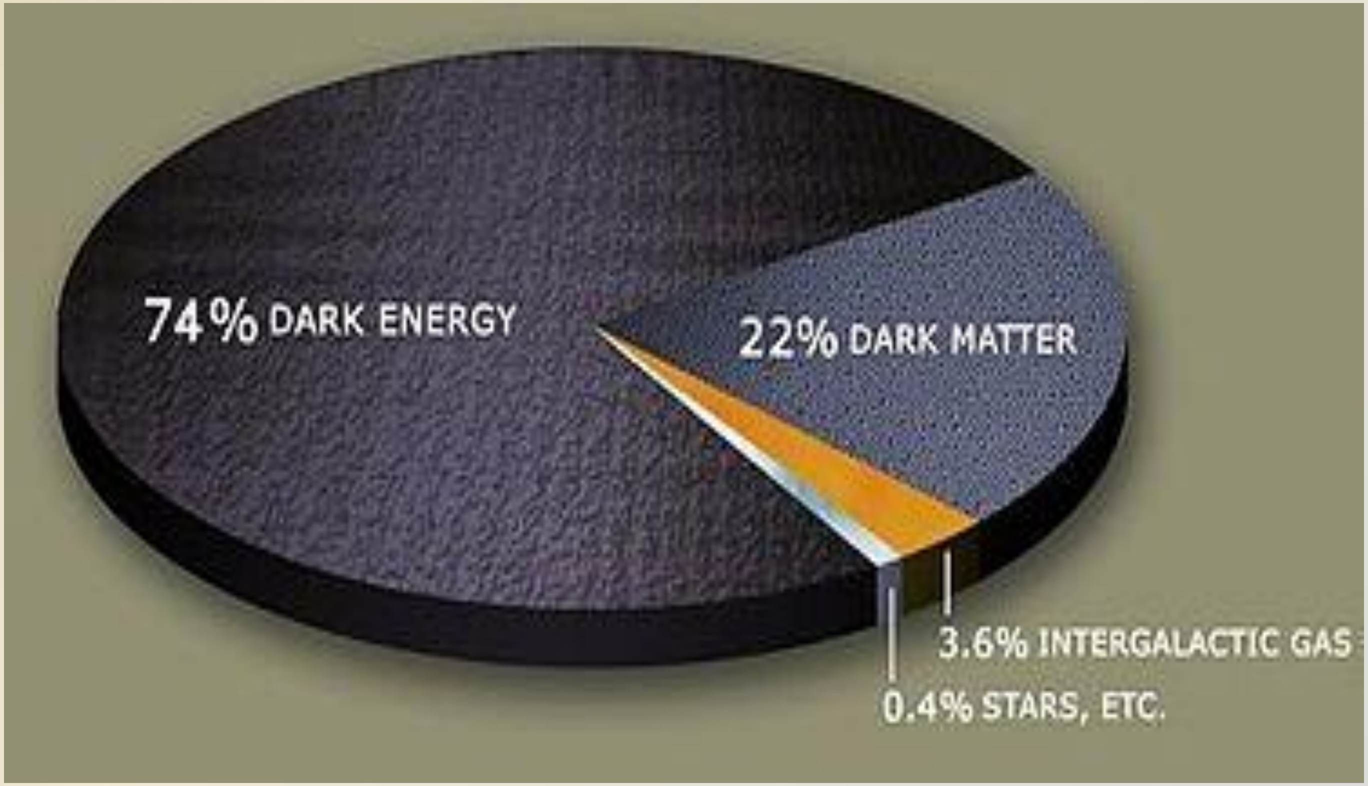
Description of Dark Energy

- The coincidence problem different densities have different evolutions with respect to *a(t).* We “happen” to live near the time when vacuum and matter densities are roughly equal. (See graph below Log(density) vs. Log (Temperature)



Description of Dark Energy

- A final oddity – "unusual" dark energy makes up most of the Universe.
- The Copernican principle pushed to the extreme.



General Relativity and Dark Energy

- Cosmological phenomenon like dark energy require general relativity (GR).

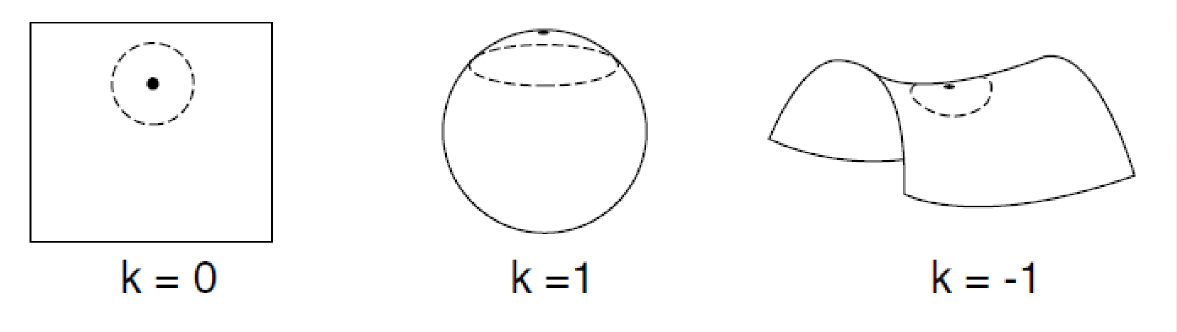
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^2} T_{\mu\nu} \qquad R_{\mu\nu} = \frac{8\pi G}{c^2}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

- Hubble's observations + Copernican ideas applied to space (not space-time) implies the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega \right]$$

- Note: Slightly different than conventional form $a(t)=R(t)/R_0$ (dimensionless scale factor); $r= R_0\, r_{FRW}$ (dimensionful distance); $\kappa=k/(R_0)^2$ (with k=0, -1, +1)*

*"An Introduction to General Relativity: Spacetime and Geometry" by Sean Carroll



General Relativity and Dark Energy

- Assume a fluid source for $T_{\mu\nu}$

$$T_{\mu\nu} = (\rho + P)U_{\mu}U_{\nu} + Pg_{\mu\nu}$$

- U^{μ} =(1,0,0,0) is 4-velocity of fluid in co-moving frame (rest frame of fluid).

- Conservation of energy using covariant derivative $\Delta_{\mu}T^{\mu}{}_{0}=0$ gives

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} = -3(1+w)H$$

- Equation of State (EoS) parameter is $w=P/\rho$ and $H=\dot{a}/a$ is Hubble Parameter

General Relativity and Dark Energy

- If we assume w=const. the EoS can be integrated as

$$\rho \propto a^{-3(1+w)}$$

- Assuming energy condition (Null Dominant) for a stable vacuum → $|w| \leq 1$.
- For dust, P(dust)=0 ; w=0 ; ρ(dust) $\sim a^{-3}$
- For radiation, P(rad)=ρ(rad)/3 ; w=1/3 ; ρ(rad) $\sim a^{-4}$
- For vacuum energy/Λ, P(Λ)=-ρ(Λ) ; w=-1 ; ρ(Λ) $\sim a^{0}$
- The coincidence problem ρ(Λ)/ρ(dust) $\sim a^{3}$. Today this ratio is of order 1

General Relativity and Dark Energy

- Two Friedmann equations coming from GR field equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = -\frac{4\pi G}{3}\rho(1+3w)$$

- 1^{st} equation → Newtonian energy conservation in 1/r potential; 2^{nd} equation Newtonian F=ma for $1/r^2$ force field.
- 2^{nd} equation implies deceleration unless w<-1/3. Dark energy must have an EoS with w<-1/3
- Observation of Cosmic Microwave background anisotropy indicate κ~0 (spatially flat Universe)

General Relativity and Dark Energy

- Now some standard definitions

- Density parameter $\Omega = 8\pi G\rho / 3H^2 = \rho / \rho_{crit}$ from 1st Friedmann equation this tells us if Universe is open (Ω<1, κ<0) ; flat (Ω=1, κ=0); closed (Ω>1, κ>0). CMB anisotropy indicates spatially flat.

- Deceleration parameter $q = -a\ddot{a} / \dot{a}^2$

General Relativity and Dark Energy

- Deceleration parameter can be used to write 2nd Friedmann equation for a multi-component source (last equation assumes $\Omega_{matter} + \Omega_{DE} = 1$)

$$q = -\frac{\ddot{a}}{aH^2} = \sum_i \frac{4\pi G\rho_i}{3H^2}(1+3w_i) = \frac{1+3w_{DE}\Omega_{DE}}{2}$$

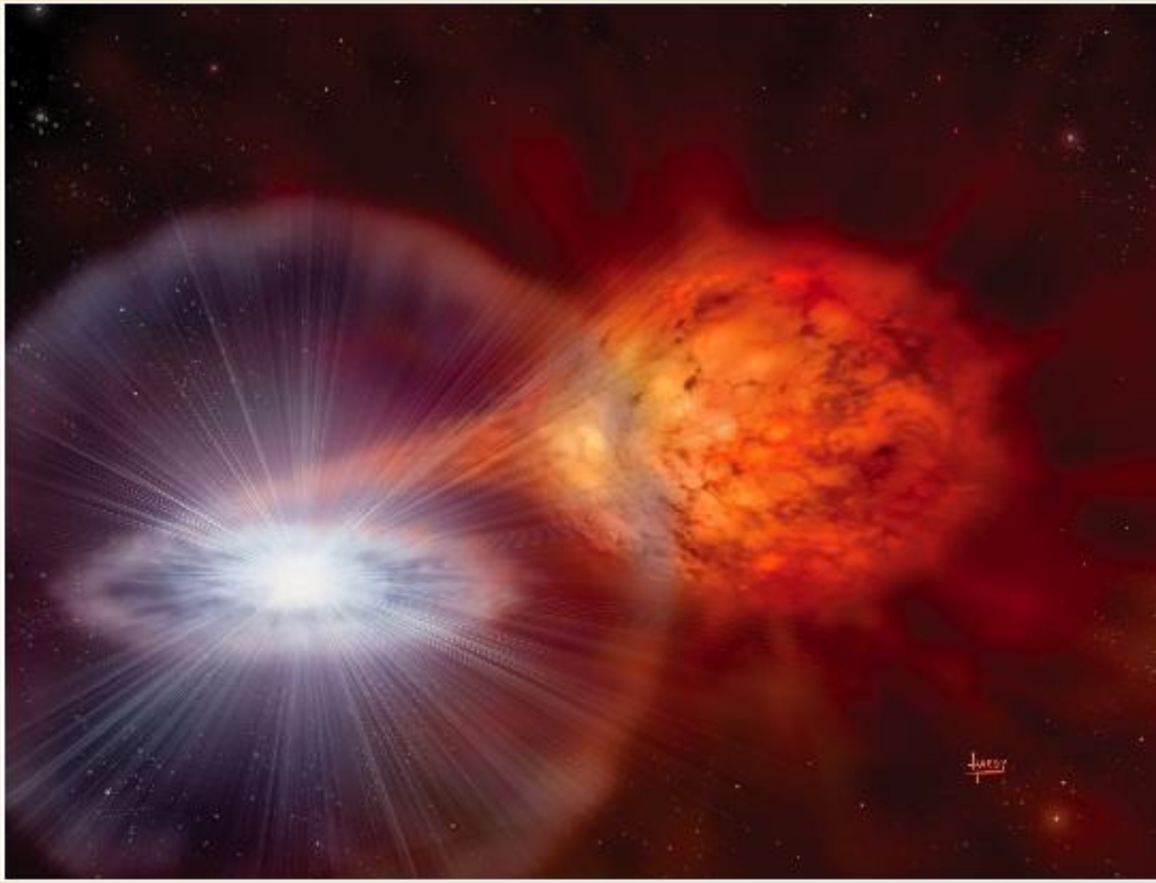
- To get acceleration ($q<0$) one needs $w_{DE} < -1/(3[1-\Omega_{matter}])$ i.e. $w_{DE} < -1/3$ for $\Omega_{matter} = 0$; $w_{DE} < -1/2$ for $\Omega_{matter} = 1/3$;

Evidence for Dark Energy

- Evidence for dark energy first came from observation of type Ia supernova

- White dwarfs accreting mass from a companion star until they are pushed over the Chandrasekhar limit.

- Very bright and very similar. They make good "standard candles".



Evidence for Dark Energy

- Two groups* measured SN Ia brightness (light flux) versus time from present (redshift z).

$$Flux = \frac{\ell}{4\pi (d_L)^2} \qquad z = \frac{\lambda_{observed} - \lambda_{emit}}{\lambda_{emit}} = \frac{a_{observed}}{a_{emit}} - 1$$

- Absolute brightness = ℓ and d_L is luminosity distance. Note $\omega_o / \omega_e = a_e / a_o$

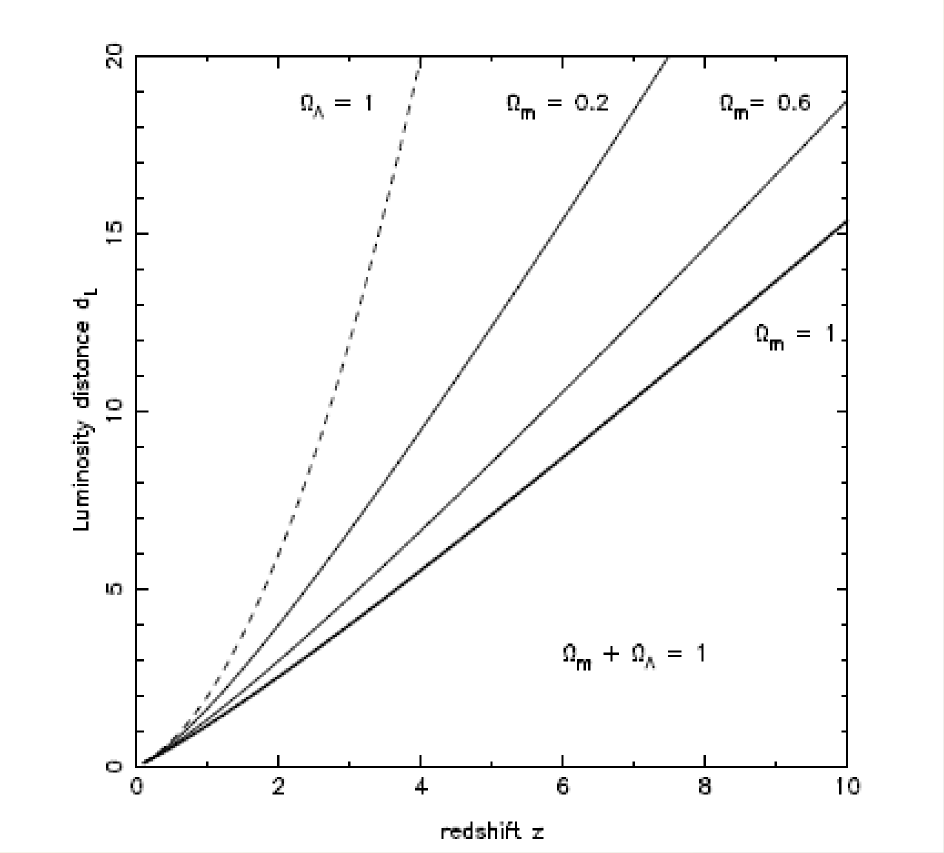
- For an expanding FRW metric d_L is $$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

with $$H(z) = H_0 \left| \Omega_{0m}(1+z)^3 + \Omega_{0DE}(1+z)^{3(1+w)} \right|^{1/2}$$

S.J. Perlmutter et al., Astroph. J.* ***517**, 565 (1999) and*
A.G. Riess et al., Astron. J. ***116****, 1009 (1998).*

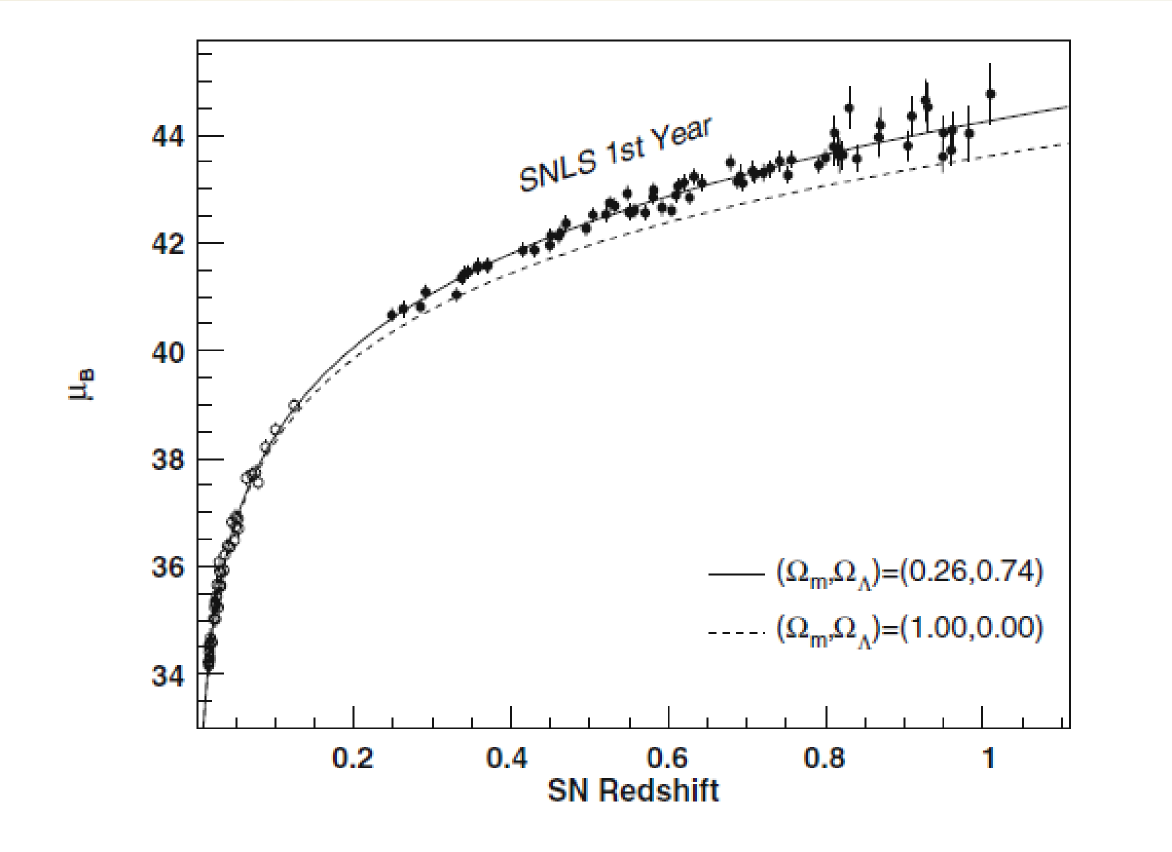
Evidence for Dark Energy

- Some sample plots of d_L versus z for various splits between matter and dark energy



Evidence for Dark Energy

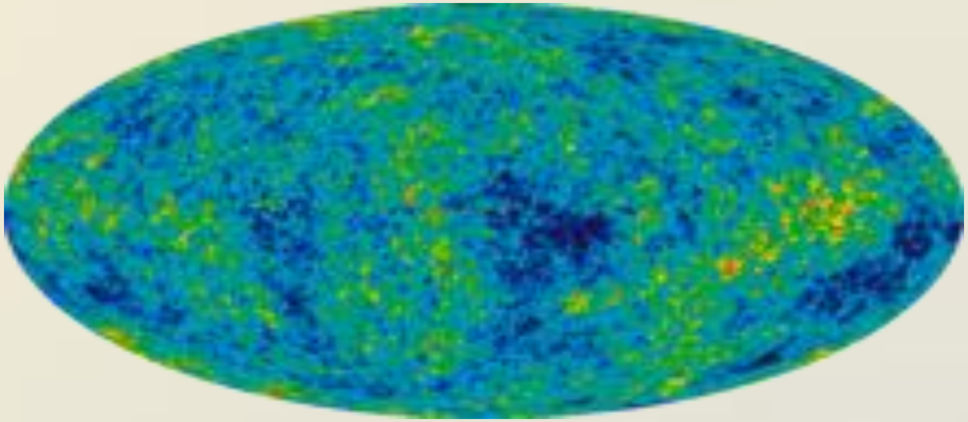
- Actual data from Super Nova Legacy Survey* $\mu_B = 5 \log(d_L / 10\ \text{pc})$



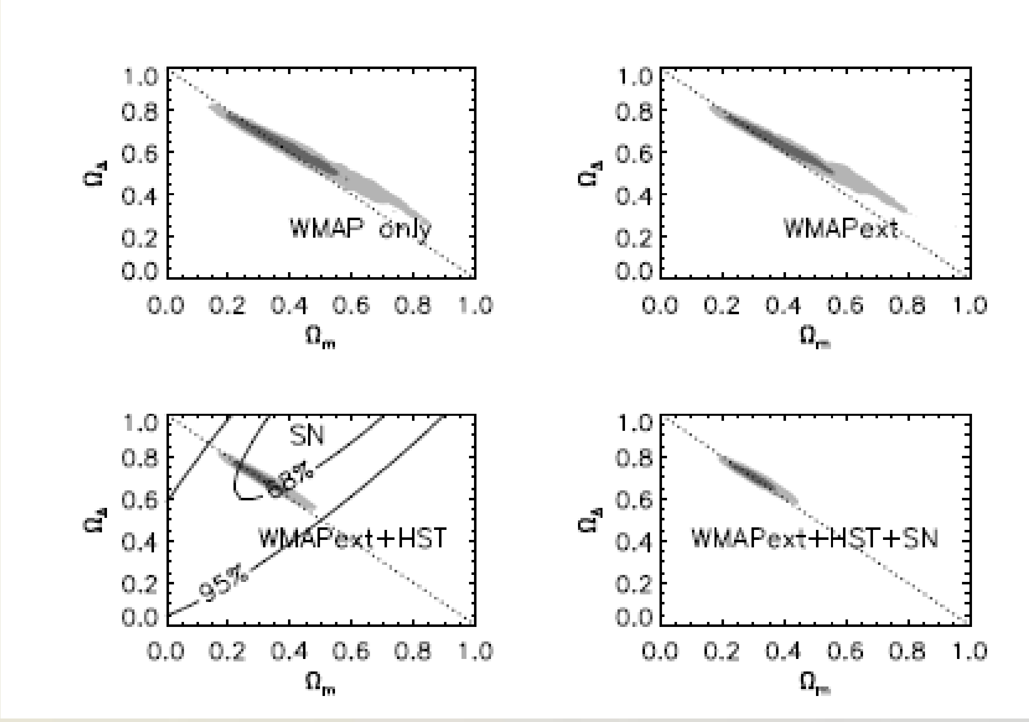
* P. Astier et al. (SNLS) Astron. Astrophys. J. Supp. **180**, 330 (2006)

Evidence for Dark Energy

- Study of the cosmic microwave background anisotropy (WMAP, Boomerang, Maxima) indicates k~0 or $\Omega_{matter} + \Omega_{DE}$ ~1.



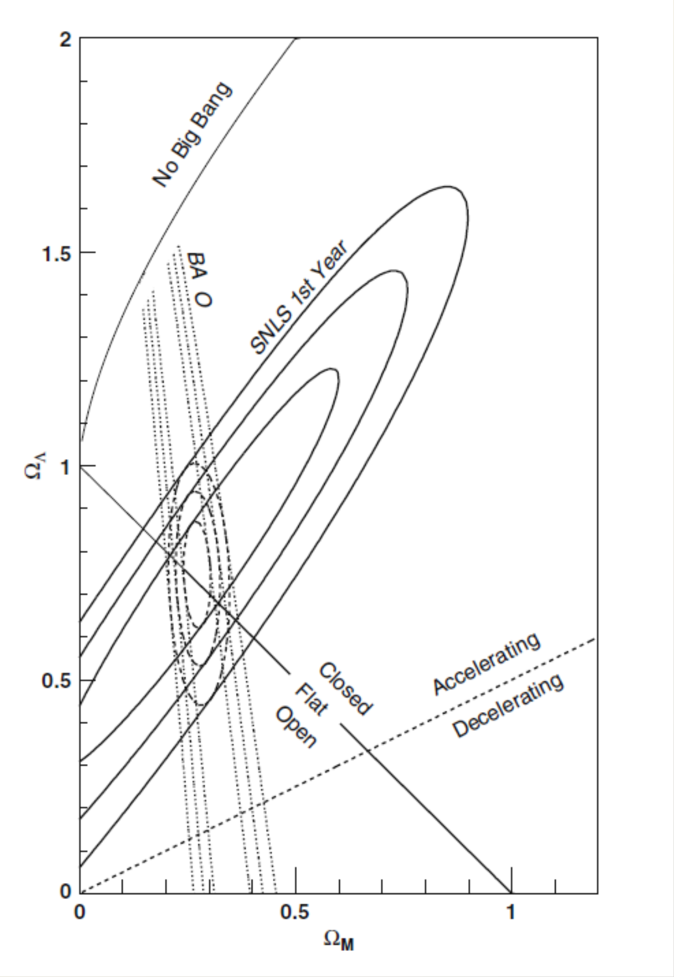
- SN data is "orthogonal" to CMB measurements which picks out a spot in the (Ω_{matter} , Ω_{DE}) plane



Evidence for Dark Energy

- Baryon acoustic oscillations (BAO) and large scale structure give an independent indications that Ω_m~0.3

- Overall conclusion: A spatially flat Universe (k~0) with $\Omega_m = \Omega_{baryon} + \Omega_{dark\text{-}matter}$~0.3 and Ω_{DE}~0.7



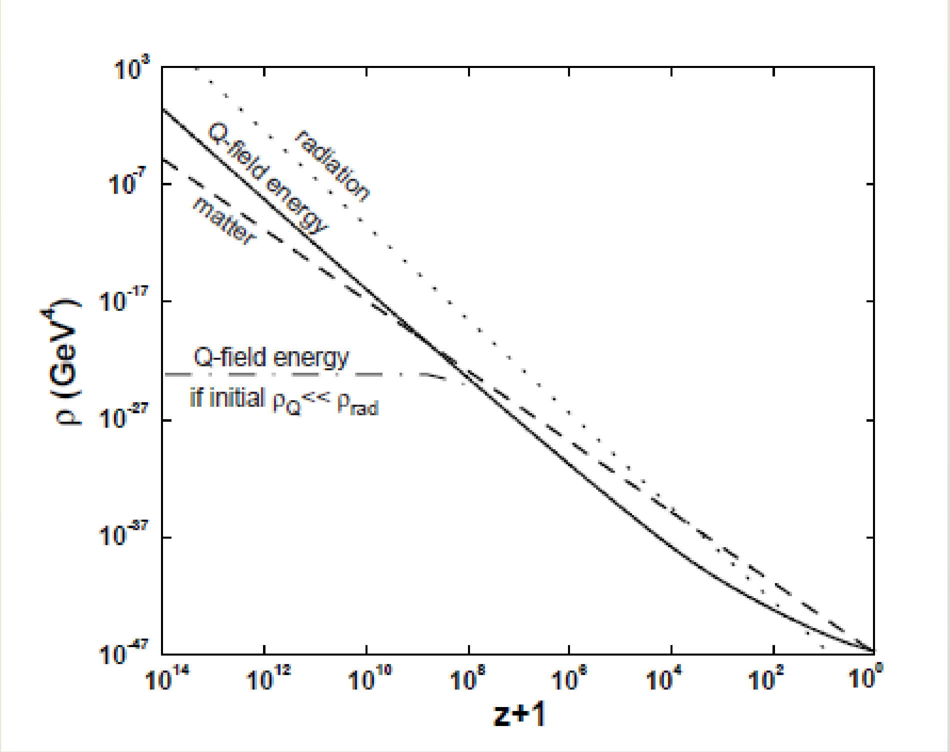
Some Models of Dark Energy

- To address coincidence, hierarchy (too fine-tuned and too small Λ) alternative models have been proposed.

- Quintessence models* postulate a spatially homogeneous scalar field

$$L = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Tuning the potential (e.g. $V \sim 1/\phi^a$) can give a quintessence field which tracks the dominate energy density (radiation or matter)



*I. Zlatev, L. Wang, and P Steinhardt, PRL **82**, 896 (1999). Named after the pervasive fifth element of Greek science/philosophy.

Some Models of Dark Energy

- Higher dimensional models such as DDG* which have 5D actions like

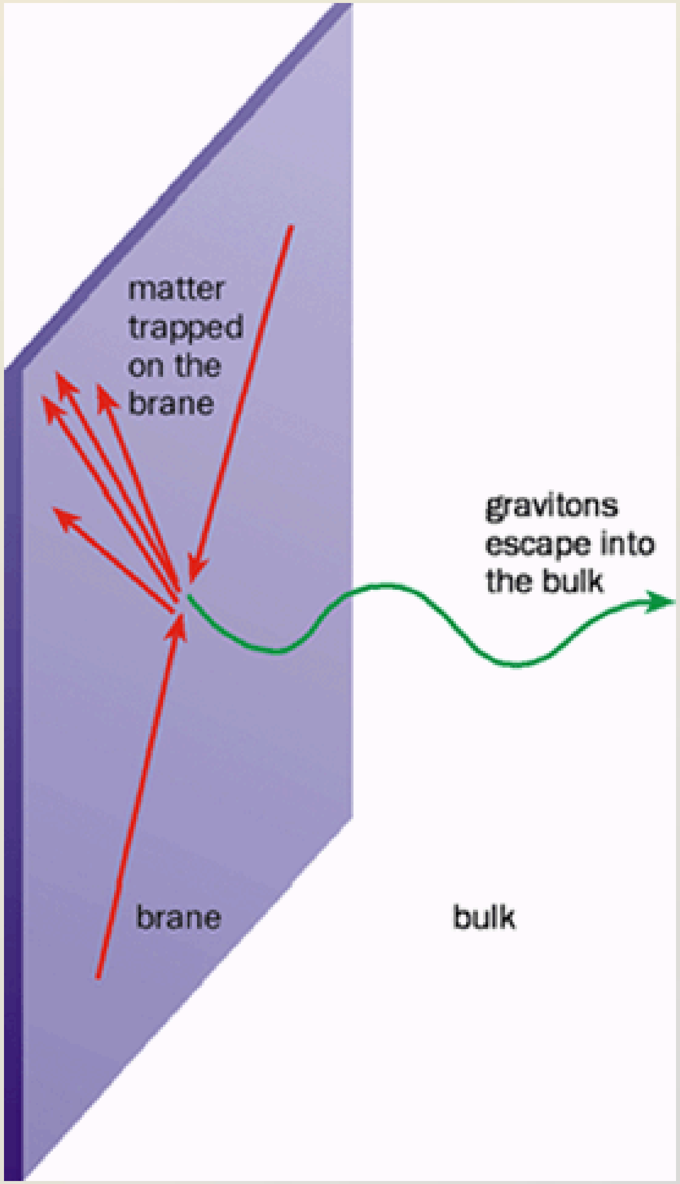
$$Action = (M_5)^3 \int_{bulk} R_5 + (m_4)^2 \int_{brane} R_4 + \int_{brane} L_{matter}$$

- These models produce a Hubble parameter

$$H = \sqrt{\frac{8\pi G\rho_m}{3} + \frac{1}{l_c^2}} + \frac{1}{l_c}$$

- At late times H→2/ℓ_c → a(t)~exp(2/ℓ_c) i.e. de Sitter type accelerated expansion

*C. Deffayet, G. Dvali, G. Gabadadze, PRD **65**, 044023 (2002); C. Deffayet et al. PRD **66**, 024019 (2002)



Some Models of Dark Energy

- Chaplygin gas model* assumes an EoS $P_c = -A/\rho_c$

- Solving energy conservation dE=-PdV → $d(\rho_c a^3) = -P_c\, d(a^3)$ gives

$$\rho_c = \sqrt{A + \frac{B}{a^6}}$$

- At early times (a<<1) Chaplygin gas behaves like dust ($\sim a^{-3}$); at late times (a>>1) it behaves like vacuum energy ($\rho_c = -P_c = \sqrt{A}$).

- Chaplygin gas models can be obtained from field theory models (quintessence and k-essence models)

* A. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001)

Some Models of Dark Energy

- Some analysis* of SN Ia data indicate one could have w<-1 which violates the Weak Energy Condition.

- This led Caldwell to propose **phantom energy****

$$L = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad \rho = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- We find $w=P/\rho<-1$ if → $0<V$ and $d\phi/dt<(2V)^{1/2}$

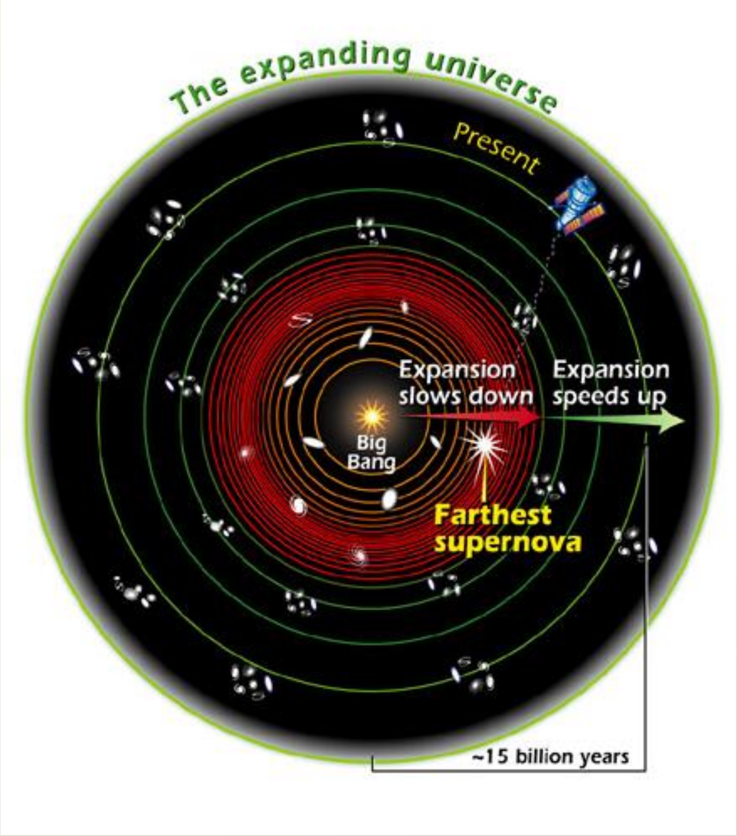
- Such a theory should have an unstable vacuum but strange observations may require strange theories.

* R. Caldwell Phys. Lett. B **545**, 23 (2002); R. Knop, et al. astro-ph/0309368

** After the Star Wars Episode I movie "Phantom Menace"

Conclusions

- Observations of type Ia Supernova indicate that the expansion rate of the universe is accelerating.
- Other observations indicate that Universe is approximately spatially flat
- The cause of this accelerated expansion → dark energy a fluid/field with large negative pressure.
- This mysterious dark energy makes up ~70% of the stuff in the Universe.
- The remaining ~ 30% is split between "ordinary" matter (~5%) and dark matter (25%)
- The nature of dark energy is one of the biggest puzzles current physics.



3-Geometry=Destiny

- In a Universe were Λ=0 (i.e. no vacuum energy) the destiny of the Universe is fixed by the density parameter Ω_m (here S(t) =a(t); time in units of $(H_0)^{-1}$