Other issues:

① Anomaly cancellation?

OK for SM.

Since we have not added any new exotic chiral fermions when embedding into $\overline{5} \oplus 10$

$\Rightarrow$ anomaly cancellation is still OK ✓

[anomaly contribution from $\overline{5}$ exactly cancels anomaly contribution from 10 ✓]

② overall hypercharge Y normalization finally fixed because it is now joined with SU(2), SU(3) non-abelian generators to fit into a single SU(5) non-abelian group!

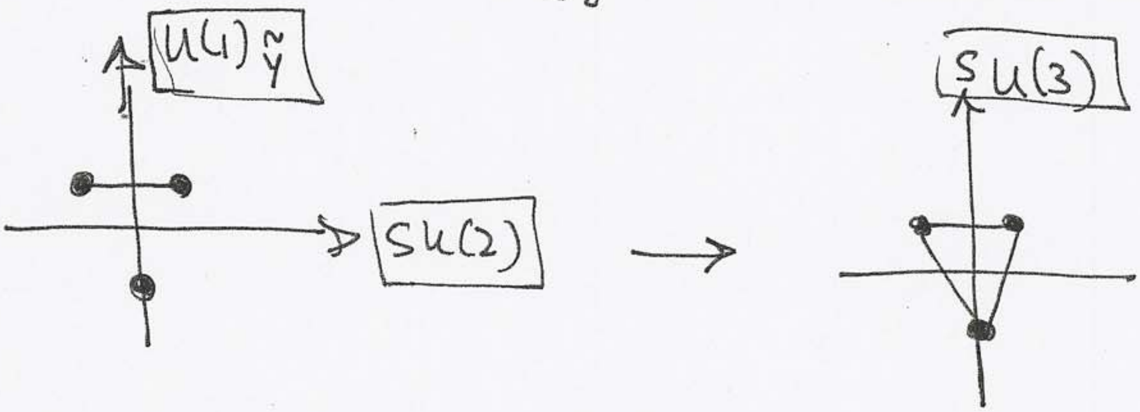
What rescaling is needed in order to accomplish this unification?

Our method

Think of the

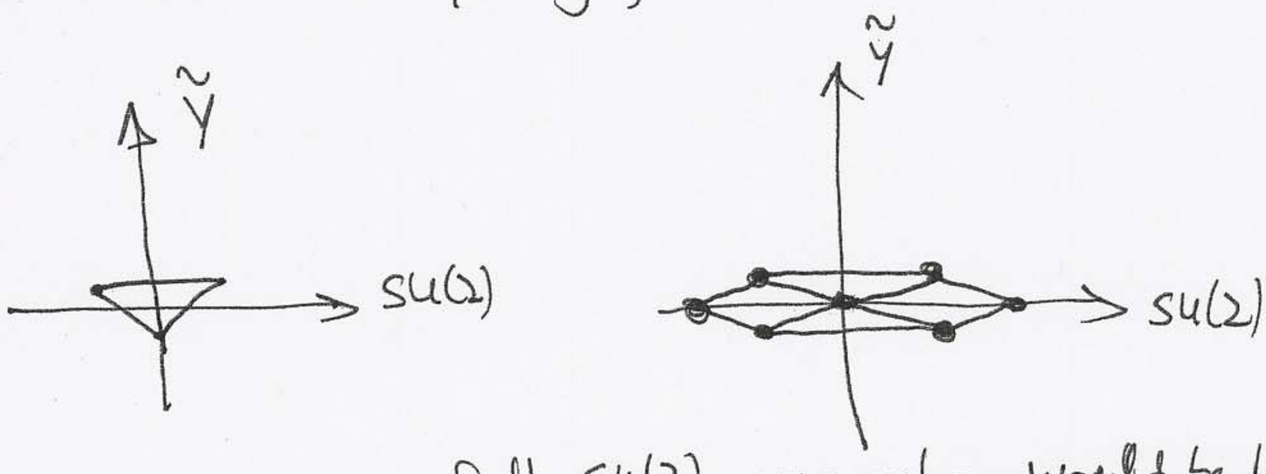
$$SU(2) \otimes U(1)_{\tilde{Y}} \subset SU(3)$$

analogy we discussed earlier



only when $U(1)_{\tilde{Y}}$ scale for vertical axis is correct relative to SU(2) scale do we get a proper SU(3) representation (equilateral triangle) !

With wrong normalization for $U(1)_Y$ vertical axis, we would obtain, e.g.,



full SU(3) symmetry would be lost !

MUST CHOOSE HYPERCHARGE NORMALIZATION RELATIVE TO SU(2), SU(3) normalizations SO THAT ALL GAUGE BOSON ROOTS HAVE SAME LENGTH [AS IN SU(5)] !

Recall normalizations in SM:

$$SU(3): \quad D_\mu = \partial_\mu - i g_3 \left(\frac{\lambda^a}{2}\right) A^a_\mu$$

$$SU(2): \quad D_\mu = \partial_\mu - i g_2 \left(\frac{\tau^a}{2}\right) A^a_\mu$$

$$U(1)_Y: \quad D_\mu = \partial_\mu - i g \left(\frac{Y}{2}\right) B_\mu$$

Thus, in SM, $\left(\frac{Y}{2}\right)$ is like $\left(\frac{\lambda^a}{2}\right), \left(\frac{\tau^a}{2}\right)$

Look at SU(2) generators

$$[\tau^i, \tau^j] = 2i\epsilon_{ijk}\tau^k \Rightarrow [J^i, J^j] = i\epsilon^{ijk} J^k$$

where $J^i = \frac{\tau^i}{2}$

$$\Rightarrow [J^z, J^\pm] = \pm J^\pm$$

so $J^\pm$ raises/lowers by <u>one</u> unit.

$\Rightarrow$ SU(2) roots have length $= 1$ in SM!

<u>SAME for SU(3)</u>: these roots also have length $= 1$ in SM!

* We therefore need to choose our normalization for $U(1)_Y$ such that <u>all*</u> of the SU(5) gauge bosons correspond to roots of length $= 1$.

(* not including the <u>diagonal</u> generators, of course)

Using the SM normalization for Y_{SM},
we already saw

$$24 \longrightarrow \underbrace{(8,1)_0 + (1,3)_0 + (1,1)_0}_{\text{SM gauge bosons: each has length}=1} + (3,2)_{-\frac{5}{3}} + (\bar{3},2)_{5/3}$$

$(3,2)_{-\frac{5}{3}}$ and $(\bar{3},2)_{5/3}$: ⇓ THESE MUST ALSO HAVE LENGTH $= 1$.

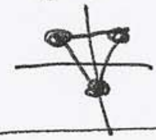
These states "point" non-trivially
in color, weak, and hypercharge directions simultaneously.
These directions are orthogonal.
Thus

[$\frac{Y}{2}$ is generator]

$$\begin{pmatrix}\text{total}\\ \text{length}\end{pmatrix}^2 = \begin{pmatrix}\text{length}\\ \text{of SU(3)}\\ \text{triplet}\end{pmatrix}^2 + \begin{pmatrix}\text{length}\\ \text{of SU(2)}\\ \text{doublet}\end{pmatrix}^2 + c^2\left(\frac{5}{6}\right)^2 \stackrel{\text{set}}{=} 1$$

c ↑ rescaling factor

SU(3) triplet: $(\text{length})^2 = 1/3$



SU(2) doublet: $(\text{length})^2 = 1/4$

thus $\frac{1}{3} + \frac{1}{4} + c^2\left(\frac{5}{6}\right)^2 = 1 \implies \boxed{c = \sqrt{3/5}}$

required rescaling
factor for U(1) Y !

Thus $Y_{SU(5)} = \sqrt{\frac{3}{5}}\, Y_{SM}$

However, since we must preserve strength of observed interactions

$$\left[D_\mu = \partial_\mu + i \frac{g_Y Y}{2} B_\mu \right] \Longrightarrow \text{product } (g_Y Y) \text{ must be preserved!}$$

$$\Longrightarrow \boxed{g_Y^{SU(5)} = \sqrt{\frac{5}{3}}\, g_Y^{(SM)}}$$

$$\Longrightarrow \boxed{\alpha_Y^{SU(5)} = \frac{5}{3}\, \alpha_Y^{(SM)}}$$

In general, $\left\{ \begin{array}{l} \alpha_Y^{SU(5)} \text{ is renamed } \alpha_1 \\ g_Y^{SU(5)} \text{ is renamed } g_1 \end{array} \right.$

<u>So GUT unification into a single GUT group such as SU(5)</u>

requires all generators to act with a <u>common coupling</u>:

$$g_5 \equiv \left(g_3 = g_2 = g_1 = \sqrt{\frac{5}{3}}\, g_Y \right) \quad *$$

or $$\alpha_5 \equiv \left(\alpha_3 = \alpha_2 = \alpha_1 = \frac{5}{3} \alpha_Y \right) \quad *$$

A priori, this does not fix the overall value of the coupling

BUT it does fix the ratios between couplings!

i.e.,

GUT PREDICTION!

$$\boxed{\frac{g_3}{g_2} = 1} \quad \text{and} \quad \boxed{\sin^2\theta_W = \frac{g_Y^2}{g_2^2+g_Y^2} = \frac{3}{8} \approx 0.375}$$

But recall

$\alpha_3^{-1} \approx 8.5$

$\alpha_2^{-1} \approx 29.6$

$\alpha_Y^{-1} \approx 98.3 \Rightarrow \alpha_1^{-1} \approx 59.0$

at Z scale

couplings are not equal!

e.g.,

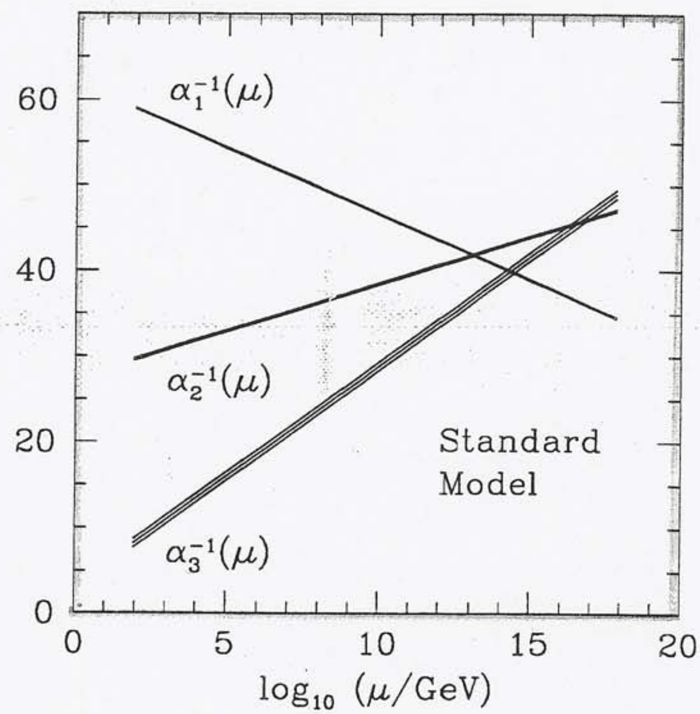
$\sin^2\theta_W \approx 0.231$ not 3/8!

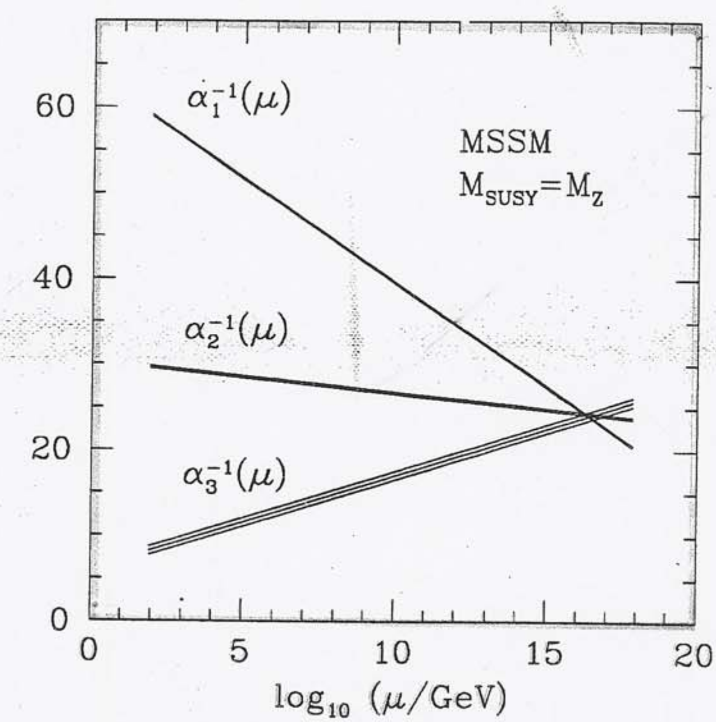
Uh, oh!

Trouble for GUT's?

No...

Unification of gauge couplings...



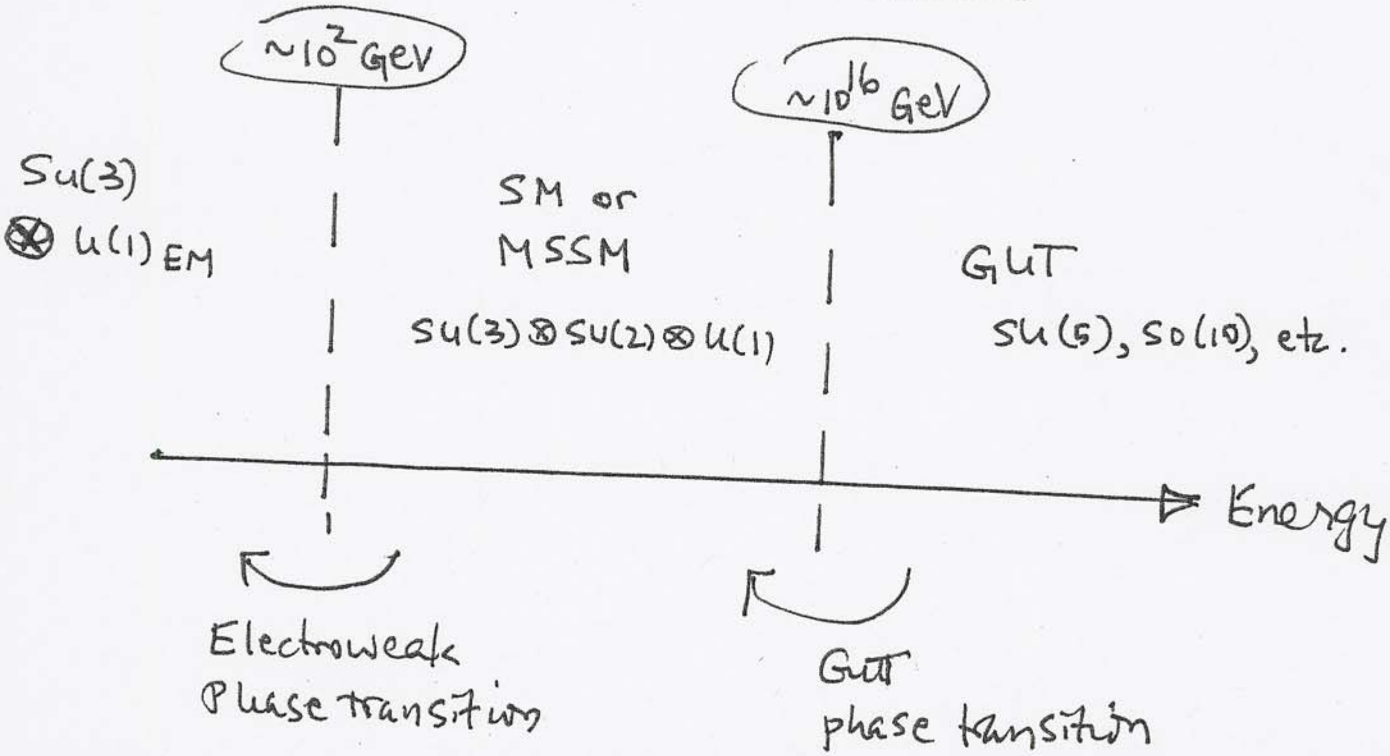


Thus, the natural energy scale for grand unification is

$$\boxed{M_{GUT} \approx 2\times 10^{16}\ \text{GeV}}$$

where the gauge couplings unify and where we have the potential to realize all gauge forces as emerging from a single GUT group [eg, SU(5)] with a single coupling constant g_5

$\Rightarrow$ **GUTs are high-scale physics!**



But how does the GUT break at M_{GUT}?

Just as in SM, one idea is to use the Higgs mechanism!

Need a Higgs $\mathbb{H}$ such that

$$SU(5) \xrightarrow{\langle H \rangle} SU(3) \otimes SU(2) \otimes U(1) \xrightarrow{\langle \phi \rangle} SU(3) \times U(1)_{EM}$$

Can we use $H \overset{?}{=} \phi_3$

↳ the "colored" Higgs which was joined ~~with~~ the EW Higgs ϕ to form the $\mathbf{5}$ of $SU(5)$?

No! Recall $\phi_3 : (\mathbf{3}, \mathbb{1})_{-2/3} \Leftrightarrow Q_{EM} = -1/3$ for each component of the triplet.

Thus, if $\langle \phi_3 \rangle \neq 0$

$\Rightarrow$ $SU(3)$ and $U(1)_{EM}$ will be broken – EXACTLY THE OPPOSITE OF WHAT WE WANT!

★ Indeed, to preserve $SU(3) \otimes SU(2) \otimes U(1)$ subgroup, need a NEW HIGGS REP OF $SU(5)$ containing a component $(\mathbb{1}, \mathbb{1})_0$ = neutral under SM!

Turns out that the smallest SU(5) rep that can do this is the <u>24 rep</u> = adjoint

Fairly big!
(But OK, since we are going to need lots of Goldstone bosons to be "eaten" by the twelve (X,Y) gauge bosons to make them massive and thereby do the breaking!)

This is a general property for all simple GUT groups [SU(5), SO(10), E_6, etc.]

$\Rightarrow$ no GUT Higgs for reps smaller than the <u>adjoint</u>!

Recall for SU(5),

$$24 \rightarrow (8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (3,2)_{-\frac{5}{3}} \oplus (\bar{3},2)_{\frac{5}{3}}$$

($(1,1)_0$ → singlet!)

In matrix notation, 24 = traceless 5×5 matrix

singlet $(\mathbb{1},\mathbb{1})_0$ representation
should look like identity for both $SU(3)$ and $SU(2)$ components

⇒ we want
$$\langle H\rangle = v_{GUT}\begin{bmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{bmatrix}$$

(this is the analogue of demanding $\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix}$ for EW)

WHAT POTENTIAL WOULD HAVE THIS MINIMUM ?

Try something familiar :

$$V(H,\phi) = -m_1^2\,\mathrm{tr}\,H^2 + \lambda_1(\mathrm{tr}\,H^2)^2 + \lambda_2(\mathrm{tr}\,H^4)$$
$$- m_2^2(\phi^+\phi) + \lambda_3(\phi^+\phi)^2$$
$$+ \lambda_4(\mathrm{tr}\,H^2)(\phi^+\phi) + \lambda_5(\phi^+H^2\phi)$$

This is a general 4^{th}-order $SU(5)$-invariant polynomial where we imposed discrete $\begin{pmatrix} H\to -H \\ \phi \to -\phi\end{pmatrix}$ symmetries to eliminate cubic terms...

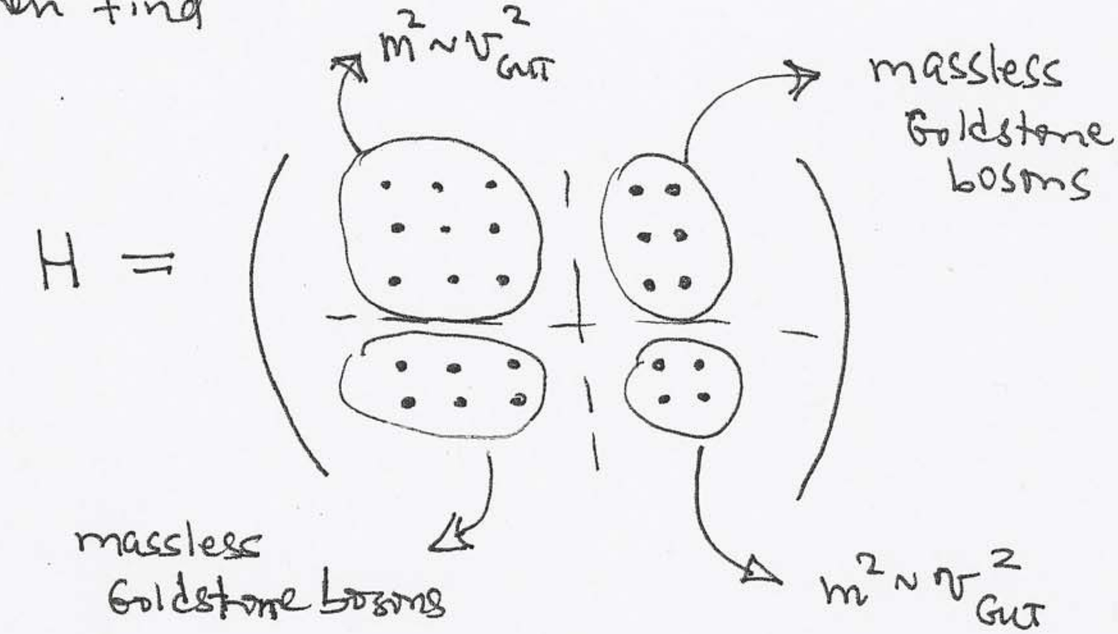
⇐ (two Mexican hats & interaction terms)

Find: for $\lambda_2 > 0$, $\lambda_1 > -\left(\frac{7}{30}\right)\lambda_2$

$\Rightarrow$ reproduces desired minimum

with $$v_{GUT}^2 = \frac{m_1^2}{60\lambda_1 + 14\lambda_2}$$

We then find



(X, Y) gauge bosons then "eat" these massless Goldstone Higgs fields, get masses

$$m_{X,Y} \sim g_{GUT}\, v_{GUT}$$

while remaining massive Higgs fields survive as physical Higgses, but at the GUT scale.

However, the presence of the cross-couplings (λ_4, λ_5) between $\underline{H}$ and $\underline{\phi}$ in the potential implies that our ϕ fields also get masses!

colored triplet ϕ_3 ---- got masses $\begin{cases} -m_2^2 + (30\lambda_4 + 4\lambda_5) v_{GUT}^2 \\ -m_2^2 + (30\lambda_4 + 9\lambda_5) v_{GUT}^2 \end{cases}$

usual Higgs doublet ϕ

Top result for ϕ_3: **OK**, triplet Higgs also will have GUT-scale mass ✓

But bottom result for EW Higgs ϕ is problematic!

ϕ needs to be light ($m \sim 10^2$ GeV) so that it can survive down to EW scale and trigger EW symmetry breaking!

⟹ MUST BE A VERY PRECISE CANCELLATION OVER 12→14 ORDERS OF MAGNITUDE BETWEEN (λ_4, λ_5) and m_2 !

Two questions:

- Where does this remarkable fine-tuned cancellation come from?

- Why / how is it stable against radiative corrections?

THESE ARE TWO PARTS of the so-called **GAUGE HIERARCHY PROBLEM**

→ solution is unknown!

- SUSY ?
- extra dimensions ?
- string landscape ?

} lots of ideas, no experimental evidence yet.

UPSHOT

★★★★ Need a lot of fine-tuning in the Higgs sector because we want to maintain two widely separated scales of gauge symmetry breaking within one UV theory emerging from one potential $V(H, \phi)$ with GUT-scale parameters.

The Higgs sector may also need to be extended for other, more phenomenological reasons.

<u>Consider fermion masses</u>:

SU(5) structure of Yukawa couplings can be shown to require certain mass relations to hold at the GUT scale:

e.g.,

$$\boxed{m_e = m_d} \; ; \; \boxed{m_\mu = m_s} \; ; \; \boxed{m_\tau = m_b.}$$

When extrapolated down to EW scale using RG equations, do these equations hold even approximately?

<u>For third generation</u>, we find $\boxed{m_b \approx 3\, m_\tau}$

which is reasonably successful ✓

But even with RG equation evolution, it is easy to see that the <u>ratios</u>

$$\frac{m_e}{m_\mu} = \frac{m_d}{m_s}, \text{ etc.} \quad \text{must remain invariant.}$$

(Same quantum #'s – thus behave the same!)

THIS IS OBVIOUSLY VIOLATED!

Are there solutions to this problem?

Yes, mostly by expanding the Higgs sector!

e.g., if we introduce a $\underline{45}$ representation as another new Higgs $\underline{h}$

then one can show that

$$\left(\frac{m_e}{m_\mu} = \frac{m_d}{m_s}\right) \xrightarrow[\text{(becomes)}]{} \boxed{\frac{m_e}{m_\mu} = \frac{1}{9}\,\frac{m_d}{m_s}}$$

This is satisfied rather well by expt., but obviously requires a non-minimal SU(5) Higgs structure!

Other similar solutions are also possible ...

But the classic signature of GUT's is PROTON DECAY!

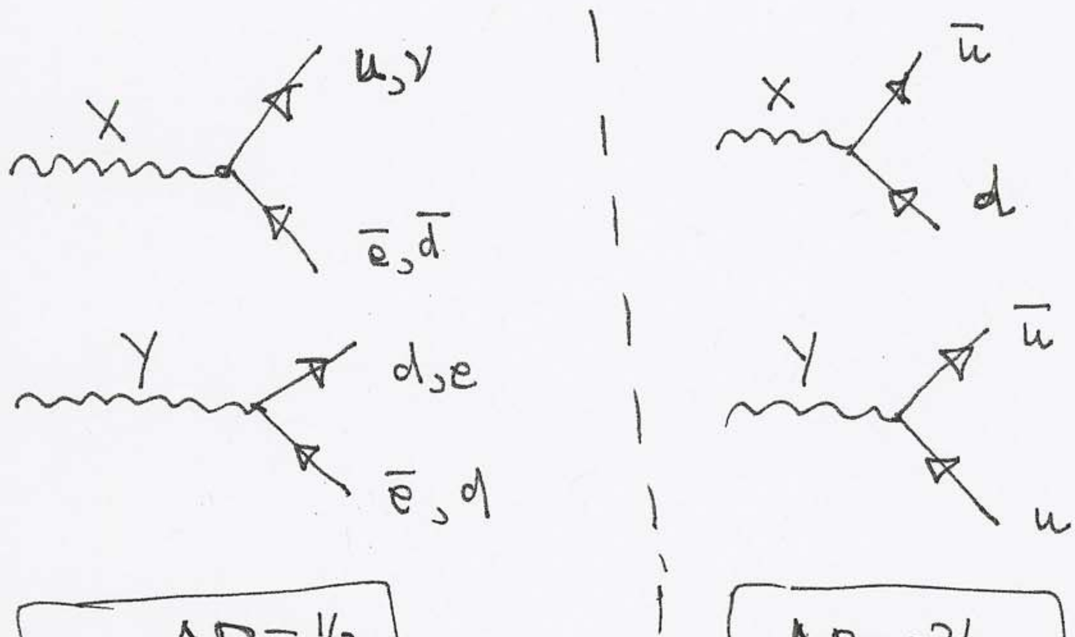
Since GUTs necessarily break

$$\begin{cases} B = \text{baryon } \# \\ L = \text{lepton } \# \end{cases},$$

the lightest baryon (= proton) is no longer stable!

⇒ NEW DECAY PROCESSES ARE MEDIATED, PRIMARILY BY (X, Y) GAUGE BOSONS!

Recall the relevant diagrams:

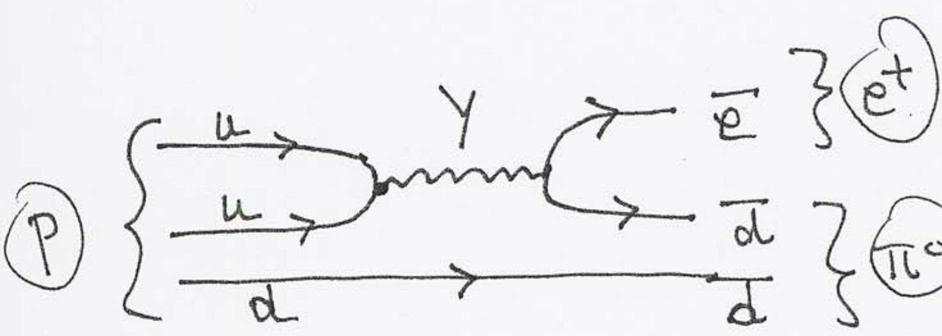


$\Delta B = 1/3$
LEPTOQUARK CHANNEL

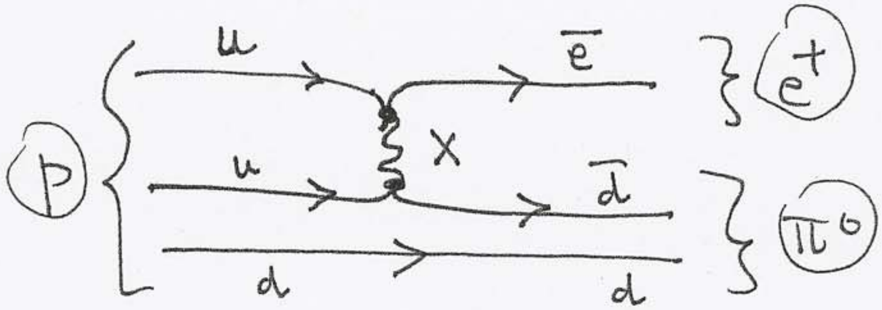
$\Delta B = -2/3$
DIQUARK CHANNEL

⇒ Therefore, through (X, Y) exchange, can build a $\Delta B = 1$ process to allow the proton to decay!

e.g.,



or



, etc.

Most lead to decay $\boxed{p \to \pi^0 e^+}$ as dominant mode in this minimal SU(5) GUT

$$\Rightarrow \boxed{M(p \to \pi^0 e^+) \approx \frac{g_{GUT}^2}{M_{X,Y}^2}} \Leftarrow \text{from } \binom{X,Y}{\text{propagator}}$$

Thus, for $M_{X,Y} \gtrsim 10^{15}$ GeV

$\Rightarrow$ proton lifetime $\gtrsim 10^{31}$ years

✳ In general, heavy X,Y gauge boson masses are critical for suppressing proton decay in GUTs!

Important observation:

Even though B and L are broken by the (X, Y) bosons in the SU(5) model, the difference

$$\boxed{B-L}$$ is conserved!

$\Rightarrow$ $\Delta B = \Delta L$ eg $p \to \pi^0 e^+$ (lose a baryon, lose a lepton)

This is an "accidental" GLOBAL symmetry in SU(5).

Miracle? Not really - it can be shown that *any* four-fermion (dim = 6) effective operators in $\mathcal{L}$ which break B yet preserve $SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$ will also preserve B-L!

Thus B-L conservation is independent of the particular unification scheme!

($\Rightarrow$ to break B-L, require operators of dim > 6!)

Aside: The breaking of $\underline{B}$ has very important COSMOLOGICAL IMPLICATIONS!

One major problem is the baryon-number asymmetry in the Universe:

$$\delta \equiv \frac{N_q - N_{\bar{q}}}{N_q + N_{\bar{q}}} \approx 10^{-9} \qquad (\text{not zero!})$$

Where does this baryon/anti-baryon asymmetry come from?

- Initial condition? (Landscape?)
- Symmetric initial condition, with subsequent dynamics generating the asymmetry?

→ Would require: (Sakharov)

- Violate C and CP conservation
- Violate B conservation
- Have these properties occur in processes which are out of thermal equilibrium

(so reverse process does not occur with equal probability!)

GUTs have all three of the required conditions !

Processes which violate B, C, and CP are those which are mediated by (X,Y) gauge bosons & colored Higgs triplets ϕ_3

⇒ These are forced out of thermal equilibrium by cosmological expansion ...

✱ Thus, GUTs may provide an explanation of the primordial baryon/antibaryon asymmetry of the universe !

Beyond the minimal SU(5) GUT:

① Add $\mathcal{N}=1$ SUSY:

- improves gauge coupling unification
- helps to stabilize (and perhaps even dynamically produce) the gauge hierarchy
- may help keep proton-decay rates within increasingly tight bounds

② Add more Higgs fields, other matter:

- helps to obtain better fermion mass relations

③ Enlarge the gauge group!

SO(10) ?

E_6 ? ...

Why do this?

Recall the SM particle content:

d^c : $(\bar{3},1)_{2/3}$
L : $(1,2)_{-1}$
} $\Longrightarrow$ $\bar{5}$ of $SU(5)$

- - - - - - - - - - - - - -

Q : $(3,2)_{1/3}$
u^c : $(\bar{3},1)_{-4/3}$
e^c : $(1,1)_2$
} $\Longrightarrow$ $\mathbb{1}0$ of $SU(5)$

- - - - - - - - - - - - - -

ν^c : $(1,1)_0$ $\longrightarrow$ if it exists ...?

Two unsolved issues:

① $\bar{5}$ and $\mathbb{1}0$ are still two different reps of $SU(5)$ $\Rightarrow$ they themselves are not unified!

② We now know neutrinos have mass!

ν^c needed for Dirac mass through Yukawa couplings!

and needed for Majorana mass through seesaw mechanism!

Thus it seems we need a third representation so that all 16 particles in each SM generation would be realized as

$$\bar{5} \oplus 10 \oplus 1$$

↖ singlet ν^c.

This is hardly a "unification"!

But if we consider the next largest GUT group $SO(10) \longrightarrow SU(5) \otimes U(1)$

we find that it has representations

1

(spinor) $16 \longrightarrow (\bar{5})_3 \oplus (10)_{-1} \oplus (1)_{-5}$

45

54

120

126 .... etc.

↑ just what we need, all bundled together into the fundamental (spinor) representation of SO(10)!

- Thus, using $SO(10)$, one entire SM generation — including the RH neutrino — can be realized as coming from a <u>single</u>, <u>unified</u>, <u>fundamental</u>, <u>anomaly-free</u>, <u>16-dimensional</u> <u>spinor</u> representation.

  ALL QUANTUM NUMBERS WORK PERFECTLY!

- Moreover, <u>the extra $U(1)$</u> ← this is <u>B-L</u>!

  Thus, in $SO(10)$ unification,

  <u>B-L conservation is no longer "accidental" but arises naturally from a gauge symmetry!</u>

  <u>Prevents processes like:</u>

  $n \to e^- \pi^+$, $n \to \nu \pi^0$
  $p \to \nu \pi^+$
  $n \to \bar{n}$

  } forbidden to all orders...

- <u>Other cosmological benefits</u>:

  e.g., <u>parity</u> is now part of the gauge symmetries of $SO(10)$ [unlike $SU(5)$]

  ⇒ <u>avoids the cosmological domain wall problem</u> normally associated with parity-breaking!

Can also extend to even larger GUT groups:

- E_6, etc ← natural "string-Theory" candidate

- $G_{GUT} \otimes G_{flavor}$

  ↑ to incorporate family (flavor) symmetries across three generations

... LOTS of possibilities exist

THE FIELD OF GUT model-building is huge & well-explored!

→ see original literature.

A2
B2
D2

The Problem of Gauge Coupling Unification

Recall situation in field theory:

W13 Experimental Data —

- Standard Model, no SUSY: no unification of couplings
- MSSM, $N = 1$ SUSY: unification occurs

  Unification scale $\boxed{M_{\text{MSSM}} \approx 2 \times 10^{16} \text{ GeV.}}$

$\Longrightarrow$ MSSM very successful!

— one of the few surviving extensions to SM which is in agreement with all experimental data!

— SUSY also provides elegant solutions for:

finiteness

gauge hierarchy problem...

Therefore, current field-theoretic scenario:

- Grand Unified Group above M_{MSSM}
- MSSM gauge group and spectrum between M_{MSSM} and M_{SUSY}
- *then* .... (soft?, spontaneous?) SUSY-breaking
  - — constrained to control Higgs mass, solve gauge hierarchy problem
  - — requires $\text{Str}(M^2) <$ some number.
- SM gauge group and spectrum below M_{SUSY}

A4
B4
J4
W15

Compelling picture except for various problems:

- M_{MSSM} close to Planck scale $\Longrightarrow$ what about gravity?
- Why the spectrum of SM and MSSM with all of these arbitrary parameters? Need to explain:
  - — three generations
  - — fermion mass matrices...
- If GUT theory above M_{MSSM}, what about proton lifetime?
  Require some sort of doublet-triplet splitting mech.
- *Why* require a GUT at all?
  Theoretical prejudice...
  Not required for consistency of model...

String theory can solve these problems.

A5

- Naturally incorporates quantized gravity
  B5 — spin-two massless particle (graviton) always appears
  D5 in spectrum.

W16

- N=1 SUSY field theories with non-abelian gauge groups appear as low-energy limits.

- May provide *uniform framework* for understanding
  - three generations
  - fermion mass matrices
  - doublet-triplet splitting mechanism, *etc...*

  — in principle, *no free parameters*!

- *Natural* unification of couplings:
  gauge and gravitational couplings *automatically* unify to form *one* coupling constant g_{string}:

$$\boxed{8\pi \frac{G_N}{\alpha'} \;=\; g_i^2\, k_i \;=\; g_{\text{string}}^2}$$

where $G_N \equiv$ gravitational (Newton) coupling
$\alpha' \equiv$ Regge slope
$k_i \equiv$ affine level of group factor G_i:

$$J^a(z)J^b(w) \;\sim\; \frac{if^{abc}}{z-w}\,J^c(w) \;+\; k\,\frac{\delta^{ab}}{(z-w)^2} \;+\; \ldots$$

"Schwinger term"

Field Theory vs. String Theory: Crucial Differences

V17

- String theory is *finite* $\Longrightarrow$ *running* of gauge couplings is within framework of low-energy *effective* theory only.

- In string theory, all couplings are *dynamical variables*, related to expectation values of *moduli fields*.

- Dependence on KM level k_i.
  - *Essentially a normalization*: analogous to hypercharge normalization [*e.g.*, $k_Y = 5/3$ for $SU(5)$ or $SO(10)$], but now also appears for non-abelian gauge factors!
  - Most easily constructed string models have $k_i = 1$.

C4

N17a

Unfortunately, this is not the unification scale expected in heterotic string theory!

- At tree level, $M_{\text{string}} = 1/\sqrt{\alpha'} = g_{\text{string}} M_{\text{Planck}}$.

- At one loop,

$$\boxed{M_{\text{string}} \approx g_{\text{string}} \times 5 \times 10^{17} \text{ GeV}}$$ in $\overline{\text{DR}}$ scheme.

Assuming $g_{\text{string}} \approx \mathcal{O}(1)$,

$$\boxed{M_{\text{string}} \approx 5 \times 10^{17} \text{ GeV}}$$

$\Longrightarrow$ factor of ≈ 20 discrepancy!!

Is this a major problem?

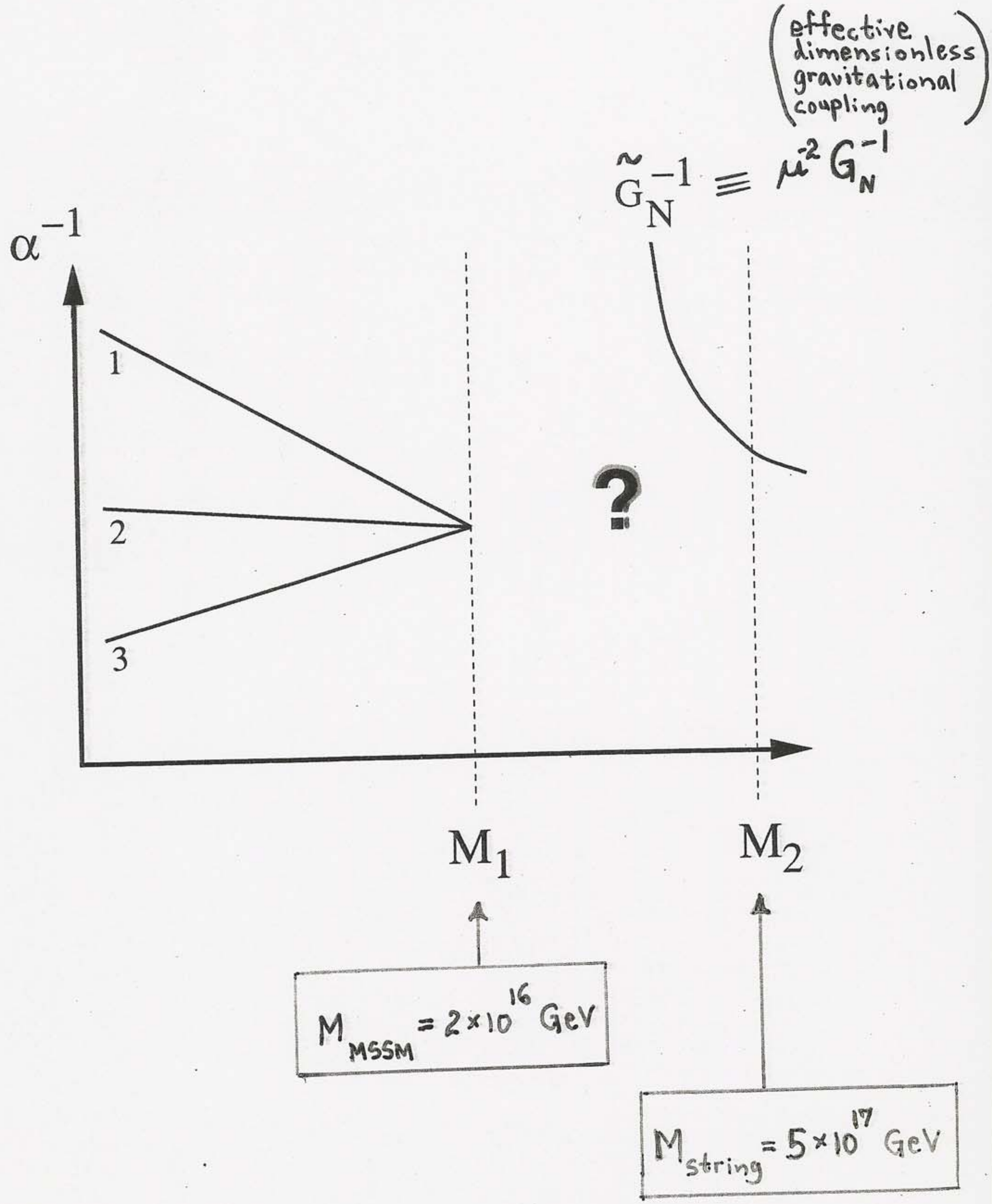
- only 10% effect in *logarithms* of mass scales, but
- leads to *wildly incorrect* values for $\sin^2\theta_W$ and α_{strong}:

$$\boxed{\frac{16\pi^2}{g_i^2(\mu)} = k_i \frac{16\pi^2}{g^2_{\text{string}}} + b_i \ln \frac{M^2_{\text{string}}}{\mu^2}}$$

$\Longrightarrow$ Major problem for string phenomenology!

W18a

The Fundamental Problem ...

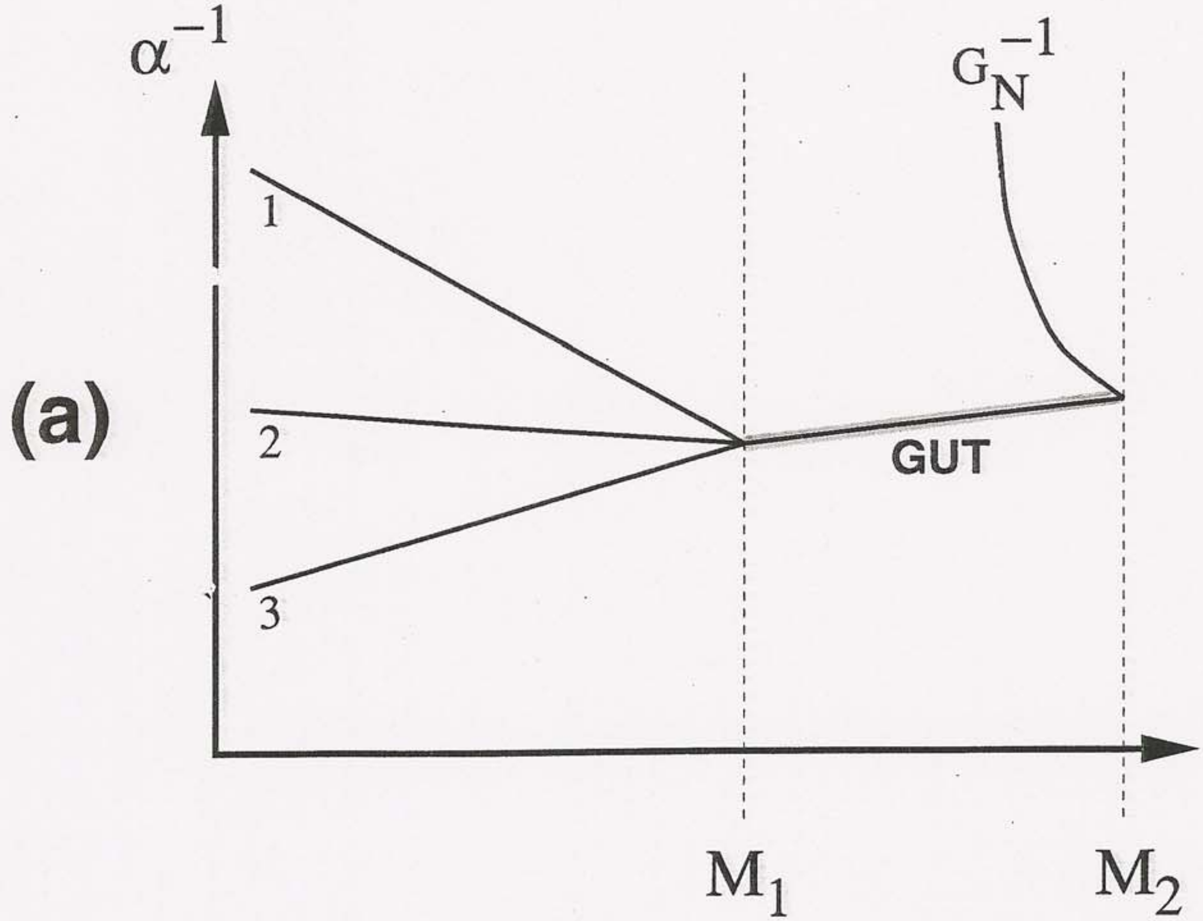


Path #1: String GUT's

Intermediate-scale gauge group:

$SU(3) \times SU(2) \times U(1)_Y \subset G$

18b For example, $G = SU(5)$, $SO(10)$, or E_6.



- KRD & J. March-Russell, *Nucl. Phys.* **B479** (1996) 113
- KRD, *Nucl. Phys.* **B488** (1997) 141

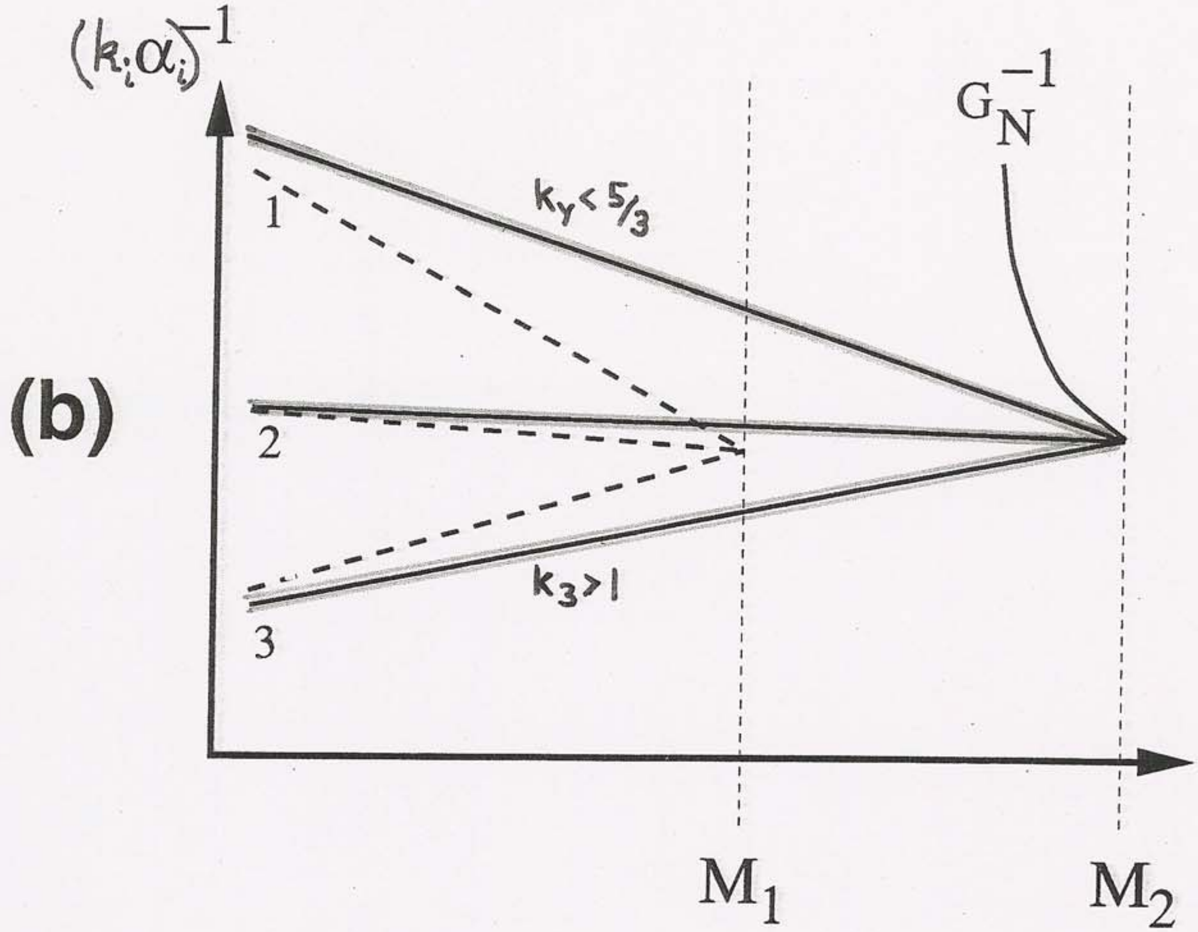
18c

Path #2: Non-Standard Levels (k_Y, k_2, k_3)

In MSSM, we have $(k_Y, k_2, k_3) = (5/3, 1, 1)$.

In string theory, however, this is not necessary! Other values are allowed.

This would be a *stringy* effect...

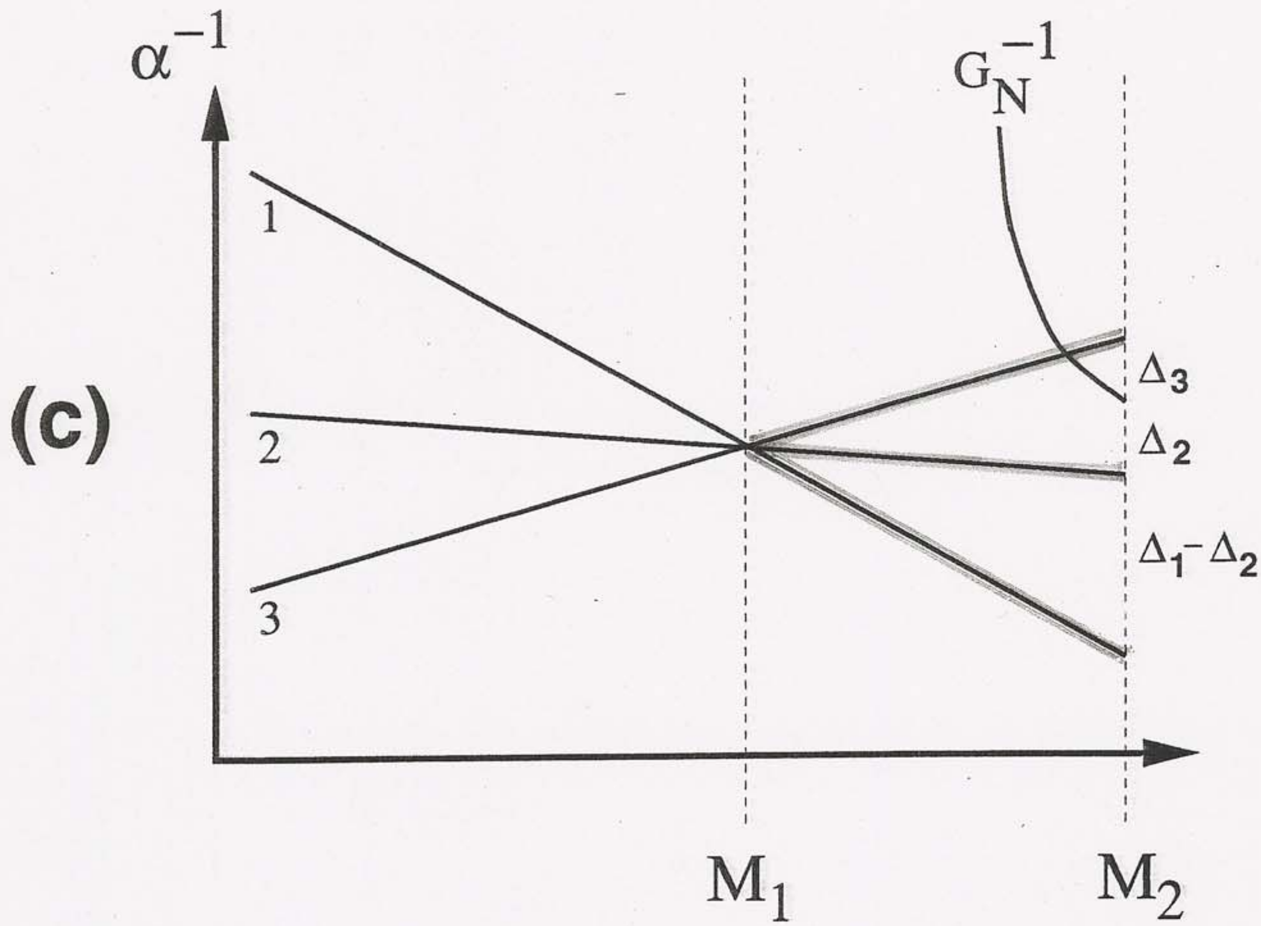


- KRD, A.E. Faraggi, & J. March-Russell, *Nucl. Phys.* **B467** (1996) 44

Path #3: Heavy String Thresholds

Heavy string threshold corrections $(\Delta_Y, \Delta_2, \Delta_3)$ from infinite towers of massive (Planck-scale) states.

Also a purely stringy effect...



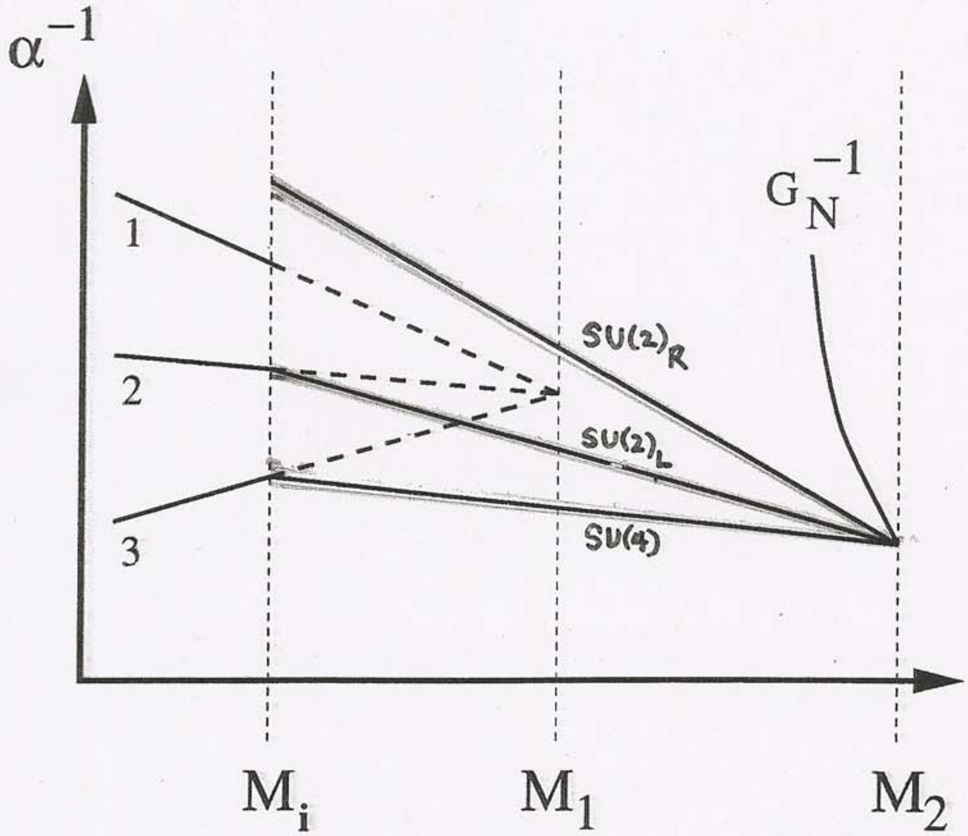
- KRD & A.E. Faraggi, *Phys. Rev. Lett.* **75** (1995) 2646
- KRD & A.E. Faraggi, *Nucl. Phys.* **B457** (1995) 409

'18e

Path #4: Intermediate-Scale Corrections

- *Light SUSY Thresholds*
  - — from breaking SUSY at intermediate scales
  - — analyzed in effective field theory
- *Intermediate Gauge Structure*
  - — *e.g.*, Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$
    flipped $SU(5) \times U(1)$
  - — also analyzed in effective field theory

(d)

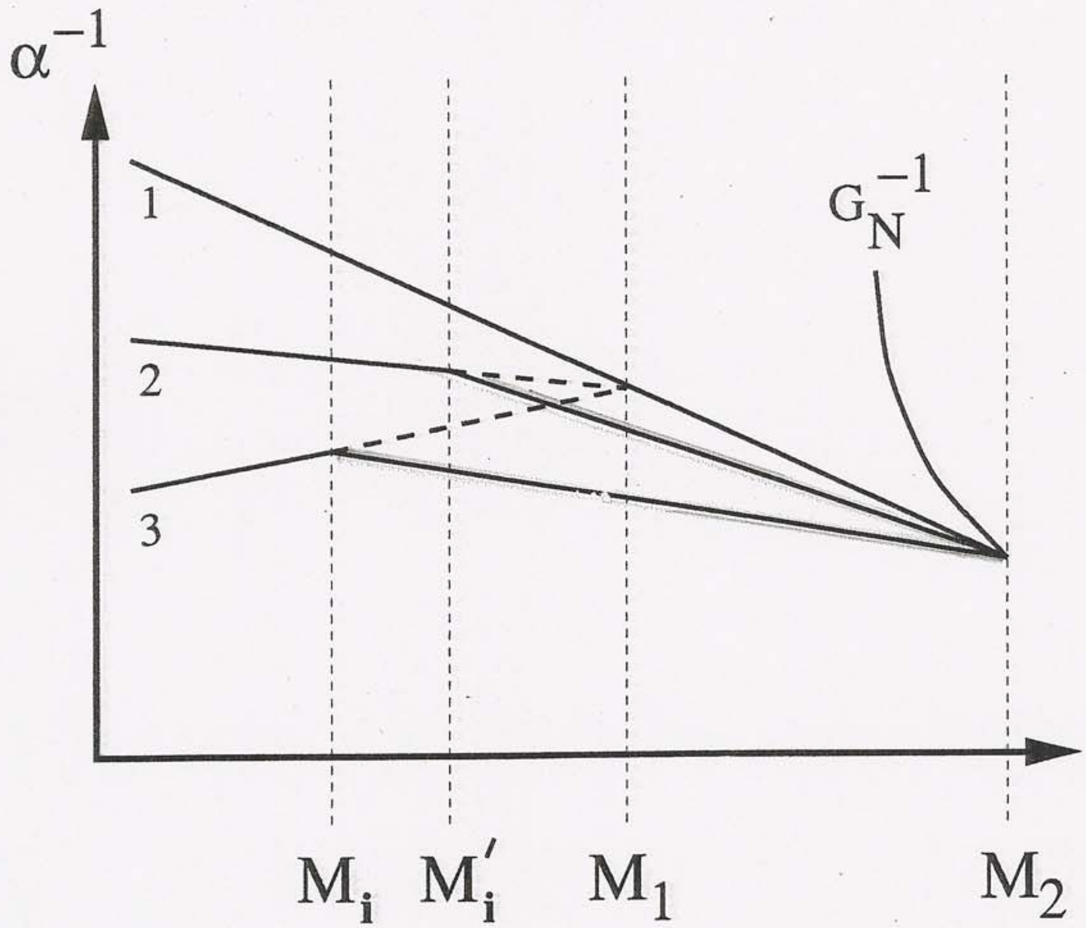


- KRD & A.E. Faraggi, *Phys. Rev. Lett.* **75** (1995) 2646
- KRD & A.E. Faraggi, *Nucl. Phys.* **B457** (1995) 409

Path #5: Extra Matter Beyond the MSSM

Such matter *ad hoc* from field-theory perspective, but *required for self-consistency* in many string models!

(e)

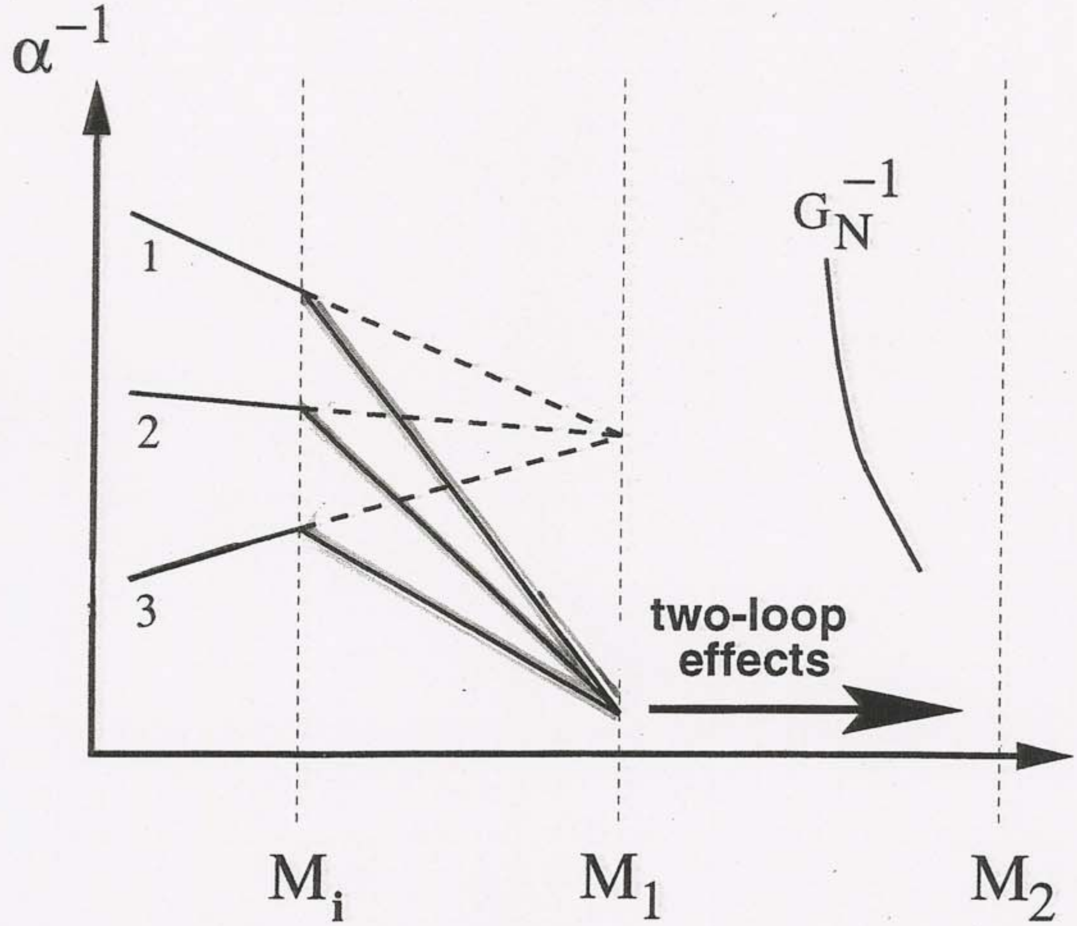


- KRD & A.E. Faraggi, *Phys. Rev. Lett.* **75** (1995) 2646
- KRD & A.E. Faraggi, *Nucl. Phys.* **B457** (1995) 409

or even...

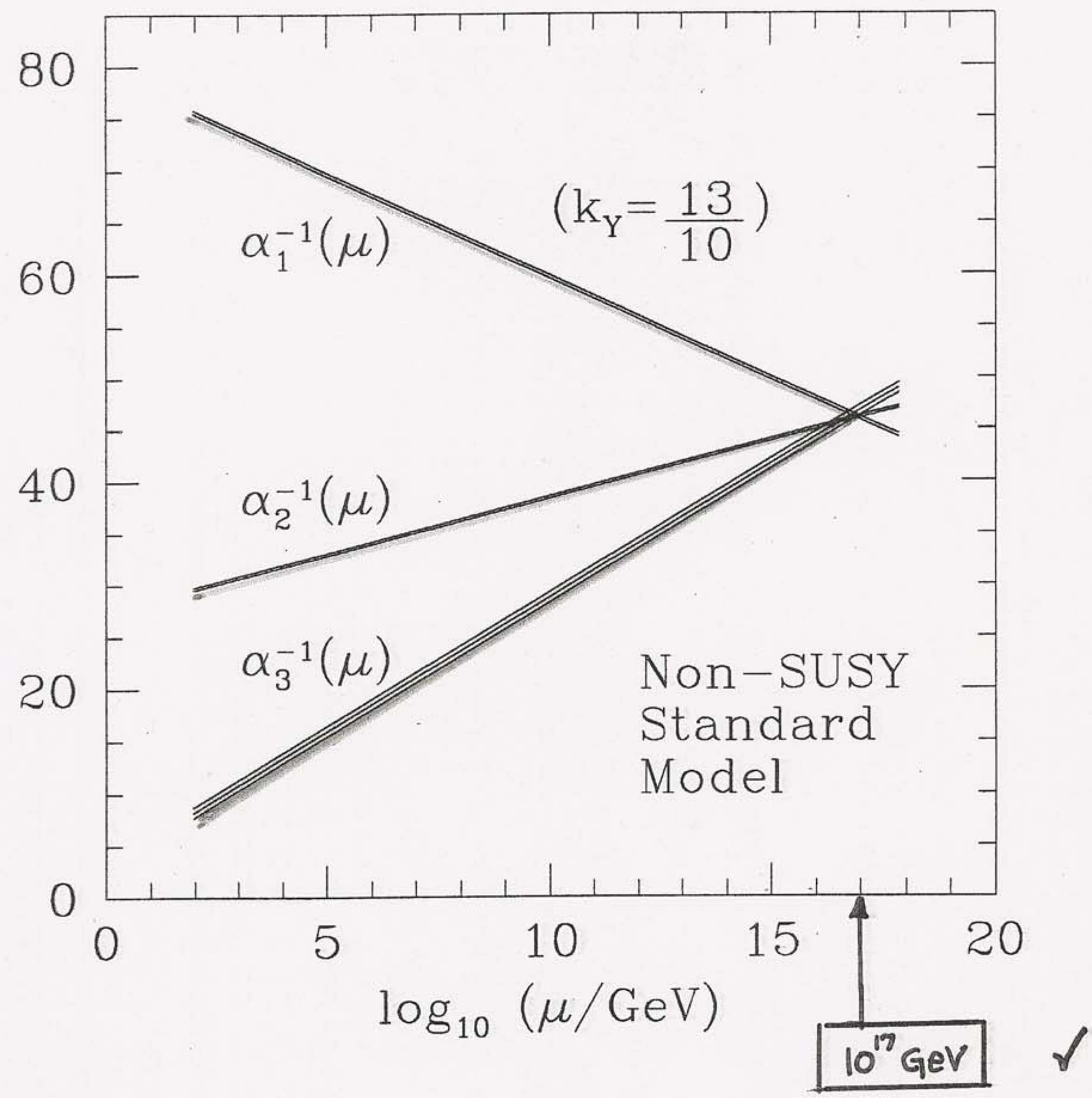
18g

(f)



Path #6: Strings without SUSY

In *field theory*, unification impossible without SUSY.
In *string theory*, however, may be possible!



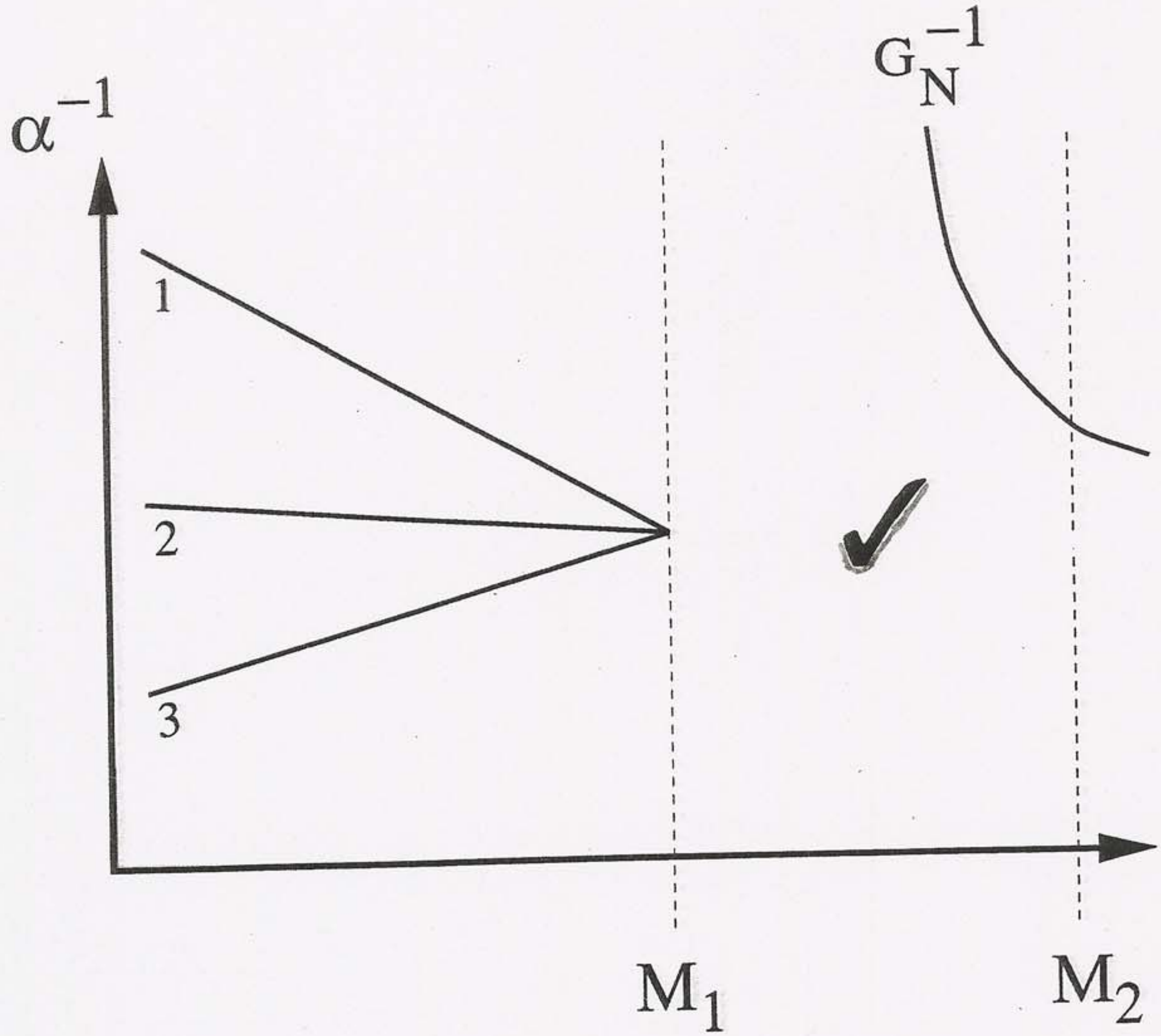
• KRD, *Nucl. Phys.* **B429** (1994) 533
• KRD, M. Moshe, & R.C. Myers, *Phys. Rev. Lett.* **74** (1995) 4767

Path #7: Strong-Coupling Effects

If underlying string coupling is strong,
$\Longrightarrow$ non-perturbative relations to new theories
e.g., M-theory or *open* string theory.

These theories have weaker unification relations.

Thus, original mismatch may not be problematic.



Thus, one over-riding question:

Which path(s) to unification does string theory actually take?

...or equivalently:

Can realistic string models be constructed which exploit these possibilities?

There has been significant recent progress in understanding/utilizing each of these paths...

- *Review Article*: KRD, hep-th/9602045 (*Physics Reports* **287** (1997) 447)

Phenomenological Lessons from String GUT's

- KRD & J. March-Russell, *Nucl. Phys.* **B479** (1996) 113
- KRD, *Nucl. Phys.* **B488** (1997) 141

135

New Rules for GUT Model-Building —

- **No 120** or **144** representations of $SO(10)$
  — typically used for
    - fixing light quark/lepton mass ratios
    - inducing GUT symmetry breaking

  — ruled out in $SO(10)$ string models!

- **No 126** representation of $SO(10)$
  — typically used for
    - heavy Majorana right-handed neutrino mass
    - GJ factor of 3 in light quark/lepton mass ratio

  — ruled out in $SO(10)$ string models!

- **ALL** larger representations of $SO(10)$ ruled out!

- All **54** representations of $SO(10)$ must transform as *singlets* under all gauge symmetries beyond $SO(10)$.
  — hard to build models with multiple **54** reps

- New restrictions on allowed couplings — *e.g.*,
  — no couplings of form $X \cdot \mathbf{54} \cdot \mathbf{54}$ where X = singlet
  — no couplings of form $X \cdot \mathbf{54} \cdot \mathbf{54}'$ where X = singlet
  — no explicit mass terms in superpotential
  — no cubic level-terms in superpotential

- All **45** representations of $SO(10)$ must transform as *singlets* under all gauge symmetries beyond $SO(10)$!
  — can only have $\mathbb{Z}_3$ or $\mathbb{Z}_6$ discrete quantum numbers
  — hard to build models with multiple **45** reps
  — hard to implement doublet/triplet splitting
  — hard to implement "fake" $\mathbf{126} \subset \mathbf{45} \times \mathbf{45'} \times \mathbf{10}$

- All **78** representations of E_6 must transform as *singlets* under all gauge symmetries beyond E_6!

- **ALL** larger representations of E_6 ruled out!

- All **24** representations of $SU(5)$ must transform as *singlets* under all gauge symmetries beyond $SU(5)$!

etc...

Guided by these results, there is much current activity —

- *Field Theory:* new efforts to build low-energy GUT models satisfying these constraints

- *String Theory:* new mechanisms for building string GUT models

Another modern topic,

potentially even more

exciting ....

GUT's

at

a TeV ??

One of the most compelling proposals for physics beyond the Standard Model is the emergence of a

SUSY GUT

SUSY GUT's would have many benefits —

- Unify strong, weak, and hypercharge forces
  - $\Longrightarrow$ explain relative gauge couplings
  - $\Longrightarrow$ explain quantization of electric charge
- Unify diverse particle representations
  - $\Longrightarrow$ explain fermion quantum numbers
  - $\Longrightarrow$ explain relative fermion masses
- Predict baryon-number violation
  - $\Longrightarrow$ explain cosmological baryon/anti-baryon asymmetry

Unfortunately, the GUT scale is very remote!

$\Longrightarrow$ hard to probe GUT physics directly

$\Longrightarrow$ must look for very *rare* processes,

e.g., proton decay

All GUT physics is suppressed by heavy scale!

Is there a way to bring GUT physics down to accessible energy scales?

Is it possible to probe GUT physics directly?

Hardly seems possible...

GUT's require *unified* gauge couplings

$\Longrightarrow$ would need to somehow achieve gauge coupling unification at lower energy scales!

But once again, this hardly seems possible...

— only possibility is to add extra matter but this generally *increases* unification scale, also pushes unification towards strong coupling.

Is there a way around this?

Why is unification scale so high?

Gauge couplings run only *logarithmically* versus energy scale μ (or *linearly* versus $\log \mu$)

$\Longrightarrow$ must extrapolate over many orders of magnitude before different low-energy gauge couplings can be reconciled!

How can we make the gauge couplings run faster?

How can we change *linear* running into *exponential* running?

Extra spacetime dimensions!

- Naturally predicted in string theory
- Radii generally unfixed by string dynamics
- Large radii play an important role in string duality

$\Longrightarrow$ Such scenarios should be easy to realize within string theory!

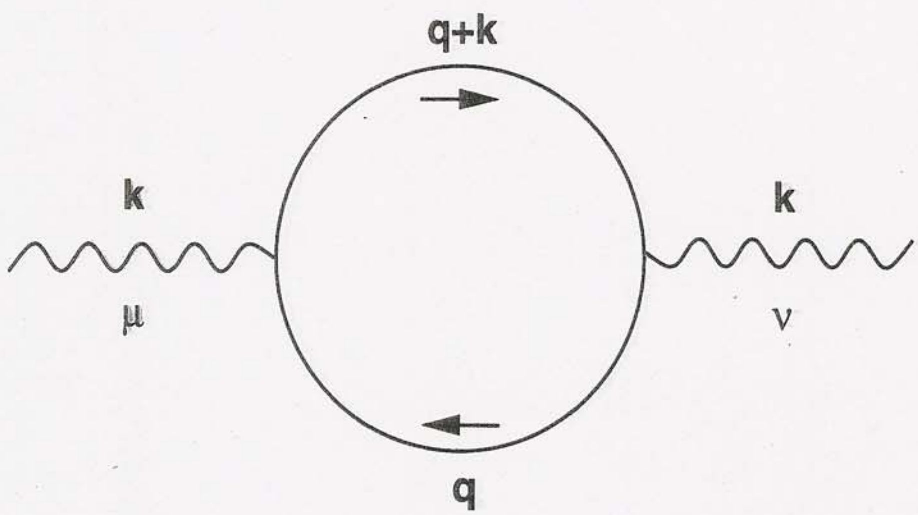
Nevertheless, extra dimensions can be discussed in purely *field-theoretic* terms...

Consider running of gauge couplings...

Recall: *Without* extra dimensions,
gauge couplings have usual logarithmic running:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z}$$

...result of evaluating the vacuum polarization diagram:



Now, imagine $\delta \equiv D - 4$ extra dimensions at scale R^{-1}.
Must also include Kaluza-Klein states in loop!

$\Longrightarrow$ Below R^{-1}, no appreciable effect.
Above R^{-1}, couplings now evolve according to:

$$\alpha_i^{-1}(\mu) \approx \alpha_i^{-1}(R^{-1}) - \frac{b_i - \tilde{b}_i}{2\pi} \ln(R\mu) - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[(R\mu)^\delta - 1\right]$$

where

- $(b_1, b_2, b_3) \equiv (33/5, 1, -3)$
- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \equiv (3/5, -3, -6)$
- $X_\delta \equiv \dfrac{2\,\pi^{\delta/2}}{\Gamma(\delta/2)}$

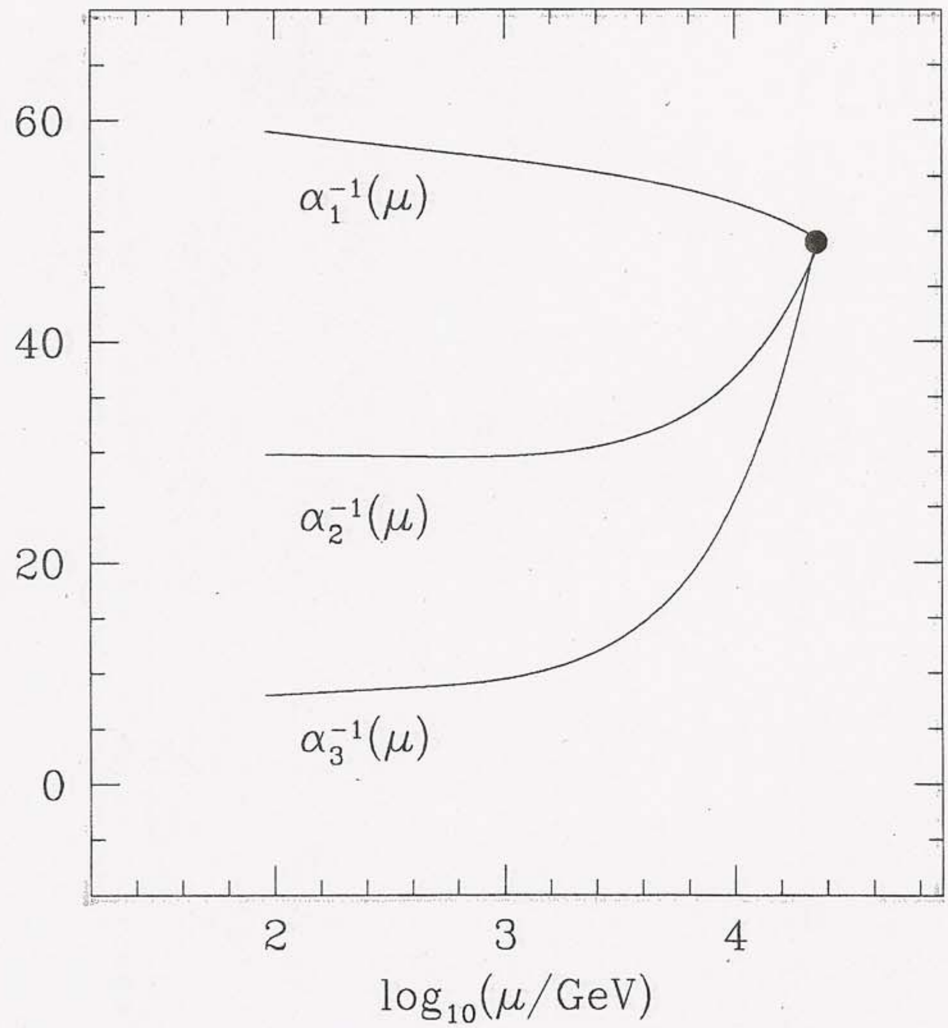
Power-law behavior is the consequence of extra dimensions!

But how does this affect gauge coupling unification?

Let us choose

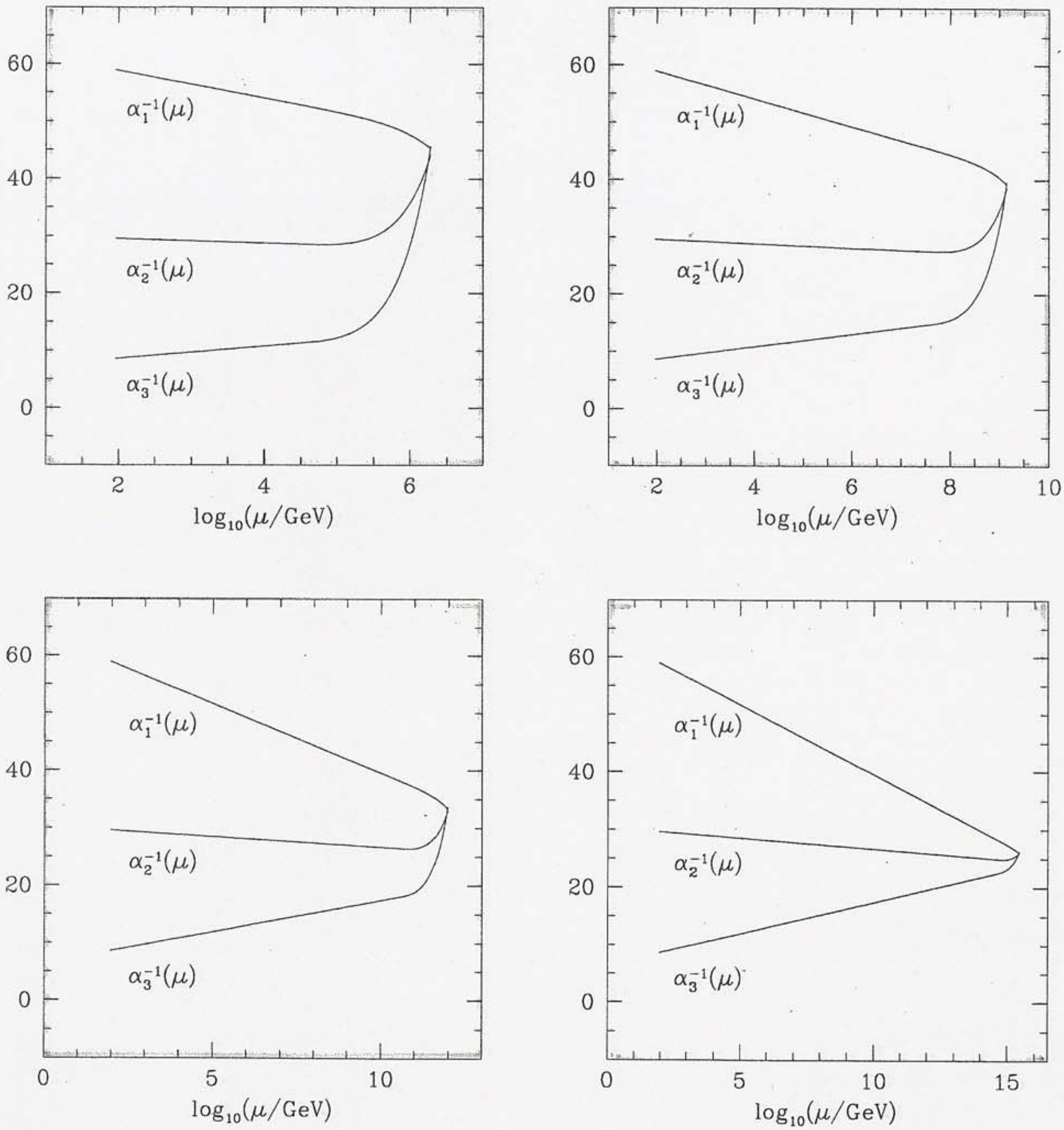
- $R^{-1} = 1$ TeV (the most *extreme* case)
- $\delta = 1$ (one extra dimension)

We then find...



- Evolution is dramatically altered, *but gauge couplings still unify*!
- Potential new scale for grand unification!

How does this depend on the chosen radius?

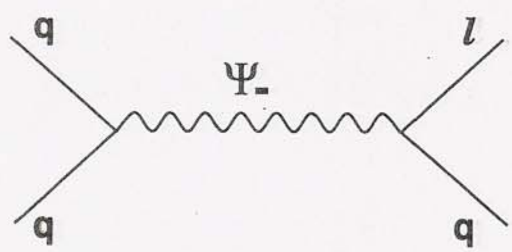


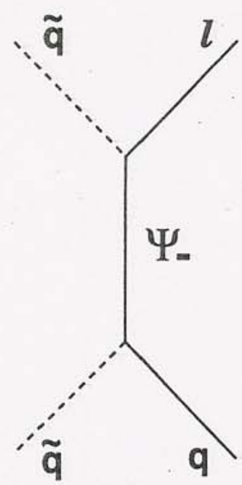
Unification is always preserved!

Extra Dimensions and Proton Decay

If we imagine a GUT theory emerging at such low energy scales, we immediately face the problem of proton decay!

Can be mediated by X-bosons, colored Higgs triplets, *etc...*





Usually, proton decay is suppressed by large GUT mass scale!

In our case, however, situation is drastically altered:

- Unified *coupling* is weaker than usual GUT value
- Unification *scale* is lower than usual GUT value

$\Longrightarrow$ Proton-decay amplitude is apparently *increased* by factor

$$\left(\frac{\alpha'_{\rm GUT}}{\alpha_{\rm GUT}}\right)\left(\frac{M_{\rm GUT}}{M'_{\rm GUT}}\right)^2 \gg 1\ .$$

Seems to be trouble!
Is there another solution?

One idea:

This is actually a GUT in *higher dimensions*!

Might there be a higher-dimensional solution to the proton-decay problem?

Yes!

Extra GUT states beyond MSSM should not be observable below unification scale!

$\Longrightarrow$ Such states should not have zero-modes!

$\Longrightarrow$ Thus, wavefunctions must be chosen *odd* under $y \to -y$:

$$\Phi^{(-)} = \sum_{n=1}^{\infty} \left[\Phi^{(n)}(\mathbf{x}) - \Phi^{(-n)}(\mathbf{x})\right] \sin\left(\frac{ny}{R}\right)$$

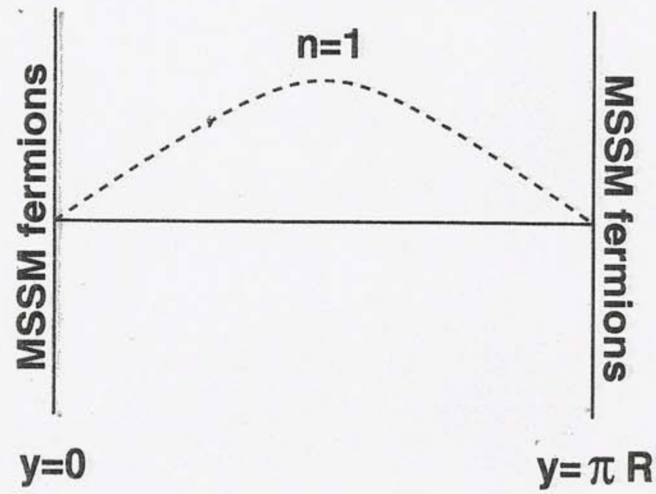
But recall that in the "minimal" scenario with $\eta = 0$, the MSSM fermions are restricted to the orbifold fixed points

$$y = 0 \quad \text{and} \quad y = \pi R\,.$$

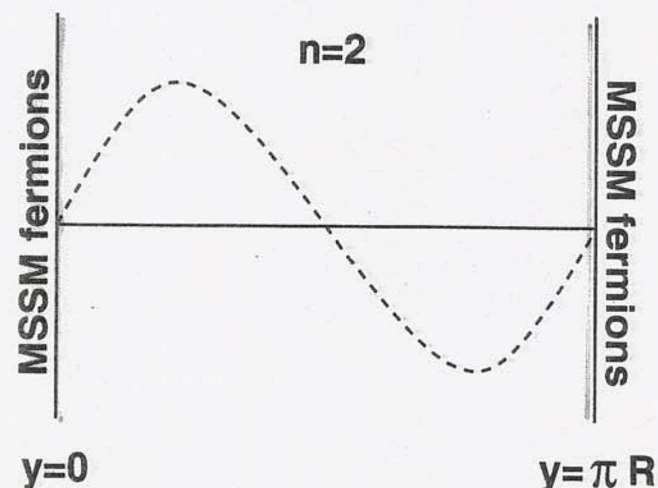
Thus, all GUT states beyond the MSSM have wavefunctions which vanish at the higher-dimensional fermion locations!

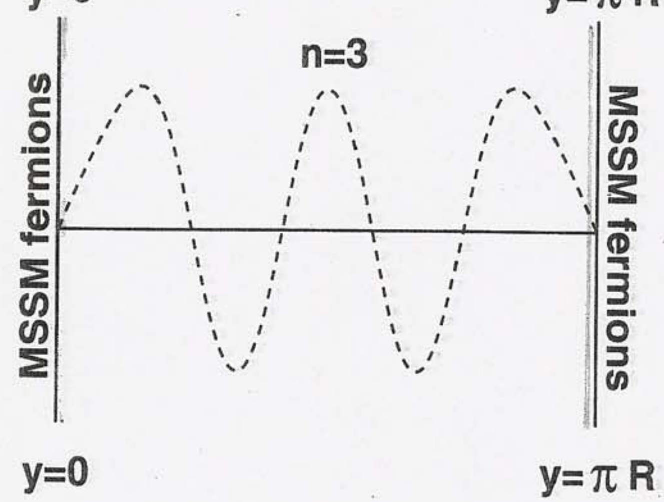
These states do not couple to the MSSM fermions!

Not only for the lowest modes of X-bosons and Higgs triplets...



but also for their infinite towers of KK excitations...





Thus, such proton-decay diagrams vanish to all orders in perturbation theory!

- This is an intrinsically *higher-dimensional* solution to the proton-decay problem.
- Valid for all radii R and all spacetime dimensions D.
- Makes use of a higher-dimensional (orbifold) symmetry.
- No analogue in usual four-dimensional GUT's.

Are there other contributions to proton decay?

Generally, yes:

- Non-perturbative effects leading to proton decay
  ($e.g.$, instantons $\sim \exp(-1/g^2)$, hence small!)
- Additional, non-renormalizable interactions
  (typical in string theory)

However, it may be possible to cancel these by making use of additional discrete symmetries (also typical in string theory).

This then becomes a model-dependent question...

Proton Stability

- KRD, E. Dudas, T. Gherghetta, hep-ph/9803466
- KRD, E. Dudas, T. Gherghetta, hep-ph/9806292
- G. Shiu, S.-H.H. Tye, hep-th/9805157
- C.P. Burgess, L.E. Ibáñez, F. Quevedo, hep-ph/9810535
- N. Arkani-Hamed, S. Dimopoulos, hep-ph/9811353
- Z. Kakushadze, hep-th/9812163

How to avoid rapid proton decay without heavy mass scales?
Two sets of ideas...

(1) higher-dimensional discrete orbifold symmetries

— suppress *vertices* of baryon number-violating processes rather than *propagators*

— many leading diagrams can be cancelled using only MSSM symmetries

— generalizations exist to prevent dangerous operators to *all* orders, assuming various kinds of extra matter

— intermediate-scale unification may not require all-order discrete symmetries

— task remains to realize such mechanisms within Type I string models

(2) approximate symmetries broken on "distant" branes

— may also be able to preserve baryon/lepton-number symmetries to desired accuracy

(3) "split fermions" inside a "fat brane"

— spatial separation between quarks & leptons in extra dimension

— prevents quark/lepton vertices leading to rapid proton decay