## Hadron Physics at the ${ }^{-1+T C}$

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Introduction

Charmed Hadron Production at the LHC

Direct Measurement of eta_b at the LHC

- Summary


## I. Introcmecion

> Although LHC was designed mainly for the study of electroweak breaking and new physics, the hadron physics capabilities of the ATLAS, CMS, and LHCb detectors are also great

At present, there are many open questions in hadron physics, which might be answered in LHC experiment

One of the key issues concerned in hadron physics is about the applicability of QCD to the description of hadrons, their production, decay nature, etc.

A wealth of return may be obtained in hadron physics study to the investigation of effective theory, new phenomena and QCD

## The $\boldsymbol{h}_{c}$

It is a recently found p-wave charmonium state, with a mass below open charm threshold

Its $J^{p c}$ is $1^{+-}$and $L=1, S=0$
Its mass is about $\mathrm{m}=3525.93 \pm 0.27 \mathrm{MeV}$, total width $\Gamma_{\text {tot }}<1 \mathrm{MeV}$
(CLEO, E760, E835, BESIII)

## The dominant decay modes of $\boldsymbol{h}_{c}$ include:

$$
\boldsymbol{h}_{c} \rightarrow J / \psi+\pi^{0}
$$

Theoretical estimate gave a branching ratio of $0.5 \%$ (Kuang. et al, Phys. Rev. D 37 (1988) 1210)

> It was observed by E760 Collab. (E760 Collaboration, Phys. Rev. D 52 (1995) 4839)

- However it was not confirmed by its successor, the $E 835$


## $\boldsymbol{h}_{c} \rightarrow \eta_{c}+\gamma$

Theoretical estimate gave a branching ratio of $50 \%$ (Y. P. Kuang. et al, Phys. Rev. D 37 (1988) 1210 ; S. Godfrey. et al, Phys. Rev. D 66 (2002) 014012 ; P. Ko. Phys. Rev D 52 (1995) 1710)

It was observed by $E 835$ Coll. (E835
Coll., Phys. Rev. D 72 (2005) 092004)

## In recently, CLEO and BESIII observed the $h_{c}$ via $\psi(2 S) \rightarrow \pi^{0}+h$

(CLEO Collaboration, Phys. Rev. L 95 (2005) 102003, Phys. Rev. D 72 (2005) 092004 , Phys. Rev. L 101 (2008) 182003; BESIII Collaboration Phys. Rev. Lett. 104 (2010) 132002)

## According to the QCD-based potential

 model prediction, to leading order of the spin-spin interaction the hyperfine splitting should be zero, i.e.,$$
\Delta M_{\mathrm{w}}\left(\mathrm{M}\left({ }^{1} \mathrm{P}_{\mathrm{t}}\right)-\mathrm{M}\left({ }^{3} \mathrm{P}_{\mathrm{J}}\right)\right) \approx 0
$$

The spin-weighted average mass of Pwave triplet states

$$
\mathrm{M}\left({ }^{3} \mathrm{P}_{\mathrm{J}}\right)=\left(\mathrm{M}\left({ }^{3} \mathrm{P}_{0}\right)+3 \mathrm{M}\left({ }^{3} \mathrm{P}_{1}\right)+5 \mathrm{M}\left({ }^{3} \mathrm{P}_{2}\right)\right) / 9=3525.30 \pm 0.04 \mathrm{MeV}
$$

## And higher order corrections to the hyperfine splitting should be less than 1 MeV

T. Appelquiat, R.M. Barnett, K.D. Lane, Annu. Rev. Nucl. Fart. Sci. 28 (1978) 387. S. Godtrey, J.L. Rosner, Hys. Rev. D 66 (z002) 014012.
D.N. Joffe, Ph.D thesis, Northwestern University, 2004; D.N. Joffe, hep-ex/0505007.

To obtain more knowledge of key point is to get enough data

## Charmonium Production Mechanism

Color-singlet model made a successful description of heavy quarkonium production and decays
[Wise, 1980; Chang, 1980; Berger \& Jones, 1981; ...]
However, its leading order result is challenged by some experimental data

O In phenomenology, non-relativistic QCD based Color-octet mechanism improves the theoretical results greatly in some cases

The NLO QCD calculations in CSM significantly enhance the color-singlet yield in lots of charmonium production processes, and hence diminish the estimated coloroctet contributions
[Chao et al., 2006, 2007, 2010; Wang et al., 2007 2008, 2010; Artoisenet, et al., 2007; ...]

In some cases, the higher order calculations in CSM can even explain the data, like in B factories

Recently complete NLO QCD calculations for $\mathrm{J} / \Psi$ production are done, and are confronted to the experimental measurements at the LHC [Chao et al., 2011; Mathias Butensch and Kniehl, 2011]

Although the transverse dependent curvature of J $/ \Psi$ production is well explained(fixed), the NLO calculation on J/ $\Psi$ polarization is still beyond the state of art


Theoretical result vs LHCb measurement [LHCb-CONF-2010-010, ICHEP 2010, Paris]


Theoretical result vs CMS measurement
[Leonardo, talk given at ICHEP 2010, Paris]

# The application of COM to charmonium physics keeps on being a debatable issue 

In all, to what degree the color-octet mechanism plays the role in quarkonium production is still an unsettled, urgent and interesting question

The J/ $\Psi$ from color-octet state ${ }^{3} \mathrm{~S}_{1}$ at the leading order of strong coupling constant, are mostly transversely polarized


Higher order corrections to color-octet state ${ }^{3} \mathrm{~S}_{1}{ }^{8}$ production and the effect of color-octet states ${ }^{1} \mathrm{~S}_{0}{ }^{8}$ and ${ }^{3} \mathrm{P}^{8}{ }^{8}$ may yield some longitudinally polarized charmonium
Beneke and Rothstein 1996; Braaten, Kniehl and lee, 2000]
The prediction of the $J / \Psi$ polarization in inclusive production is impaired in certain degree by the large higher order corrections

- No well-confirmed model yet



## $\eta_{b}$ Production and Decays <br> The lowest energy state in $Y$ family, the $\eta_{v}$ is very elusive

About thirty year after it spin triplet partner being found, recently it was observed for the first time by Babar through $Y(3 \mathrm{~s})$--> $\quad \eta_{b} \mathrm{Y}$

[Aubert, et al., Babar Collaboration, 2008]



The existence of the $\eta_{b}$ is a solid prediction of the quark model

In recent years, the search for $\eta_{b}$ has been conducted at CLEO, LEP, and CDF, B-factories, using both inclusive and exclusive methods

Search $\eta_{b}$ at CDF [Tseng, CDF, 2002] $\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$
A small cluster of 7 events can be seen, where 1.8 events are expected from background
If this cluster is due to decay, then the product of its production cross-section and decay branching fractions are near the lower limit of expectation from Braaten et al.

According Braaten, Fleming and Leibocich (hep-ph/0008091) , though helicity suppressed,

$$
\operatorname{Br}\left[\eta_{0} \rightarrow J / \psi+J / \psi\right]=7 \times 10^{-3} \sim 7 \times 10^{-5}
$$

Which seems to be overestimated, since

$$
\operatorname{Br}\left[\eta_{b} \rightarrow C+C+\bar{C}+\bar{C}\right] \sim 10^{-5}
$$

[Maltoni and Polosa,hep-ph/0405082]

## A recent analysis shows:

$$
\begin{aligned}
& \operatorname{Br}\left[\eta_{b} \rightarrow \phi+\phi\right] \approx(0.9-1.4) \times 10^{-9} \\
& \operatorname{Br}\left[\eta_{b} \rightarrow J / \psi+J / \psi\right]=2.4_{-1.9}^{+4.2} \times 10^{-8}
\end{aligned}
$$

[Jia, Phys.Rev.D78:054003,2008]
If so, such a rare decay mode perhaps will not be observed in the foreseeable future in experiment

# Following we will not further enumerate such kind of open questions, but show the possibilities of answering them through LHC experiments 

## II. Charmes

 Fon-Production at
## The leadin hc Hadror ufdition

 It is found that the LHC will produce copious data, and enables people to perform precise study on its nature
## In hadron-hadron collision, dominant processes for $h_{c}$ production include

1) 

$$
\mathrm{g}+\mathrm{g} \rightarrow \mathrm{~h}_{\mathrm{C}}\left({ }^{1} \mathrm{~S}_{0}^{[8]}\right)+\mathrm{g}
$$

2) $\mathrm{g}+\mathrm{q}(\overline{\mathrm{q}}) \rightarrow \mathrm{h}_{\mathrm{C}}\left({ }^{1} \mathrm{~S}_{0}^{[8]}\right)+\mathrm{q}(\overline{\mathrm{q}})$
3) $\mathrm{q}+\overline{\mathrm{q}} \rightarrow \mathrm{h}_{\mathrm{C}}\left({ }^{1} \mathrm{~S}_{0}^{[8]}\right)+\mathrm{g}$
4) 

$$
g+g \rightarrow h_{C}\left({ }^{1} P_{1}^{[1]}\right)+g
$$

5) $\mathrm{g}+\mathrm{c}(\overline{\mathrm{c}}) \rightarrow \mathrm{h}_{\mathrm{C}}\left({ }^{1} \mathrm{P}_{1}^{[1]}\right)+\mathrm{c}(\overline{\mathrm{c}})$

## The typ

## Feynnian diag



The differential cross section for $h_{c}$ hadroproduction is formulated in a standard way,

$$
\begin{align*}
& \frac{d \sigma}{d p_{T}}\left(p p \rightarrow h_{c}+X\right) \\
& \quad=\sum_{a, b} \int d x_{0} d y f_{0 / p}\left(x_{0}\right) f_{b / p}\left(x_{b}\right) \frac{4 p_{T} x_{0} x_{b}}{2 x_{a}-\bar{x}_{T} e^{y}} \\
& \quad \times \frac{d \hat{\sigma}}{d t}\left(a+b \rightarrow h_{c}+X\right) \tag{1}
\end{align*}
$$

where $f_{o / p}$ and $f_{b / p}$ denote the parton densities; $s, t$, and $u$ are Mandelstam variables at the parton level, $y$ stands for the rapidity of produced $h_{c} ; \bar{x}_{T} \equiv \frac{2 m_{T}}{\sqrt{S}}$ with $m_{T}=\sqrt{M^{2}+p_{T}^{2}}$; and the capital $\sqrt{S}$ and $M$ denote the total energy of incident beam and the mass of $h_{C}$, respectively.

## The processes 1)--4) were numerically calculated

dhar, PLB674, 36(2009); Qiao and Yuan, PRD63,014007(2001), lao, Ren and Sun, PLB680, 159(2009)]

And, it was found that the intrinsic charm process 5) is very important in the hc production at the LHC

[QIAO, Ren and Sun, PLB680, 159 (2009)]

## The differentiarcruss sceumanorproces

$$
\begin{aligned}
\frac{d \hat{\sigma}}{d t}= & \frac{16 \alpha_{s}^{3} \pi\left|R^{\prime}(0)\right|^{2}}{27 m_{c}\left(s-m_{c}^{2}\right)^{2}}\left(\frac{9 t}{\left(s-m_{c}^{2}\right)^{2} m_{c}^{2}}+\frac{96\left(3 m_{c}^{2}-5 s\right) m_{c}^{4}}{\left(s-m_{c}^{2}\right)\left(t-m_{c}^{2}\right)^{4}}\right. \\
& +\frac{32\left(39 m_{c}^{4}-16 s m_{c}^{2}-6 s^{2}\right) m_{c}^{2}}{\left(s-m_{c}^{2}\right)^{2}\left(t-m_{c}^{2}\right)^{3}} \\
& -\frac{6\left(57 m_{c}^{4}+14 s m_{c}^{2}-7 s^{2}\right) m_{c}^{2}}{\left(s+t-2 m_{c}^{2}\right)\left(s-m_{c}^{2}\right)^{4}} \\
& +\frac{880 m_{c}^{8}-631 s m_{c}^{6}+119 s^{2} m_{c}^{4}-201 s^{3} m_{c}^{2}+25 s^{4}}{\left(s-m_{c}^{2}\right)^{4}\left(t-m_{c}^{2}\right) m_{c}^{2}} \\
& +\frac{1177 m_{c}^{8}-856 s m_{c}^{6}-82 s^{2} m_{c}^{4}-88 s^{3} m_{c}^{2}+9 s^{4}}{\left(s-m_{c}^{2}\right)^{3}\left(t-m_{c}^{2}\right)^{2} m_{c}^{2}} \\
& +\frac{2}{\left(s+t-2 m_{c}^{2}\right)^{2}}-\frac{256 m_{c}^{6}}{\left(t-m_{c}^{2}\right)^{5}} \\
& +\frac{118 m_{c}^{8}-379 s m_{c}^{6}+141 s^{2} m_{c}^{4}-161 s^{3} m_{c}^{2}+25 s^{4}}{\left(s-m_{c}^{2}\right)^{5} m_{c}^{2}} \\
& \left.-\frac{8 m_{c}^{2}}{\left(s+t-2 m_{c}^{2}\right)^{3}}\right) .
\end{aligned}
$$

## With

$$
\begin{aligned}
& \sqrt{S}=14 \mathrm{TeV}, m_{C}=M / 2=1.78 \mathrm{GeV} \\
& \left|\eta\left(h_{c}\right)\right|<2.2 \quad\left|R^{\prime}(0)\right|=\sqrt{\frac{2 \pi}{27}\left\langle\left(0\left|\mathcal{O}_{1}^{h_{c}}\left({ }^{1} P_{1}\right)\right| 0\right\rangle\right.}
\end{aligned}
$$

$$
\langle 0| D_{1}^{h_{c}}\left({ }^{1} P_{1}\right)|0\rangle=0.32 \mathrm{GeV}^{5}
$$

$$
\left.\langle 0| O_{8}^{h_{c}}\left({ }^{1} S_{0}\right)|0\rangle=9.8 \times 10^{-3} C_{e v}\right)^{3}
$$

PL. Cho, A.K. Leibovich, Phys. Rev. D 53 (1996) 150; P.L. Cho, A.K. Leibovich, Phys. Rev. D 53 (1996) 53.

## We obtain(

## right, solid foi



## The resultenows.

## The color-octet process contributes more to hc hadroprodution at the LHC

color-singlet mechanism, the intrinsic charm quark induced process dominates over the other one

## From

$$
\begin{aligned}
& (\mathrm{A}) \mathrm{h}_{\mathrm{C}} \rightarrow \mathrm{~J} / \psi+\pi^{0} \rightarrow \mu^{+} \mu^{-}+\gamma \gamma \\
& (\mathrm{B}) \mathrm{h}_{\mathrm{C}} \rightarrow \eta_{\mathrm{C}}+\gamma \rightarrow \mathrm{p} \overline{\mathrm{p}}+\gamma \\
& (\mathrm{C}) \mathrm{h}_{\mathrm{C}} \rightarrow \eta_{\mathrm{C}}+\gamma \rightarrow \gamma \gamma+\gamma \\
& \operatorname{} \operatorname{Br}[\mathrm{A}]=0.5 \% \times 5.9 \% \times 100 \%=2.95 \times 10^{-4}
\end{aligned}
$$

$-\operatorname{Br}[\mathrm{B}]=50 \% \times 0.13 \%=6.5 \times 10^{-4}$
$-\operatorname{Br}[\mathrm{C}]=50 \% \times 0.024 \%=1.2 \times 10^{-4}$

## That means

## Color-singlet event

| PTcut | 5 GeV | 10 GeV | 20 GeV | 30 GeV |
| :--- | :---: | :---: | :---: | :---: |
| Total | $1.65 \times 10^{8}$ | $4.32 \times 10^{6}$ | $8.14 \times 10^{4}$ | $7.57 \times 10^{3}$ |
| Chain $[\mathrm{A}]$ | $4.49 \times 10^{4}$ | $1.30 \times 10^{3}$ | $2.44 \times 10$ | 2.27 |
| Chain $[\mathrm{B}]$ | $1.07 \times 10^{5}$ | $2.81 \times 10^{3}$ | $5.29 \times 10$ | 4.92 |
| Chain $[\mathrm{C}]$ | $1.97 \times 10^{4}$ | $5.19 \times 10^{2}$ | 9.76 | 0.91 |

## Color-octet event

| PTcut | 5 GeV | 10 GeV | 20 GeV | 30 GeV |
| :--- | :---: | :---: | :---: | :---: |
| Total | $3.78 \times 10^{9}$ | $1.56 \times 10^{8}$ | $3.67 \times 10^{6}$ | $3.54 \times 10^{5}$ |
| Chain $[\mathrm{A}]$ | $1.13 \times 10^{6}$ | $4.68 \times 10^{4}$ | $1.10 \times 10^{3}$ | $1.06 \times 10^{2}$ |
| Chain $[\mathrm{B}]$ | $2.45 \times 10^{6}$ | $1.01 \times 10^{5}$ | $2.38 \times 10^{3}$ | $2.30 \times 10^{2}$ |
| Chain $[\mathrm{C}]$ | $4.53 \times 10^{5}$ | $1.87 \times 10^{4}$ | $4.40 \times 10^{2}$ | $4.42 \times 10$ |

## hc productiontenm



## Fragmentation function for heavy quarkonium

- Generally, fragmentation function for heavy quarkonium has factorized formation



## Fragmentation function for hc

- Charm quark
- Gluon fragmentation fragmentation



## Gluon fragmentauriruncuon for he

## G.Hao \& C.F.Q, Y.B.Zuo, arXiv:0911.5539

- At LO of $\alpha_{s}$ and NLO of $v$
$D_{g \rightarrow h_{c}}(z)=d_{1}(z, \Lambda)\left\langle\mathcal{O}^{h_{c}}\left({ }^{1} P_{1}^{(1)}\right)\right\rangle+d_{8}(z)\left\langle\mathcal{O}^{h_{c}}\left({ }^{1} S_{0}^{(8)}\right)\right\rangle(\Lambda)$
- $\Lambda$ is factorization scale.
- Only $d_{1}(z)$ and Matrix element of octet operator depends on $\Lambda$ :

$$
\Lambda \frac{d}{d \Lambda}\langle 0| \mathcal{O}_{8}^{h_{c}}\left({ }^{1} S_{0}\right)|0\rangle=\frac{4 C_{F} \alpha_{s}(\Lambda)}{3 N_{c} \pi m_{c}^{2}}\langle 0| \mathcal{O}_{1}^{h_{c}}\left({ }^{1} P_{1}\right)|0\rangle .
$$

Braaten and Yuan PRD50,3176,1994

## Calculation of $\mathrm{d}_{8}$

- Calculation of $\mathrm{d}_{8}(\mathrm{z})$ is easy


$$
d_{8}(z)=\frac{5}{24} \frac{\alpha_{s}^{2}}{m_{c}^{3}}\left[3 z-2 z^{2}+2(1-z) \ln (1-z)\right]
$$

## Calculation of $\mathrm{d}_{1}$

- Infrared divergence in color-singlet process
- set a cutoff $\Lambda$ for gluon's energy
- $\mathrm{d}_{1}(\mathrm{z})$ depends on $\Lambda$


$$
d_{1}(z, \Lambda)
$$

$=f(z)-\frac{10 \alpha_{s}^{3}}{81 \pi m_{c}^{5}}\left[3 z-2 z^{2}+2(1-z) \ln (1-z)\right] \ln \frac{\Lambda}{m_{c}}$

- $f(z)$ has no $\Lambda$ dependence, but very complicated expression.


## Fragmentation function for hc

- To avoid large logarithms, we choose

$$
\Lambda=m_{c}
$$

- Gluon fragmentation function is

$$
D_{g \rightarrow h_{c}}(z)=f(z)\left\langle\mathcal{O}^{h_{c}}\left({ }^{1} P_{1}^{(1)}\right)\right\rangle+d_{8}(z)\left\langle\mathcal{O}^{h_{c}}\left({ }^{1} S_{0}^{(8)}\right)\right\rangle\left(m_{c}\right)
$$

- Probability of gluon fragmentation into hc

$$
P_{g \rightarrow h_{c}}=\int_{0}^{1} d z D_{g \rightarrow h_{c}}(z)
$$

# $\mathrm{c} \rightarrow$ hc probability is about $10^{\wedge}-5$ 

$\mathrm{g} \mathrm{->}$ hc probability is about $10^{\wedge}-7$

Therefore, for near future experiment, the hc fragmentation production is negligible

## Polarized J/ $\Psi$ Pair Production at the LHC

It was observed by NA3 collaboration in pion-platinum interaction, and found sigma $(\mathrm{J} / \Psi \mathrm{J} / \Psi) /$ sigma $(\mathrm{J} / \Psi) \sim 10^{\wedge}-4$

In very recently it is observed by LHCb collaboration and measured by CMS collaboration at the LHC
[LHCb-CONF-2011-009]

## In exclusive process the higher order contributions are relatively suppressed

Therefore, to measure the J/ $\Psi$ pair production at the LHC is possibly a feasible way to detect the charmonium production mechanism

## owthe LHC with

The luminosity : $10^{32}-10^{34} \mathrm{~cm}^{-2} / \mathrm{s}$

Center mass of energy: 7-10-14 TeV

The pseudorapidity : $\eta<2.2$

## The polarized J/ $\Psi$ pair hadroproduction is estimated

[Barger et al. 1996; CFQ 2002; Li, Zhang and Chao 2009; Sun, Sun and QCF 2010]

Both the color-singlet and color-octet effects are considered, at the leading order of strong coupling constant

## Fock State Configuration of the J/ $/ \Psi$ pair

## The typucanteymane grams

## in color singlet case



(4)


( B

$+$

$+\quad . \quad=$

## In color octet case










## With input parameters



## The integrated cross sections of J/ $/ \Psi$ proauction in color-singlet model

## The integrated cross sections of J/ $\Psi$ -production in color-octet model

## The differential cross-section of J/Чpair

 production versus $p_{T}$ at the LHC in CSM
a, b, c represent $\perp \perp,\| \|, \| \perp$, respectively

## The differential cross-section of J/Чpair production versus $p_{T}$ at the LHC in COM


a , b, c represent $\perp \perp,\| \|, \| \perp$, respectively

The results
For the case of colliding energy up to 10 TeV and luminosity $1 \mathrm{fb}^{\wedge}-1$, the data with at least one $J / \Psi$ in a pair being longitudinally polarized are about fifty percent of the total yield with the lower transverse momentum bound of 5 GeV
In the pair production, both color-singlet and octet schemes exhibit that the J/ $\Psi$ should be dominated by the transversely polarized yield in large transverse momentum region

## III. Direct Measurement of $\eta_{b}$ at the LHC

LHC can produce $10^{\wedge} 8 \sim 10^{\wedge} 9 \eta_{b}$ per year

Recently, the $\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$ process was calculated at the next-to-leading order accuracy and find the NLO correction many enhance the branching fraction to the same level of relativistic correction
[Bin Gong, Yu Jia, and J.X.Wang, PLB, 2009]
[Braguta \& Kartvelishvili, PRD, 2010] [Sun, Hao, Qiao, 2010]


FIG. 1: Typical Feynman diagrams of the exclusive process $\eta_{b}\left(P_{\eta_{b}}\right) \rightarrow J / \psi\left(P_{J / \psi_{1}}\right)+J / \psi\left(P_{J / \psi_{2}}\right)$ at the one-loop level.

Beeause of parity and Lorentz invariance, the decay amplitude possesses the following unique tensor structure

$$
\mathcal{M}\left(\lambda_{1}, \lambda_{2}\right)=\mathcal{A} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{J / \psi_{1}}^{* \mu}\left(\lambda_{1}\right) \varepsilon_{J / \psi_{2}}^{* \nu}\left(\lambda_{2}\right) P_{J / \psi_{1}}^{\rho} P_{J / \psi_{1}}^{\sigma}
$$

## After a lengthy calculation, we obtain:

## [Sun, Hao, QCF, 2010]

$$
\mathcal{A}=\frac{512 \sqrt{2} \pi \alpha_{s}^{3} m_{c} \psi_{\eta_{b}}(0) \psi_{J / \psi}^{2}(0)}{9 \sqrt{3} m_{b}^{9 / 2}\left(m_{b}^{2}-4 m_{c}^{2}\right)} F\left(m_{c}^{2}, m_{b}^{2}\right)
$$

with the real and imaginary parts reading as

$$
\begin{aligned}
\operatorname{Re}\left(F\left(m_{c}^{2}, m_{b}^{2}\right)\right)_{a s y}= & \frac{19}{32} \log ^{2}(a)-\frac{1}{8} \log (2) \log (a)+\frac{5}{4} \log (a)+\frac{5}{16} \log ^{2}(2) \\
& +\frac{1}{2} \log (2)+\frac{29 \pi^{2}}{96}-\frac{3 \sqrt{3}}{8} \pi+\frac{3}{4}
\end{aligned}
$$

$$
\operatorname{Im}\left(F\left(m_{c}^{2}, m_{b}^{2}\right)\right)_{a s y}=\frac{19 \pi}{16} \log (a)+\frac{7 \pi}{16} \log (2)+\pi
$$

## in small mc limit

The double and single logarithmic terms agree with Gong et al. result, but not the finite terms

## With the following inputs

$$
\psi_{J / \psi}(0)=0.263 \mathrm{GeV}^{3 / 2}, m_{c}=1.5 \mathrm{GeV}, m_{b}=4.7 \mathrm{GeV}, \alpha_{s}=0.18 \sim 0.26
$$

## we have

$$
\operatorname{Br}\left[\eta_{b} \rightarrow J / \psi J / \psi\right]=5.93 \times 10^{-8} \sim 2.58 \times 10^{-7}
$$

## Higher twist contributions

In the light-cone formalism, the leading twist term has no contribution to the $\eta_{b} \rightarrow \mathrm{~J} / \psi \mathrm{J} / \psi$ process

We expand the LCDA projector in momentum space given by Beneke and Feldmann(2001) to twist-4, which yields more terms than what employed by Braguta et al.

With the asymptotic form for twist-2 distribution amplitudes, i.e.

$$
\phi_{\perp}(u)=\phi_{\|}(u)=\phi_{A S}(u)=6 v(1-u)
$$

The analytical decay amplitude turns to be pretty simple, it reads

$$
\begin{aligned}
\mathcal{M}_{\perp \perp}= & T_{0} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{1 \perp}^{* \mu} \varepsilon_{2 \perp}^{* \nu} n_{-}^{\rho} n_{+}^{\sigma} \frac{9}{256 E_{1}^{2} E_{2}^{2}} \times\left[\left(\pi^{2}-4\right) m_{V_{1}} m_{V_{2}}\left(f_{V_{1}} \hat{f}_{V_{2}}+f_{V_{1}} \tilde{f}_{V_{2}}\right)\right. \\
& \left.+2 \pi^{2}\left(m_{V_{2}}^{2} f_{V_{1}}^{T} \tilde{f}_{V_{2}}^{T}+m_{V_{1}}^{2} f_{V_{2}}^{T} \tilde{f}_{V_{1}}^{T}\right)\right]
\end{aligned}
$$

## Here,

$$
\tilde{f}_{V}=f_{V}-f_{V}^{T} \frac{m_{1}+m_{2}}{m_{V}}, \quad \tilde{f}_{V}^{T}=f_{V}^{T}-f_{V} \frac{m_{1}+m_{2}}{m_{V}}
$$

and decay constants $f_{J / \psi}$ and $f_{J / \phi}^{T}$ can be obtained through experiment and NRQCD

$$
f_{J / \psi}=416 \mathrm{MeV} \quad \boldsymbol{\quad} \quad f_{J / \psi}^{T}=379 \mathrm{MeV}
$$

Then the numerical result reads

$$
B r\left[\eta_{b} \rightarrow J / \psi J / \psi\right]=(1.1 \sim 2.3) \times \mathbf{1 0}^{-6} .
$$

-If so, the $\eta_{b}$ is measurable by its double J/ $\Psi$ decay mode

## IV. Sump

The LHC may be suitable for the precise hc study

By the study of hc production, the charmonium production mechanism may also be elucidated in some sense

The early running of the LHC will supply numerous event numbers of J/ $\Psi$ pair production

At the LHC, we expect the experimental measurement on double $J / \Psi$ production may tell us more about the charmonium production mechanism

To further study the nature of recently observed state $\eta_{b}$, direct measurement of its decay products is necessary

In the light cone formalism, expanding the LCDAs of final vector mesons to twist4 , we find that the higher twist terms contribute more to the $\eta_{0}$ decay width than what from the NLO corrections

# According to our calculation up to twist-4 , the branching fraction of process can be as large as $10^{\wedge}-6$, which enables the direct search of in Tevatron Run II or LHC 

Besides what we mentioned, there are still more topics on hadron physics should be performed at the LHC, like heavy meson indirect production, doubly heavy baryon production, exotic states study, etc.

The LHC may have a rich hadron physics program


