Hadron Physics at the LHC

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• Summary
I. Introduction

• Although LHC was designed mainly for the study of electroweak breaking and new physics, the hadron physics capabilities of the ATLAS, CMS, and LHCb detectors are also great.

• At present, there are many open questions in hadron physics, which might be answered in LHC experiment.
• One of the key issues concerned in hadron physics is about the applicability of QCD to the description of hadrons, their production, decay nature, etc.

• A wealth of return may be obtained in hadron physics study to the investigation of effective theory, new phenomena and QCD.
The $h_c$

- It is a recently found p-wave charmonium state, with a mass below open charm threshold

- Its $J^{pc}$ is $1^{+-}$ and $L=1$, $S=0$

- Its mass is about $m = 3525.93 \pm 0.27\text{MeV}$, total width $\Gamma_{tot} < 1\text{ MeV}$

(CLEO, E760, E835, BESIII)
The dominant decay modes of $h_c$ include:

$$h_c \rightarrow J/\psi + \pi^0$$

- Theoretical estimate gave a branching ratio of 0.5% (Kuang et al., Phys. Rev. D 37 (1988) 1210)

- It was observed by E760 Collab. (E760 Collaboration, Phys. Rev. D 52 (1995) 4839)

- However it was not confirmed by its successor, the E835
$h_c \rightarrow \eta_c + \gamma$


- It was observed by E835 Coll. (E835 Coll., Phys. Rev. D 72 (2005) 092004)
In recently, CLEO and BESIII observed the \( h_c \) via \( \psi(2S) \rightarrow \pi^0 + h_c \rightarrow \pi^0 + \eta_c + \gamma \)

According to the QCD-based potential model prediction, to leading order of the spin–spin interaction the hyperfine splitting should be zero, i.e.,

$$\Delta M_{hf} (M(^1P_1) - M(^3P_J)) \approx 0$$

The spin-weighted average mass of P-wave triplet states

$$M(^3P_J) - (M(^3P_0) + 3M(^3P_1) + 5M(^3P_2))/9 = 3525.30 \pm 0.04 \text{MeV}$$
And higher order corrections to the hyperfine splitting should be less than 1 MeV.

To obtain more knowledge of $h_v$, a key point is to get enough data.
Charmonium Production Mechanism

- Color-singlet model made a successful description of heavy quarkonium production and decays
  
  [Wise, 1980; Chang, 1980; Berger & Jones, 1981; ...]

- However, its leading order result is challenged by some experimental data

- In phenomenology, non-relativistic QCD based Color-octet mechanism improves the theoretical results greatly in some cases
The NLO QCD calculations in CSM significantly enhance the color-singlet yield in lots of charmonium production processes, and hence diminish the estimated color-octet contributions

[Chao et al., 2006, 2007, 2010; Wang et al., 2007 2008, 2010; Artoisenet, et al., 2007; ...]

In some cases, the higher order calculations in CSM can even explain the data, like in B factories
Recently complete NLO QCD calculations for $J/\Psi$ production are done, and are confronted to the experimental measurements at the LHC [Chao et al., 2011; Mathias Butensch and Kniehl, 2011]

Although the transverse dependent curvature of $J/\Psi$ production is well explained (fixed), the NLO calculation on $J/\Psi$ polarization is still beyond the state of art
Theoretical result vs LHCb measurement

[LHCb-CONF-2010-010, ICHEP 2010, Paris]
Theoretical result vs CMS measurement

[Leonardo, talk given at ICHEP 2010, Paris]
The application of COM to charmonium physics keeps on being a debatable issue.

In all, to what degree the color-octet mechanism plays the role in quarkonium production is still an unsettled, urgent and interesting question.
The $J/\Psi$ from color-octet state $^3S_1$ at the leading order of strong coupling constant, are mostly transversely polarized.
Higher order corrections to color-octet state $^3S_1^8$ production and the effect of color-octet states $^1S_0^8$ and $^3P_J^8$ may yield some longitudinally polarized charmonium

[Beneke and Rothstein 1996; Braaten, Kniehl and Lee, 2000]

The prediction of the $J/\Psi$ polarization in inclusive production is impaired in certain degree by the large higher order corrections
• No well-confirmed model yet
**$\eta_b$ Production and Decays**

- The lowest energy state in Y family, the $\eta_b$ is very elusive.

- About thirty years after it spin triplet partner being found, recently it was observed for the first time by Babar through $Y (3s) \rightarrow \eta_b \gamma$

[Aubert, et al., Babar Collaboration, 2008]
The bottomonium energy level spectrum

(CLEO Collaboration July 19, 2002)
The existence of the $\eta_b$ is a solid prediction of the quark model.

In recent years, the search for $\eta_b$ has been conducted at CLEO, LEP, and CDF, B-factories, using both inclusive and exclusive methods.
Search $\eta_b$ at CDF [Tseng, CDF, 2002]

- $\eta_b \rightarrow J/\psi J/\psi$

- A small cluster of 7 events can be seen, where 1.8 events are expected from background.

- If this cluster is due to $\eta_b$ decay, then the product of its production cross-section and decay branching fractions are near the lower limit of expectation from Braaten et al.
According Braaten, Fleming and Leibocich (hep-ph/0008091), though helicity suppressed,

\[
Br[\eta_b \rightarrow J/\psi + J/\psi] = 7 \times 10^{-3} \sim 7 \times 10^{-5}
\]

Which seems to be overestimated, since

\[
Br[\eta_b \rightarrow C + C + \bar{C} + \bar{C}] \sim 10^{-5}
\]

A recent analysis shows:

\[ Br[\eta_b \to \phi + \phi] \approx (0.9 - 1.4) \times 10^{-9} \]

\[ Br[\eta_b \to J/\psi + J/\psi] = 2.4^{+4.2}_{-1.9} \times 10^{-8} \]


If so, such a rare decay mode perhaps will not be observed in the foreseeable future in experiment
Following we will not further enumerate such kind of open questions, but show the possibilities of answering them through LHC experiments.
II. Charmed Hadron Production at the LHC

The leading order calculation for charmed hadron production.

- It is found that the LHC will produce copious $h_c$ data, and enables people to perform precise study on its nature.
In hadron-hadron collision, dominant processes for $h_c$ production include

1) $g + g \rightarrow h_c (^{1}S_0^{[8]}) + g$

2) $g + q(\bar{q}) \rightarrow h_c (^{1}S_0^{[8]}) + q(\bar{q})$

3) $q + \bar{q} \rightarrow h_c (^{1}S_0^{[8]}) + g$

4) $g + g \rightarrow h_c (^{1}P_1^{[1]}) + g$

5) $g + c(\bar{c}) \rightarrow h_c (^{1}P_1^{[1]}) + c(\bar{c})$
The typical Feynman diagrams
The differential cross section for $h_c$ hadroproduction is formulated in a standard way,

$$
\frac{d\sigma}{dp_T} (pp \to h_c + X) = \sum_{a,b} \int dx_a \, dy \, f_{a/p}(x_a) f_{b/p}(x_b) \frac{4p_T x_a x_b}{2x_a - \bar{x}_T e^y} \left( a + b \to h_c + X \right),
$$

where $f_{a/p}$ and $f_{b/p}$ denote the parton densities; $s$, $t$, and $u$ are Mandelstam variables at the parton level; $y$ stands for the rapidity of produced $h_c$; $\bar{x}_T \equiv \frac{2m_T}{\sqrt{S}}$ with $m_T = \sqrt{M^2 + p_T^2}$; and the capital $\sqrt{S}$ and $M$ denote the total energy of incident beam and the mass of $h_c$, respectively.
• The processes 1)--4) were numerically calculated

[Sridhar, PLB674, 36(2009); Qiao and Yuan, PRD63, 014007(2001), Qiao, Ren and Sun, PLB680, 159(2009)]

• And, it was found that the intrinsic charm process 5) is very important in the hc production at the LHC

[QIAO, Ren and Sun, PLB680, 159 (2009)]
The differential cross-section for process 5) is:

\[
\frac{d\hat{\sigma}}{dt} = \frac{16\alpha_s^3 \pi |R'(0)|^2}{27 m_c (s - m_c^2)^2} \left( \frac{9t}{(s - m_c^2)^2 m_c^2} + \frac{96 (3m_c^2 - 5s) m_c^4}{(s - m_c^2) (t - m_c^2)^4} \right) \\
+ \frac{32 (39m_c^4 - 16sm_c^2 - 6s^2)m_c^2}{(s - m_c^2)^2 (t - m_c^2)^3} \\
- \frac{6 (57m_c^4 + 14sm_c^2 - 7s^2)m_c^2}{(s + t - 2m_c^2)(s - m_c^2)^4} \\
+ \frac{880m_c^8 - 631sm_c^6 + 119s^2m_c^4 - 201s^3m_c^2 + 25s^4}{(s - m_c^2)^4 (t - m_c^2)m_c^2} \\
+ \frac{1177m_c^8 - 856sm_c^6 - 82s^2m_c^4 - 88s^3m_c^2 + 9s^4}{(s - m_c^2)^3 (t - m_c^2)^2 m_c^2} \\
+ \frac{2}{(s + t - 2m_c^2)^2} - \frac{256m_c^6}{(t - m_c^2)^5} \\
+ \frac{118m_c^8 - 379sm_c^6 + 141s^2m_c^4 - 161s^3m_c^2 + 25s^4}{(s - m_c^2)^5 m_c^2} \\
- \frac{8m_c^2}{(s + t - 2m_c^2)^3} \right).
\]
With the inputs of:

\[ \sqrt{s} = 14 \text{ TeV}, \quad m_c = M/2 = 1.78 \text{ GeV} \]

\[ |\eta(h_c)| < 2.2 \]

\[ |R'(0)| = \sqrt{\frac{2\pi}{27}} \langle 0 | O_1^{hc} (1 P_1) | 0 \rangle \]

\[ \langle 0 | O_1^{hc} (1 P_1) | 0 \rangle = 0.32 \text{ GeV}^5 \]

\[ \langle 0 | O_8^{hc} (1 S_0) | 0 \rangle = 9.8 \times 10^{-3} \text{ GeV}^3 \]

We obtain (a-e for process 1-5 on the left; on right, solid for CO and dashed line for CS)
The result shows:

• **The** color-octet process contributes more to hc hadroproduction at the LHC

• **In** color-singlet mechanism, the intrinsic charm quark induced process dominates over the other one
From PDG and theoretical calculation

\[(A) \ h_c \rightarrow J/\psi + \pi^0 \rightarrow \mu^+\mu^- + \gamma\gamma\]
\[(B) \ h_c \rightarrow \eta_c + \gamma \rightarrow p\bar{p} + \gamma\]
\[(C) \ h_c \rightarrow \eta_c + \gamma \rightarrow \gamma\gamma + \gamma\]

- \(\text{Br}[A] = 0.5\% \times 5.9\% \times 100\% = 2.95 \times 10^{-4}\)
- \(\text{Br}[B] = 50\% \times 0.13\% = 6.5 \times 10^{-4}\)
- \(\text{Br}[C] = 50\% \times 0.024\% = 1.2 \times 10^{-4}\)
That means:

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<tr>
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<th>20GeV</th>
<th>30GeV</th>
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hc production through fragmentation

\[ d\sigma_H(p) = \sum_i \int_0^1 dz \ d\hat{\sigma}_i(p/z) \ D_{i\to H}(z) \]

Cross section for parton production

Fragmentation Function: How parton i fragments into hadron H
Fragmentation function for heavy quarkonium

Generally, fragmentation function for heavy quarkonium has factorized formation

\[ D_{i\rightarrow H}(z) = \sum_n d_n(z) \langle O^H_n \rangle \]

- Production of quark pair in \( n \) state
- pQCD calculable due to large mass of heavy quark

- NRQCD operator
- Quark pair hadronization into heavy quarkonium
- non-perturbative quantity

\( n \): quantum number
Color, spin
Fragmentation function for $hc$

- Charm quark fragmentation
- Gluon fragmentation

![Diagram showing charm quark and gluon fragmentation processes with labels for color singlet and octet.]
Gluon fragmentation function for hc


- At LO of $\alpha_s$ and NLO of $\nu$

\[ D_{g \to hc}(z) = d_1(z, \Lambda) \langle \mathcal{O}^{hc}(1P_1^{(1)}) \rangle + d_8(z) \langle \mathcal{O}^{hc}(1S_0^{(8)}) \rangle(\Lambda) \]

- $\Lambda$ is factorization scale.

- Only $d_1(z)$ and Matrix element of octet operator depends on $\Lambda$:

\[
\Lambda \frac{d}{d\Lambda} \langle 0 | \mathcal{O}^{hc}_8(1S_0) | 0 \rangle = \frac{4C_F \alpha_s(\Lambda)}{3N_c \pi m_c^2} \langle 0 | \mathcal{O}^{hc}_1(1P_1) | 0 \rangle.
\]

Braaten and Yuan PRD50,3176,1994
Calculation of $d_8$

- Calculation of $d_8(z)$ is easy

$$d_8(z) = \frac{5}{24} \frac{\alpha_s^2}{m_c^3} [3z - 2z^2 + 2(1 - z) \ln(1 - z)]$$
Calculation of $d_1$

- Infrared divergence in color-singlet process
  - set a cutoff $\Lambda$ for gluon’s energy
  - $d_1(z)$ depends on $\Lambda$

$$d_1(z, \Lambda) = f(z) - \frac{10\alpha_s^3}{81\pi m_c^5} [3z - 2z^2 + 2(1 - z) \ln(1 - z)] \ln \frac{\Lambda}{m_c}$$

- $f(z)$ has no $\Lambda$ dependence, but very complicated expression.
Fragmentation function for hc

- To avoid large logarithms, we choose

\[ \Lambda = m_c \]

- Gluon fragmentation function is

\[ D_{g \rightarrow h_c}(z) = f(z)\langle \mathcal{O}_{h_c}^{1} P_1^{(1)} \rangle + d_8(z)\langle \mathcal{O}_{h_c}^{1} S_0^{(8)} \rangle(m_c) \]

- Probability of gluon fragmentation into hc

\[ P_{g \rightarrow h_c} = \int_0^1 dz D_{g \rightarrow h_c}(z) \]
• c -> hc probability is about $10^{-5}$

• g -> hc probability is about $10^{-7}$

• Therefore, for near future experiment, the hc fragmentation production is negligible
Polarized $J/\Psi$ Pair Production at the LHC

- It was observed by NA3 collaboration in pion-platinum interaction, and found $\sigma(J/\Psi J/\Psi)/\sigma(J/\Psi) \sim 10^{-4}$

- In very recently it is observed by LHCb collaboration and measured by CMS collaboration at the LHC

[LHCb-CONF-2011-009]
In exclusive process the higher order contributions are relatively suppressed.

Therefore, to measure the $J/\Psi$ pair production at the LHC is possibly a feasible way to detect the charmonium production mechanism.
For the LHC with

- The luminosity: $10^{32} - 10^{34}$ cm$^{-2}$/s
- Center mass of energy: 7 – 10 – 14 TeV
- The pseudorapidity: $\eta < 2.2$
The polarized $J/\Psi$ pair hadroproduction is estimated

[Barger et al. 1996; CFQ 2002; Li, Zhang and Chao 2009; Sun, Sun and QCF 2010]

Both the color-singlet and color-octet effects are considered, at the leading order of strong coupling constant
Fock State Configuration of the $J/\Psi$ pair

\[
J/\psi = O(1)|cc[3S^{(1)}_1]\rangle + O(v)|cc[3P^{(8)}_j]g\rangle + O(v^2)|cc[3S^{(1,8)}_1]gg\rangle + O(v^2)|cc[1S^{(8)}_0]g\rangle + \ldots
\]

\[
|J/\psi\rangle|J/\psi\rangle = |cc[3S^{(1)}_1]\rangle|cc[3S^{(1)}_1]\rangle + \left|cc[3S^{(1)}_1]\rangle|cc[3S^{(8)}_1]gg\rangle\right|_{Part_1}
\]

\[
+|cc[3S^{(1)}_1]\rangle(|cc[3P^{(8)}_j]g\rangle + |cc[1S^{(8)}_0]g\rangle) + |cc[3S^{(8)}_1]gg\rangle|cc[3S^{(8)}_1]gg\rangle\right|_{Part_3}
\]

\[
+|cc[3S^{(8)}_1]gg\rangle(|cc[3P^{(8)}_j]g\rangle + |cc[1S^{(8)}_0]g\rangle)\right|_{Part_4}
\]

\[
+\left(|cc[3P^{(8)}_j]g\rangle + |cc[1S^{(8)}_0]g\rangle\right)(|cc[3P^{(8)}_j]g\rangle + |cc[1S^{(8)}_0]g\rangle)\right|_{Part_5}
\]
The typical Feynman diagrams in color singlet case
In color octet case
With input parameters

\[ m_c = 1.5 \text{ GeV} \]

\[ |R(0)|_{cs}^2 = 0.8 \text{ GeV}^3 \]

\[ < O_8^{J/\psi} (3 S_1) >= 0.012 \text{ GeV}^3 \]

\[ B(J/\psi \rightarrow \mu^+ \mu^-) = 0.0597 \]

\[ \eta(J/\psi) < 2.2 \]
The integrated cross sections of $J/\Psi$ pair production in color-singlet model

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The integrated cross sections of $J/\Psi$ pair production in color-octet model

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BCVSPIN2011, HUE

Cong-Feng Qiao
The differential cross-section of $J/\Psi$ pair production versus $p_T$ at the LHC in CSM

$a$, $b$, $c$ represent $\perp \perp$, $|| ||$, $|| \perp$, respectively
The differential cross-section of $J/\Psi$ pair production versus $p_T$ at the LHC in COM

$$d\sigma(gg \to J/\Psi + X)/dp_T$$ (nb/GeV)

- $a$, $b$, $c$ represent $\perp \perp$, $|| ||$, $|| \perp$, respectively

BCVSPIN2011, HUE

CONG-FENG QIAO
The results show:

- For the case of colliding energy up to 10 TeV and luminosity 1 fb$^{-1}$, the data with at least one J/$\Psi$ in a pair being longitudinally polarized are about fifty percent of the total yield with the lower transverse momentum bound of 5 GeV.

- In the pair production, both color-singlet and octet schemes exhibit that the J/$\Psi$ should be dominated by the transversely polarized yield in large transverse momentum region.
III. Direct Measurement of $\eta_b$ at the LHC

- LHC can produce $10^8 \sim 10^9$ $\eta_b$ per year

- Recently, the $\eta_b \rightarrow J/\psi J/\psi$ process was calculated at the next-to-leading order accuracy and find the NLO correction many enhance the branching fraction to the same level of relativistic correction

  [Bin Gong, Yu Jia, and J.X.Wang, PLB, 2009]

  [Braguta & Kartvelishvili, PRD, 2010]

  [Sun, Hao, Qiao, 2010]
Because of parity and Lorentz invariance, the decay amplitude possesses the following unique tensor structure:

\[ \mathcal{M}(\lambda_1, \lambda_2) = A \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{J/\psi_1}^* (\lambda_1) \varepsilon_{J/\psi_2}^* (\lambda_2) P_{J/\psi_1}^\rho P_{J/\psi_1}^\sigma. \]
After a lengthy calculation, we obtain:

\[ A = \frac{512\sqrt{2\pi}a_s^3m_c\psi_{\eta_b}(0)\psi_{J/\psi}^2(0)}{9\sqrt{3}m_b^{9/2}(m_b^2 - 4m_c^2)} F(m_c^2, m_b^2) \]

with the real and imaginary parts reading as

\[
\text{Re}(F(m_c^2, m_b^2))_{\alpha\gamma} = \frac{19}{32}\log^2(a) - \frac{1}{8}\log(2)\log(a) + \frac{5}{4}\log(a) + \frac{5}{16}\log^2(2) \\
+ \frac{1}{2}\log(2) + \frac{29\pi^2}{96} - \frac{3\sqrt{3}}{8}\pi + \frac{3}{4}
\]
in small $m_c$ limit

- The double and single logarithmic terms agree with Gong et al. result, but not the finite terms

\[
\text{Im}(F (m_c^2, m_b^2))_{asy} = \frac{19\pi}{16} \log(a) + \frac{7\pi}{16} \log(2) + \pi
\]
With the following inputs

\[ \psi_{J/\psi}(0) = 0.263 \text{ GeV}^{3/2}, \quad m_c = 1.5 \text{ GeV}, \quad m_b = 4.7 \text{ GeV}, \quad \alpha_s = 0.18 \sim 0.26 \]

we have

\[ Br[\eta_b \rightarrow J/\psi J/\psi] = 5.93 \times 10^{-8} \sim 2.58 \times 10^{-7} \]
Higher twist contributions

- In the light-cone formalism, the leading twist term has no contribution to the $\eta_b \rightarrow J/\psi \ J/\psi$ process.

- We expand the LCDA projector in momentum space given by Beneke and Feldmann (2001) to twist-4, which yields more terms than what employed by Braguta et al.
With the asymptotic form for twist-2 distribution amplitudes, i.e.:

\[ \phi_\perp(u) = \phi_\parallel(v) = \phi_{AS}(u) = 6u(1 - u) \]

The analytical decay amplitude turns to be pretty simple, it reads:

\[ M_{\perp} = T_{0} \varepsilon_{\mu \nu \rho \sigma} \varepsilon_{1 1 }^{* \mu} \varepsilon_{2 1 }^{* \nu} n_{-}^{\rho} n_{+}^{\sigma} \frac{9}{256 E_{1}^{2} E_{2}^{2}} \times \left[ (\pi^2 - 4)m_{V_{1}} m_{V_{2}} (f_{V_{1}} \tilde{f}_{V_{2}} + f_{V_{1}} \tilde{f}_{V_{2}}) \\
+ 2\pi^2 (m_{V_{2}}^{2} f_{V_{1}}^{T} \tilde{f}_{V_{2}}^{T} + m_{V_{1}}^{2} f_{V_{2}}^{T} \tilde{f}_{V_{1}}^{T}) \right]. \]
\[ f_V = f_V - f_V^T \frac{m_1 + m_2}{m_V}, \quad f_V^T = f_V - f_V \frac{m_1 + m_2}{m_V} \]

and decay constants \( f_{J/\psi} \) and \( f^T_{J/\psi} \) can be obtained through experiment and NRQCD

\[ f_{J/\psi} = 416 \text{ MeV}, \quad f^T_{J/\psi} = 379 \text{ MeV} \]

Then the numerical result reads

\[ Br[\eta_b \rightarrow J/\psi J/\psi] = (1.1 \sim 2.3) \times 10^{-6}. \]

If so, the \( \eta_b \) is measurable by its double \( J/\Psi \) decay mode
IV. Summary

• The LHC may be suitable for the precise hc study

• By the study of hc production, the charmonium production mechanism may also be elucidated in some sense
● The early running of the LHC will supply numerous event numbers of $J/\Psi$ pair production

● At the LHC, we expect the experimental measurement on double $J/\Psi$ production may tell us more about the charmonium production mechanism
To further study the nature of recently observed state $\eta_b$, direct measurement of its decay products is necessary.

In the light cone formalism, expanding the LCDAs of final vector mesons to twist-4, we find that the higher twist terms contribute more to the $\eta_b$ decay width than what from the NLO corrections.
According to our calculation up to twist-4, the branching fraction of $\eta_b \rightarrow J/\psi J/\psi$ process can be as large as $10^{-6}$, which enables the direct search of $\eta_b$ in Tevatron Run II or LHC.
• Besides what we mentioned, there are still more topics on hadron physics should be performed at the LHC, like heavy meson indirect production, doubly heavy baryon production, exotic states study, etc.

• The LHC may have a rich hadron physics program
Thank you for your attention