Dynamical Dark Matter

A New Framework for Dark-Matter Physics

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Dark Matter = ??

- Situated at the nexus of particle physics, astrophysics, and cosmology
- Dynamic interplay between theory and current experiments
- Of fundamental importance: literally 23% of the universe!
- Necessarily involves physics beyond the Standard Model

⇒ One of the most compelling mysteries facing physics today!

Many theoretical proposals for physics beyond the SM give rise to suitable dark-matter candidates --- e.g.,

- LSP in supersymmetric theories
- LKP in (universal) higher-dimensional theories in which the SM propagates in the extra dimensions

In all cases, the ability of these particles to serve as dark-matter candidates rests squarely on their stability. This in turn is usually the consequence of a stabilizing symmetry --- e.g.,

- R-parity in supersymmetric theories
- "KK parity" in higher-dimensional theories

Indeed, any particle which decays too rapidly into SM states is likely to upset BBN and light-element abundances, and also leave undesirable imprints in the CMB and diffuse photon/X-ray backgrounds.

There is, of course, one important exception to this argument:

A given dark-matter candidate need not be stable if its abundance at the time of its decay is sufficiently small.

A sufficiently small abundance assures that the disruptive effects of the decay of such a particle will be minimal, and that all constraints from BBN, CMB, etc. will continue to be satisfied.

In this talk, we will consider a new framework for dark-matter physics which takes advantage of this possibility.

- *Multi-component framework*: dark matter comprises a vast collection of interacting fields with varying masses, mixings, and abundances.
- Rather than impose stability for each field individually (or even for the collection of fields as a whole), we ensure the phenomenological viability of this scenario by requiring that states with larger masses and SM decay widths have correspondingly smaller abundances, and vice versa.
- *In other words, stability is not an absolute requirement in such a scenario: stability is balanced against abundance!*
- As we shall see, this leads to a highly dynamical scenario in which cosmological quantities such as Ω_{CDM} experience non-trivial time-dependences beyond those associated with the expansion of the universe.

→ **“Dynamical Dark Matter”**

At first glance, it might seem difficult (or at best fine-tuned) to arrange a collection of states which are not only suitable candidates for dark matter but in which the abundances and SM decay widths are precisely balanced in this manner...

However, it turns out that there is one group of states for which such a balancing act occurs naturally:

An infinite tower of Kaluza-Klein (KK) states living in the bulk of large extra spacetime dimensions!

- SM restricted to brane $\Longrightarrow$ all bulk states interact with SM only gravitationally $\Longrightarrow$ natural candidates for dark matter!
- From 4D perspective, this "dark matter" appears as infinite tower of KK states.
- As we shall see, a suitable balancing of abundances and lifetimes occurs even if the stability of the KK tower itself is entirely unprotected!

Thus, theories of large extra dimensions --- and by extension, certain limits of string theory --- naturally and unavoidably give rise to “dynamical dark matter”.

Moreover, as we shall demonstrate, such scenarios also generically give rise to a rich set of collider and astrophysical pheonomena which transcend those usually associated with dark matter. New and unique signature patterns are possible!

Thus, by studying dynamical dark matter and its phenomenological viability, we are not only exploring a new candidate for dark matter but also providing new phenomenological constraints on large extra dimensions and certain limits of string theory.

Outline of this talk

- Dynamical dark matter: General scenario
- Dynamical dark matter meets the incredible bulk: KK towers
- New collider/astrophysics phenomenon:

  "decoherence" --- a new way to help dark matter stay dark

Up to this point, this talk will merely present a broad, theoretical overview of the general scenario. We will not specify any particular dark-matter fields, neither restricting ourselves to specific numbers nor subjecting ourselves to specific phenomenological bounds.

- A concrete example
  - Results from a detailed investigation of one particular possibility within this framework, demonstrating that this possibility actually satisfies all known collider and astrophysical constraints.

This, then, will constitute an "existence proof" that dynamical dark matter is a viable dark-matter framework, and must be considered alongside other approaches in the overall dark-matter discussion.

Dynamical Dark Matter: General scenario

Let's begin by discussing our dynamical dark-matter scenario in its most general form, without reference to the specific example of KK towers...

Overall issue faced when proposing a dark-matter candidate: must constrain its abundance, its lifetime, or the relation between the two.

Suppose only a single dark-matter particle χ .

- Must carry entire DM abundance: $\Omega_\chi = \Omega_{CDM} = 0.23$ (WMAP).
- Given this large abundance, consistency with BBN, CMB, etc. requires that χ have a lifetime which meets or exceeds the current age of the universe ("minimally stable").
- Actually, because of the quantum-mechanical nature of the decay process (not all DM decays at once), the lifetime of χ must *exceed the age of universe* by at least a few orders of magnitude ("hyperstable").
- Most DM scenarios take this form.

Hyperstability is the only way in which a single DM candidate can satisfy the competing constraints on abundance and lifetime. Resulting theory is essentially "frozen in time": Ω_{CDM} is constant, etc.

But why should dark matter consist of only one particle?
After all, the *visible* matter has much smaller abundance, yet is teeming with a diversity and complexity known as the Standard Model.

Let's suppose the dark matter of the universe consists of N states, with N>>1.

- No state individually needs to carry the full Ω_{CDM} so long as the sum of their abundances matches Ω_{CDM}.
- In particular, each state can have a very small abundance.
- If all states have the same lifetime, then they must continue to be hyperstable in order to evade problems with BBN, CMB, ...
- However, states can carry different lifetimes! As long as those with larger abundances have larger lifetimes (and vice versa), phenomenological constraints can be satisfied.

Usual dark-matter scenarios are nothing but a limiting N=1 case of this more general framework. However, taking N>>1 leaves room for our states to exhibit a whole spectrum of decay widths (lifetimes) without running afoul of phenomenological and cosmological constraints.

Can outline the salient features of this scenario more quantitatively...

In general, universe progresses through four distinct phases

- Inflation
- Reheating (matter-dominated, where matter = inflaton)
- Radiation-dominated
- Matter-dominated (current epoch)

In general, consider "stuff" with equation of state $\rho = w\,p$.
This "stuff" will have an abundance $\Omega = \rho / \rho_{crit}$ which scales with time as...

- w= 0 for matter
- w= -1 for vacuum energy
- w= +1/3 for radiation
- w= -1/3 for curvature

$$\Omega \sim \begin{cases} t^{(1-3w)/2} & \text{RD phase} \\ t^{-2w} & \text{MD and reheating phases} \\ \exp[-3H(1+w)t] & \text{inflationary phase .} \end{cases}$$

For concreteness, assume individual DM components in our scenario are described by scalars ϕ_i , $i=1,...,N$ with

- masses m_i
- decay widths Γ_i describing decays into SM states.

In FRW universe, these fields will evolve according to...

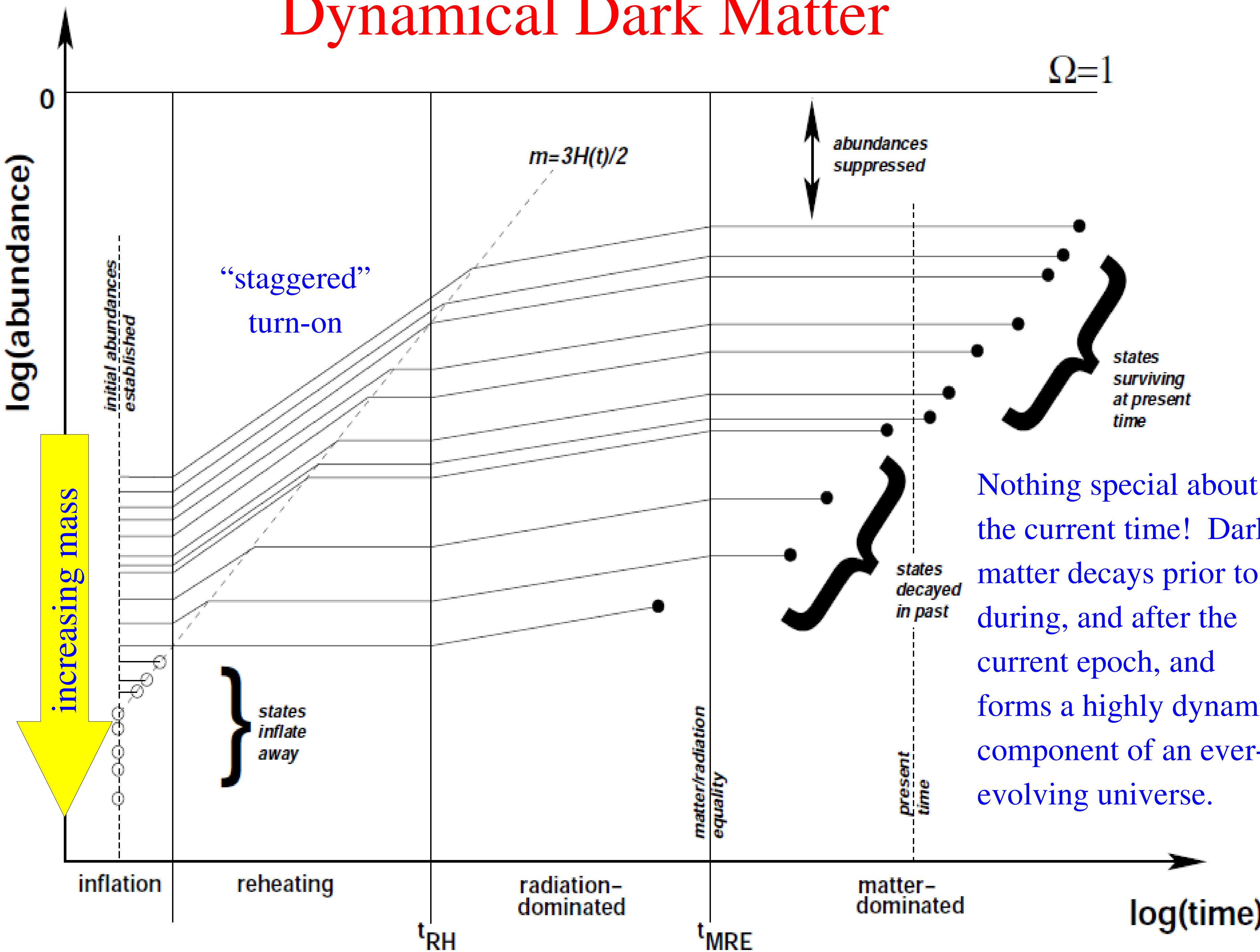
$$\ddot{\phi}_i + [3H(t) + \Gamma_i]\dot{\phi}_i + m_i^2 \phi_i \;=\; 0$$

Hubble parameter:
$H(t) \sim 1/t$ (FRW)

Transition from overdamped to underdamped oscillation...

- Transition from vacuum energy (w= -1) to matter (w=0).
- Occurs when $3H(t) = 2m_i$ ⇨ $t \sim 1/m_i$
- Heavier states “turn on” first, lighter states later.

Dynamical Dark Matter



Nothing special about the current time! Dark matter decays prior to, during, and after the current epoch, and forms a highly dynamic component of an ever-evolving universe.

How to characterize a particular dynamical dark matter configuration?

Introduce two "complementary" parameters:

- Total abundance at any moment: $\Omega_{\rm tot}(t) \equiv \sum_i \Omega_i(t)$
- *Distribution* of that total abundance: how much is $\Omega_{\rm tot}$ shared between a dominant component and all others?

  Define

  $$\eta \equiv 1 - \frac{\Omega_0}{\Omega_{\rm tot}}$$

  where $\Omega_0 \equiv \max_i\{\Omega_i\}$

  Thus

  $0 \le \eta \le 1$

  - $\eta=0$ signifies one dominant component (standard picture)
  - $\eta>0$ quantifies departure from standard picture

Each of these quantities will have a unique time-dependence in the dynamical dark matter framework.

Start with η :

- Initial value of η is set when initial abundances established
- If during inflation, heavy modes inflate away ⇨ η decreases
- If staggered turn-on occurs, then this inflates abundances of light modes relative to heavy modes ⇨ η decreases
- Dark-matter decay widths are larger for heavier states which have smaller abundances ⇨ η decreases

Thus, η can only decrease monotonically from its original value.
It is nevertheless regarded to be a fundamental property of our scenario that η is presumed significantly different from zero at the present time.

Now look at time-dependence of Ω_{tot} .

- Indeed, one important signature of the dynamical nature of dark matter in this framework is that Ω_{tot} is a time-evolving quantity ---- even during the current matter-dominated epoch!
- Within such a framework, it is therefore only to be regarded as an accident that Ω_{tot} happens to match the observed Ω_{CDM}=0.23 at the present time.
- Moreover, the time-dependence of Ω_{tot} in this framework will essentially give us an "effective" equation of state for our decaying ensemble of dark-matter states.

Let's focus on the final, MD era:

$$\Omega_i(t) \;=\; \Omega_i\,\Theta(\tau_i - t)$$ where $\tau_i \equiv \Gamma_i^{-1}$

$$\frac{d\Omega_{\rm tot}(t)}{dt} \;=\; \sum_i \Omega_i \frac{d}{dt}\Theta(\tau_i - t) \;=\; -\sum_i \Omega_i \delta(\tau_i - t)$$

Now let's replace sum over states by an integral

$$\sum_i \;\Longrightarrow\; \int d\tau\, n_\tau(\tau)$$ ← density of states per unit τ .

$$\begin{aligned}\frac{d\Omega_{\rm tot}(t)}{dt} &= -\int d\tau\,\Omega(\tau)\,n_\tau(\tau)\,\delta(\tau - t)\\ &= -\Omega(t)n_\tau(t)\ .\end{aligned}$$

To go further, let us parametrize the spectrum of components in terms of their scaling behavior as function of decay width ---

$$\Omega(\Gamma) \sim A\Gamma^{\alpha}$$

$$\alpha < 0$$

$$\eta_{\Gamma}(\Gamma) \sim B\Gamma^{\beta}$$

density of states per unit Γ

$$n_{\tau} = n_{\Gamma}\left|\frac{d\Gamma}{d\tau}\right| = \Gamma^{2}\, n_{\Gamma}$$

We then have

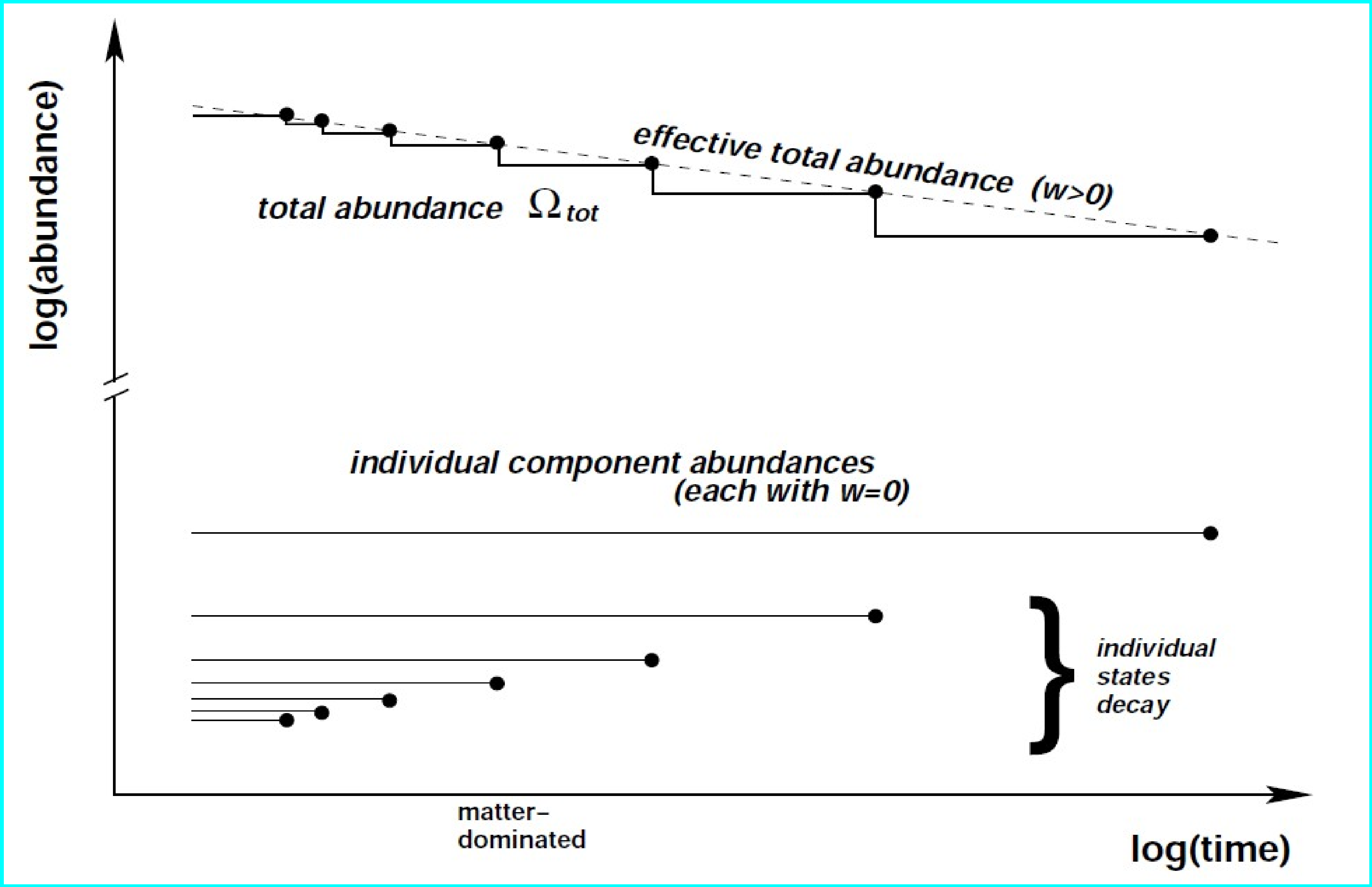
$$\Omega(\Gamma)n_{\tau}(\Gamma) \sim AB\Gamma^{\alpha+\beta+2}$$

or

$$\Omega(\tau)n_{\tau}(\tau) \sim AB\tau^{-\alpha-\beta-2}$$

$$\frac{d\Omega_{\text{tot}}(t)}{dt} = -AB\, t^{-\alpha-\beta-2}$$

General result!



Sketch shown for $\alpha + \beta > -1$, with $\alpha < 0$ and $\beta > 0$

This means that our ensemble of states has a non-zero "effective" equation-of-state parameter $w_{eff}(t)$. In general, we can define...

$$w_{\rm eff}(t) \equiv -\left(\frac{1}{3H}\frac{d\log\rho_{\rm tot}}{dt}+1\right)$$
$$= \begin{cases} -\frac{1}{2}\left(\frac{d\log\Omega_{\rm tot}}{d\log t}\right) & \text{for RH/MD eras} \\ -\frac{2}{3}\left(\frac{d\log\Omega_{\rm tot}}{d\log t}\right)+\frac{1}{3} & \text{for RD era} \end{cases} .$$

We then find the results

- **For** $x \equiv \alpha+\beta \neq -1$ **:**

$$w_{\rm eff}(t) = \frac{(1+x)w_*}{2w_* + (1+x-2w_*)(t/t_{\rm now})^{1+x}}$$

where

$$w_* \equiv w_{\rm eff}(t_{\rm now}) = \frac{AB}{2\Omega_{\rm CDM} t_{\rm now}^{1+x}}$$

- **For** $x = -1$ **:**

$$w_{\rm eff}(t) = \frac{w_*}{1-2w_*\log(t/t_{\rm now})}$$

where

$$w_* \equiv w_{\rm eff}(t_{\rm now}) = \frac{AB}{2\Omega_{\rm CDM}}$$

These are effective equations of state for our entire DM ensemble!

If our dynamical dark matter scenario is to be in rough agreement with cosmological observations, we expect that w_* today should be fairly small (since traditional dark "matter" has $w = 0$).

We also expect that the function $w_{eff}(t)$ should not have experienced strong variations within the recent past.

Situations with $x = \alpha + \beta < -1$ are likely to be phenomenologically preferred over those with $x > -1$, since having $x < -1$ ensures that

$$0 \leq w_{\text{eff}}(t) \leq w_* \text{ for all } t < t_{\text{now}}$$

However, depending on the detailed properties of the particular dynamical dark-matter scenario under study, values of x slightly above -1 may also be phenomenologically acceptable.

Thus far, we have only presented a general scenario. In particular, we have not yet demonstrated that such collections of dark-matter states can be easily assembled in which the individual component abundances are balanced against lifetimes in a well-motivated way.

However, it turns out that an infinite tower of KK states propagating in the bulk of large extra dimensions has exactly the desired properties!

As we shall see, this feature ultimately emerges as the consequence of the non-trivial interplay between physics in the bulk and physics on the brane.

To see this, let us consider a very simple "bare-bones" setup:

Universe has a single, flat extra dimension of length R, one bulk field Φ, and SM lives on a brane located at y=0...

SM fields

$$S = \int d^4x dy \left[\mathcal{L}_{\rm bulk}(\Phi) + \delta(y)\mathcal{L}_{\rm brane}(\psi_i, \Phi)\right]$$

where

$$\mathcal{L}_{\rm bulk} = \frac{1}{2}\partial_M \Phi^* \partial^M \Phi - \frac{1}{2} M^2 |\Phi|^2$$

$$\mathcal{L}_{\rm int} \supset -\frac{1}{2} m^2 |\Phi|^2$$

Now do KK reduction for Z_2 orbifold (line segment) of radius R:

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{k=0}^{\infty} r_k\, \phi_k(x^\mu) \cos\left(\frac{ny}{R}\right)$$

$$r_k \equiv \begin{cases} 1 & \text{for } k=0 \\ \sqrt{2} & \text{for } k>0 \end{cases}$$

We then find

$$S = \int d^4x dy \left[\tfrac{1}{2}\partial_M \Phi^* \partial^M \Phi - \tfrac{1}{2} M^2 |\Phi|^2 - \tfrac{1}{2}\delta(y) m^2 \Phi^2\right]$$

$$= \int d^4x \left(\frac{1}{2}\sum_{k=0}^{\infty} \partial_\mu \phi_k^* \partial^\mu \phi_k - \frac{1}{2}\sum_{k,\ell=0}^{\infty} \mathcal{M}^2_{k\ell}\, \phi_k \phi_\ell^* \right)$$

KK mass matrix

$$\mathcal{M}^2_{k\ell} = \left(\frac{k\ell}{R^2} + M^2\right)\delta_{k\ell} + r_k r_\ell\, m^2$$

for M=0:

$$\begin{pmatrix} 0 & \sqrt{2}m^2 & \sqrt{2}m^2 & \sqrt{2}m^2 & \dots \\ \sqrt{2}m^2 & 1/R^2 & 2m^2 & 2m^2 & \dots \\ \sqrt{2}m^2 & 2m^2 & 4/R^2 & 2m^2 & \dots \\ \sqrt{2}m^2 & 2m^2 & 2m^2 & 9/R^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Brane mass induces non-diagonality!
- KK mass eigenstates are different from KK momentum eigenstates!

Define

$$y \equiv \frac{1}{mR}$$

to parametrize degree of non-diagonality:

- $y \longrightarrow \infty$ 4D limit (diagonal)
- $y \longrightarrow 0$ Full tower participates equally

Mass eigenvalues λ are solutions to transcendental equation

$$\pi m^2 R \cot\left(\pi R\sqrt{\lambda^2 - M^2}\right) = \sqrt{\lambda^2 - M^2}$$

This has two distinct limiting behaviors...

$$\lambda_n^2 = M^2 + \frac{n^2}{R^2}, \qquad n \in \mathbb{Z}$$

"Top" of tower $n \gg \pi/y^2$

... as expected for periodic boundary conditions

$$\lambda_n^2 = M^2 + \frac{(n+\frac{1}{2})^2}{R^2}, \qquad n \in \mathbb{Z}$$

Bottom of tower $n \ll \pi/y^2$

... as expected for *anti*-periodic boundary conditions!

This is the result of the non-trivial interplay between brane physics and bulk physics.

Can also diagonalize the mass matrix...

Obtain *exact* result

$$|\phi_\lambda\rangle \;=\; A_\lambda \sum_{k=0}^{\infty} \frac{r_k \tilde{\lambda}^2}{\tilde{\lambda}^2 - k^2 y^2} |\phi_k\rangle$$

KK mass eigenstates

KK momentum eigenstates

where...

$$\tilde{\lambda} \;\equiv\; \sqrt{\lambda^2 - M^2}/m$$

dimensionless eigenvalue

$$A_\lambda \;\equiv\; \frac{\sqrt{2}}{\tilde{\lambda}} \frac{1}{\sqrt{1 + \pi^2/y^2 + \tilde{\lambda}^2}}$$

matrix element

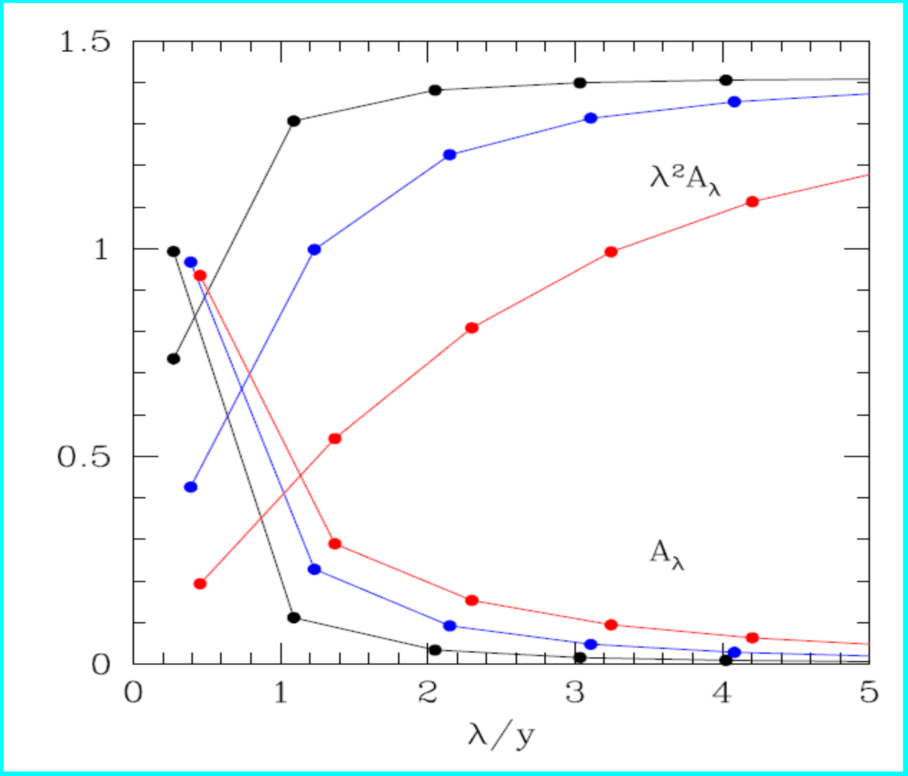
There are two matrix elements of particular importance...

1 $$\langle \phi_\lambda | \phi_{k=0} \rangle = A_\lambda$$ overlaps with KK zero mode

2 Let us define $$\phi' \equiv \Phi(y)\big|_{y=0} = \sum_{k=0}^{\infty} r_k \phi_k$$ projection of bulk field onto SM brane

Then

$$\langle \phi_\lambda | \phi' \rangle = A_\lambda \sum_{k=0}^{\infty} \frac{r_k^2 \tilde{\lambda}^2}{\tilde{\lambda}^2 - k^2 y^2} = \frac{\pi \tilde{\lambda}^2}{y} \cot\left(\frac{\pi \tilde{\lambda}}{y}\right) A_\lambda = \tilde{\lambda}^2 A_\lambda$$



Now let's look at abundances...

Henceforth let us assume bulk mass M=0 (ok for moduli fields, axions, etc.)

Prior to the brane dynamics that establishes the brane mass

⇒ $\Phi \longrightarrow \Phi + c$ (shift symmetry for bulk field)

⇒ Any vev for Φ is equally likely: $\langle\Phi\rangle \sim f_\phi^{3/2}$

⇒ $$\begin{cases} \langle\phi_0\rangle = \theta \hat{f}_\phi \\ \langle\phi_k\rangle = 0 \quad \text{for all} \quad k > 0 \end{cases}$$

5D mass scale for Φ (decay constant)

where $$\hat{f}_\phi^2 \equiv 2\pi R f_\phi^3$$

4D mass scale for each individual KK mode

After the brane dynamics that establishes the brane mass, this shift symmetry is broken.

KK momentum basis is no longer appropriate. Switch to mass eigenstate basis...

$$\langle \phi_\lambda \rangle = \theta A_\lambda \hat{f}_\phi \qquad \text{for all} \quad \lambda$$

Turn-on of brane mass establishes a potential for the KK modes and thus an energy density associated with the above configuration... $\rho_\lambda = \frac{1}{2}\lambda^2 \langle \phi_\lambda \rangle^2$

$$\rho_\lambda = \frac{1}{2}\theta^2 \lambda^2 A_\lambda^2 \hat{f}_\phi^2$$

This then leads to an initial abundance $\Omega_\lambda \equiv \rho_\lambda / \rho_{\text{crit}}$

where $\rho_{\text{crit}} = 3 M_P^2 H^2$

$M_P \equiv (8\pi G_N)^{-1/2}$

reduced Planck mass

$$\Omega_\lambda^{(0)} = \frac{\theta^2}{6} \tilde{\lambda}^2 A_\lambda^2 \left(\frac{m \hat{f}_\phi}{M_P H} \right)^2$$

initial abundances

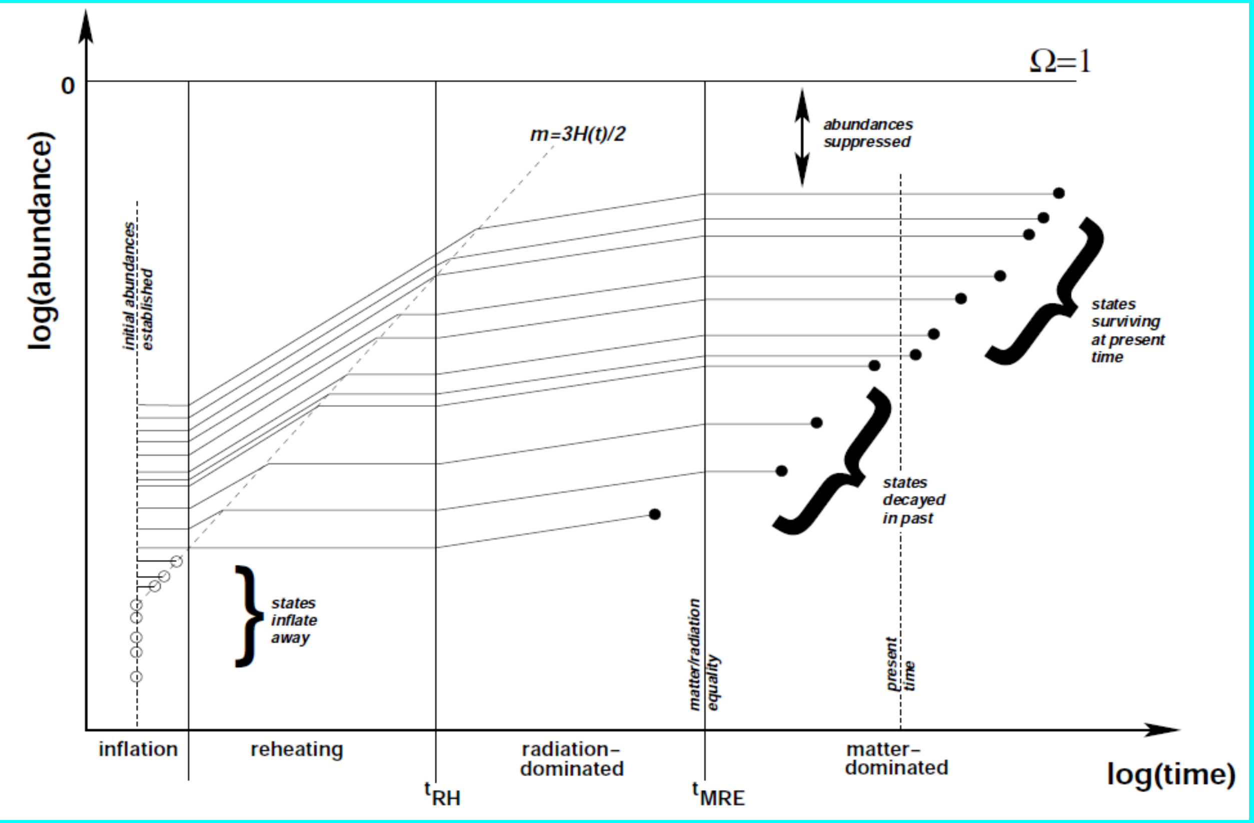
These are the initial abundances at time T when the brane mass is generated: $\Omega_\lambda(T) = \Omega_\lambda^{(0)}$

"misalignment production"

But we seek to know the corresponding abundances *today!*

$$\Omega_\lambda(t_{\rm now}) \quad = \quad ?$$

Recall:



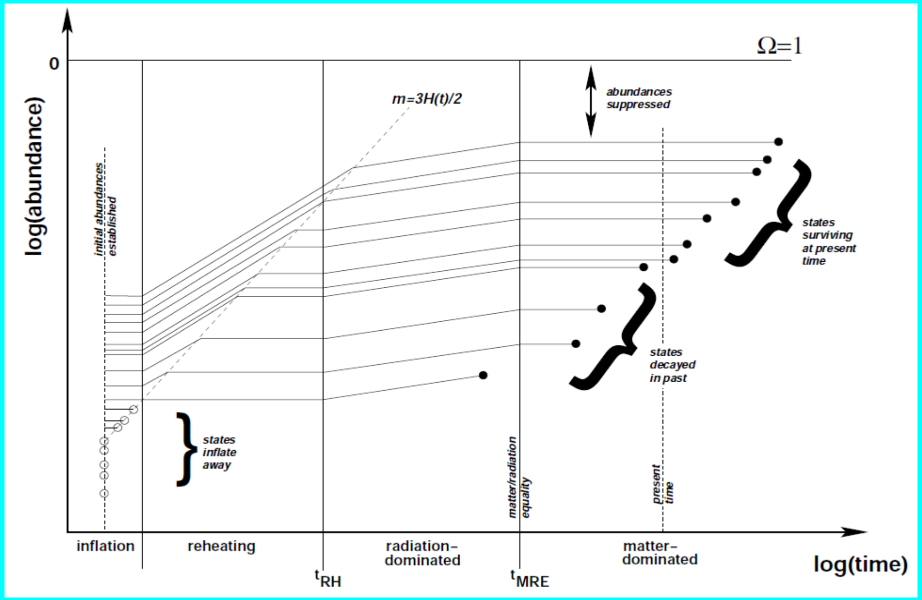
Answer depends on the era for T and whether the mode in question has instantaneous or staggered "turn-on"!

Recall:

Turn-on occurs when $3H(t_\lambda) = 2\lambda$

Since $H(t) = c/3t$ ⇒ $t_\lambda = c/2\lambda$

turn-on time for each mode



$$X_\lambda \equiv e^{-\Gamma_\lambda(t_{\rm now} - T)}$$

Damping factor from dark-matter decays

For $T > t_{\rm MRE}$:

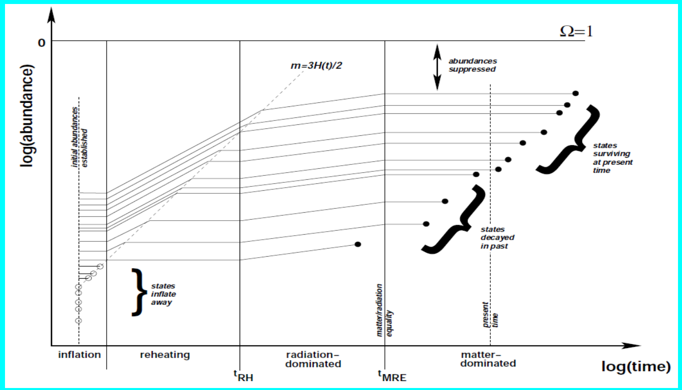
$$\Omega_\lambda(t_{\rm now}) \sim \tilde{\lambda}^2 A_\lambda^2 X_\lambda \left(\frac{\hat{f}_\phi}{M_P}\right)^2 (mT)^2$$

$t_\lambda \leq T$ instantaneous

$$\Omega_\lambda(t_{\rm now}) \sim X_\lambda \Omega_\lambda^{(0)} \left(\frac{t_\lambda}{T}\right)^2 \sim A_\lambda^2 X_\lambda \left(\frac{\hat{f}_\phi}{M_P}\right)^2$$

$t_\lambda > T$ staggered

For $t_{\mathrm{RH}} \lesssim T \lesssim t_{\mathrm{MRE}}$:



$$\Omega_\lambda(t_{\mathrm{now}}) \sim \Omega_\lambda^{(0)} X_\lambda \left(\frac{t_{\mathrm{MRE}}}{T}\right)^{1/2}$$

$$\sim \tilde{\lambda}^2 A_\lambda^2 \, X_\lambda \left(\frac{\hat{f}_\phi}{M_P}\right)^2 (mT)^{3/2} (m t_{\mathrm{MRE}})^{1/2}$$

$t_\lambda \leq T$

instantaneous

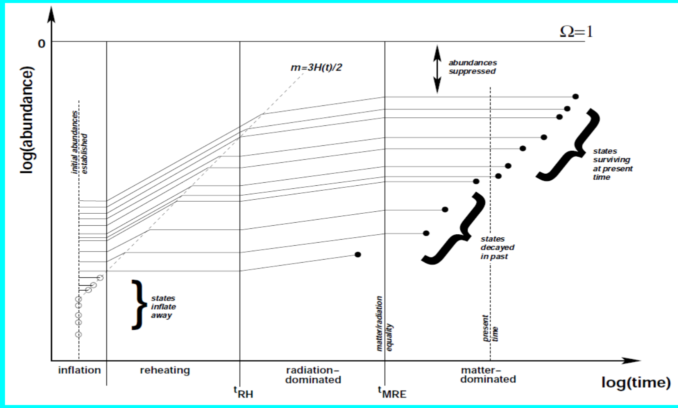
$$\Omega_\lambda(t_{\mathrm{now}}) \sim \Omega_\lambda^{(0)} X_\lambda \left(\frac{t_\lambda}{T}\right)^2 \left(\frac{t_{\mathrm{MRE}}}{t_\lambda}\right)^{1/2}$$

$$\sim \tilde{\lambda}^{1/2} A_\lambda^2 \, X_\lambda \left(\frac{\hat{f}_\phi}{M_P}\right)^2 (m t_{\mathrm{MRE}})^{1/2}$$

$t_\lambda > T$

staggered

Finally, for $T \lesssim t_{\rm RH}$:



$$\Omega_\lambda(t_{\rm now}) \sim \Omega_\lambda^{(0)}\, X_\lambda \left(\frac{t_{\rm MRE}}{t_{\rm RH}}\right)^{1/2}$$
$$\sim \tilde\lambda^2 A_\lambda^2\, X_\lambda \left(\frac{\hat f_\phi}{M_P}\right)^2 (mT)^2 \left(\frac{t_{\rm MRE}}{t_{\rm RH}}\right)^{1/2}$$

$t_\lambda \leq T$

instantaneous

$$\Omega_\lambda(t_{\rm now}) \sim \Omega_\lambda^{(0)}\, X_\lambda \left(\frac{t_\lambda}{T}\right)^2 \left(\frac{t_{\rm MRE}}{t_{\rm RH}}\right)^{1/2}$$
$$\sim A_\lambda^2\, X_\lambda \left(\frac{\hat f_\phi}{M_P}\right)^2 \left(\frac{t_{\rm MRE}}{t_{\rm RH}}\right)^{1/2}$$

$t_\lambda > T$

staggered

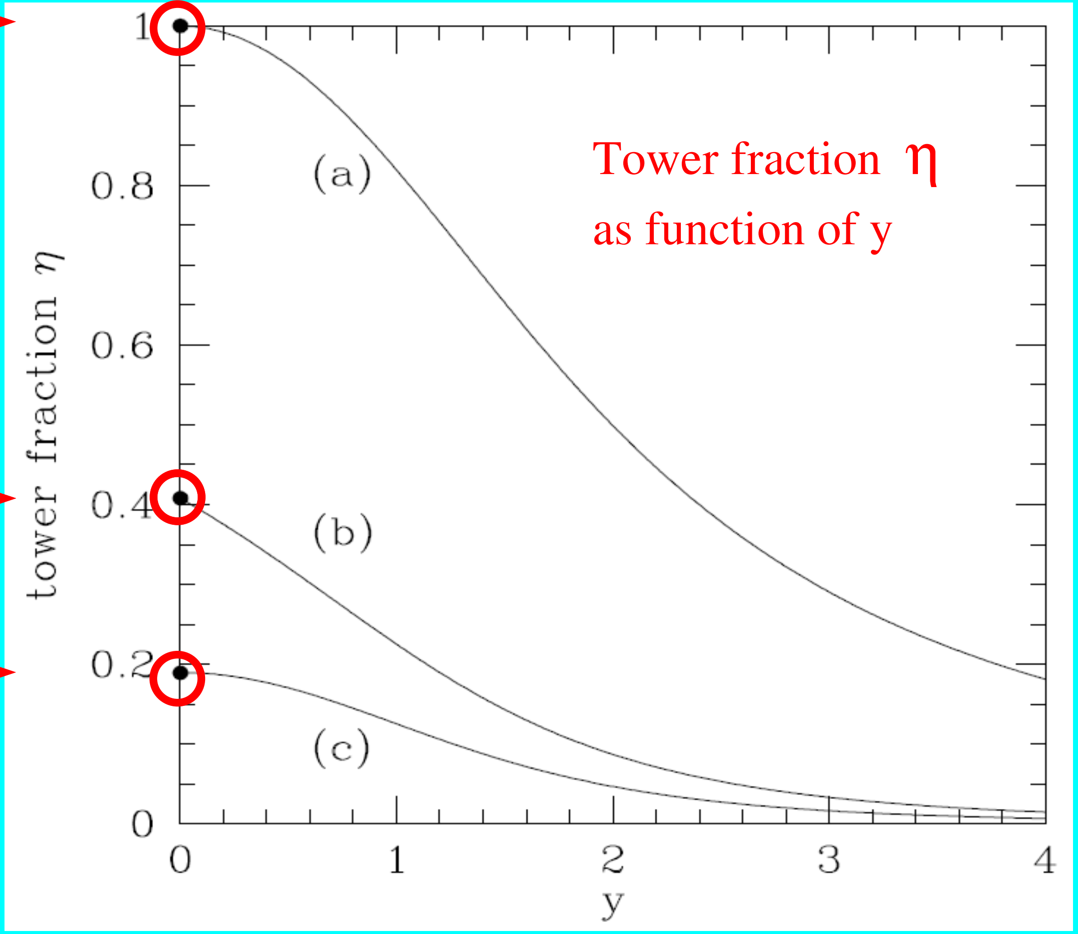
These six different results exhibit three different dependences on the masses λ :

$$\Omega_\lambda \sim \begin{cases} \tilde{\lambda}^2 A_\lambda^2 & \text{instantaneous} \\ \tilde{\lambda}^{1/2} A_\lambda^2 & \text{staggered (RD era)} \\ A_\lambda^2 & \text{staggered (reheating/MD era)} \end{cases}$$

maximum values in each case

$$\eta \to 1 - 2\sqrt{2}\left[\sum_{n=0}^{\infty}(n+1/2)^{-3/2}\right]^{-1} = 1 - \frac{2\sqrt{2}}{(2\sqrt{2}-1)\zeta(3/2)} \approx 0.408$$

$$\eta \to 1 - 4\left[\sum_{n=0}^{\infty}\frac{1}{(n+1/2)^2}\right]^{-1} = 1 - \frac{8}{\pi^2} \approx 0.189$$



For large masses λ , these behaviors scale as

$$\Omega_\lambda \sim \begin{cases} \tilde{\lambda}^{-2} & \text{instantaneous} \\ \tilde{\lambda}^{-7/2} & \text{staggered (RD era)} \\ \tilde{\lambda}^{-4} & \text{staggered (reheating/MD era)} \end{cases}$$

Thus, for a KK tower, we have the scaling coefficients:

$$\alpha = \begin{cases} -2/3 & \text{instantaneous} \\ -7/6 & \text{staggered (RD era)} \\ -4/3 & \text{staggered (reheating/MD era)} \end{cases}$$

Likewise, for a KK tower, all modes are approximately equally spaced: $\beta = -2/3$.

What about decay widths?

- Need to understand interactions between bulk field Φ and SM brane fields.
- To be conservative, consider operators of lowest dimension which are Lorentz invariant, gauge invariant, etc.
- Lowest dimension: consider only linear in Φ .

$$\frac{1}{\hat{f}}(\partial_\mu \phi')\,\overline{\psi}\gamma^\mu\psi\ , \qquad \frac{1}{\hat{f}}\,\phi'\,F_{\mu\nu}F^{\mu\nu}$$

if Φ is CP-even
e.g., moduli...

$$\frac{1}{\hat{f}}(\partial_\mu \phi')\,\overline{\psi}\gamma^\mu\gamma^5\psi\ , \qquad \frac{1}{\hat{f}}\,\phi'\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

if Φ is CP-odd
e.g., axions...

where ϕ' is projection of Φ onto brane:

$$\phi' \equiv \Phi(y)\big|_{y=0} = \sum_{k=0}^{\infty} r_k \phi_k$$

$$\Gamma_\lambda \sim \frac{\lambda^3}{\hat{f}^2}(\tilde{\lambda}^2 A_\lambda)^2 \sim \frac{\lambda^3}{\hat{f}^2}$$

Thus, combining our results for Ω_λ and Γ_λ ,

we obtain the following *product relations*

across our KK towers:

$$\begin{aligned} \text{instantaneous}: \quad & \Omega_\lambda \Gamma_\lambda^{2/3} \sim \text{constant} \\ \text{staggered (RD era)}: \quad & \Omega_\lambda \Gamma_\lambda^{7/6} \sim \text{constant} \\ \text{staggered (reheating/MD era)}: \quad & \Omega_\lambda \Gamma_\lambda^{4/3} \sim \text{constant} \end{aligned}$$

Decay widths are balanced against abundances, as promised!

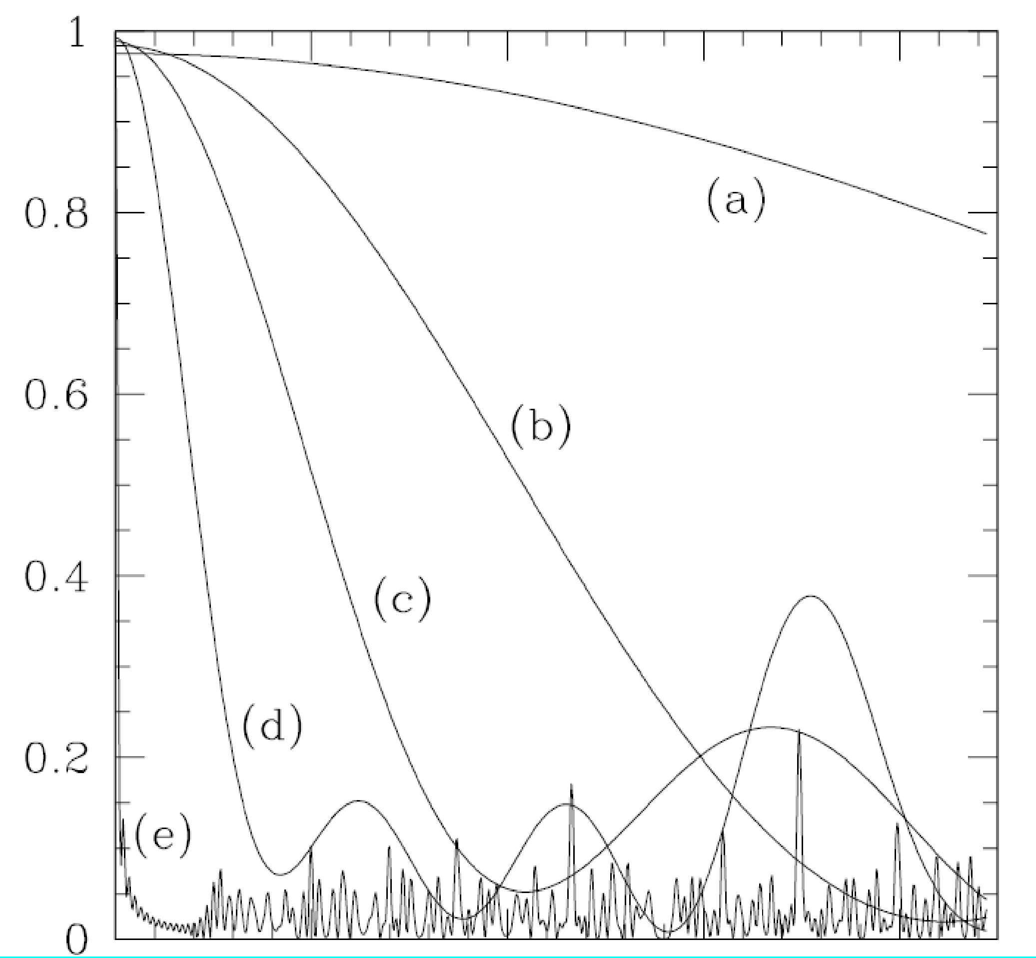
Finally, another feature which helps the dark matter stay dark!

Recall that the bulk field only couples to brane through its brane projection:

$$\phi' \equiv \Phi(y)\Big|_{y=0} = \sum_{k=0}^{\infty} r_k \phi_k$$

However, once ϕ' is produced (in laboratory, in distant astrophysical sources, etc.), it rapidly ***decoheres*** and does not reconstitute in finite time...

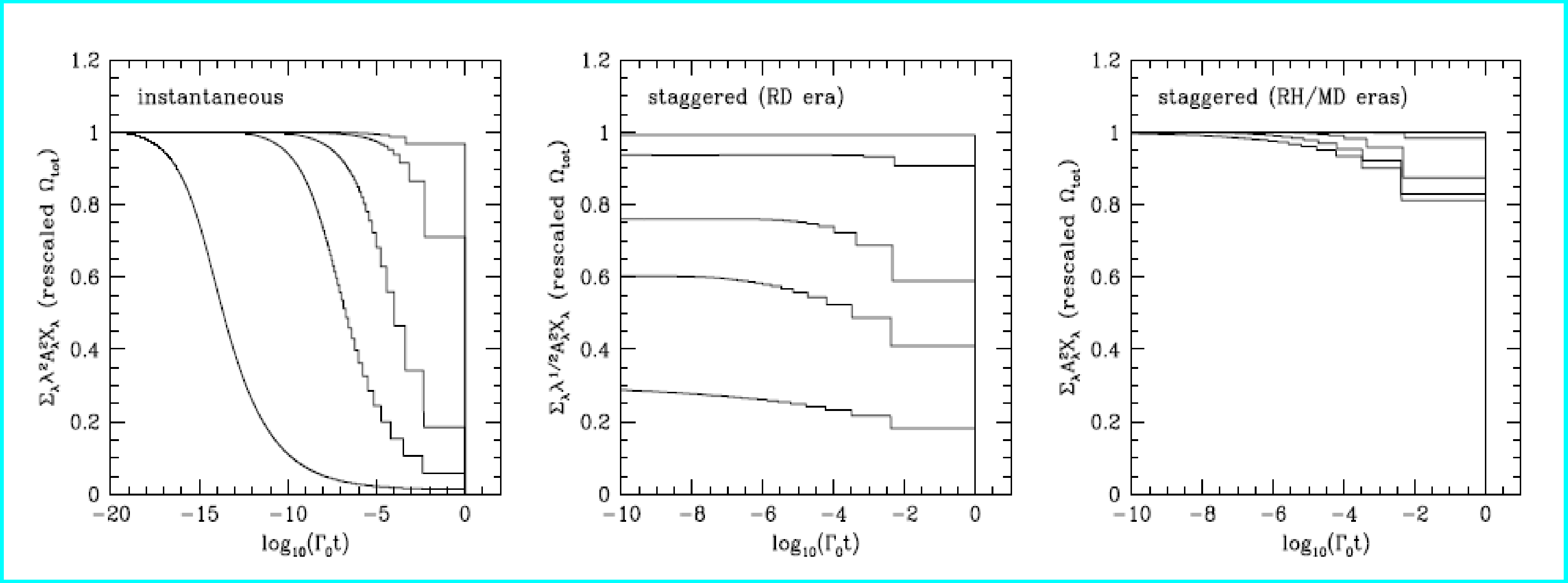
This novel effect provides yet another mechanism which may help dark matter stay dark, and leads to different signature patterns from those which characterize purely 4D scalars and traditional single-component dark-matter candidates.



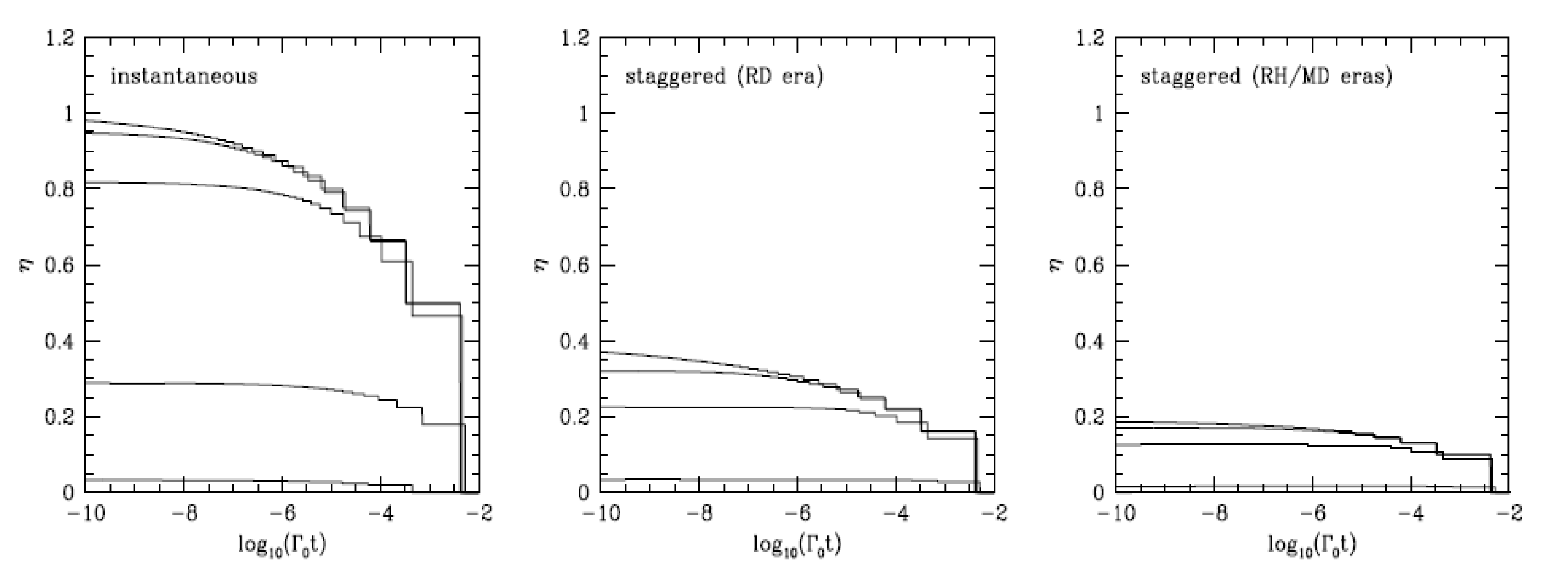
DDG (1999)

So a generic KK tower gives us the following behaviors...

Total abundance Ω_{tot}

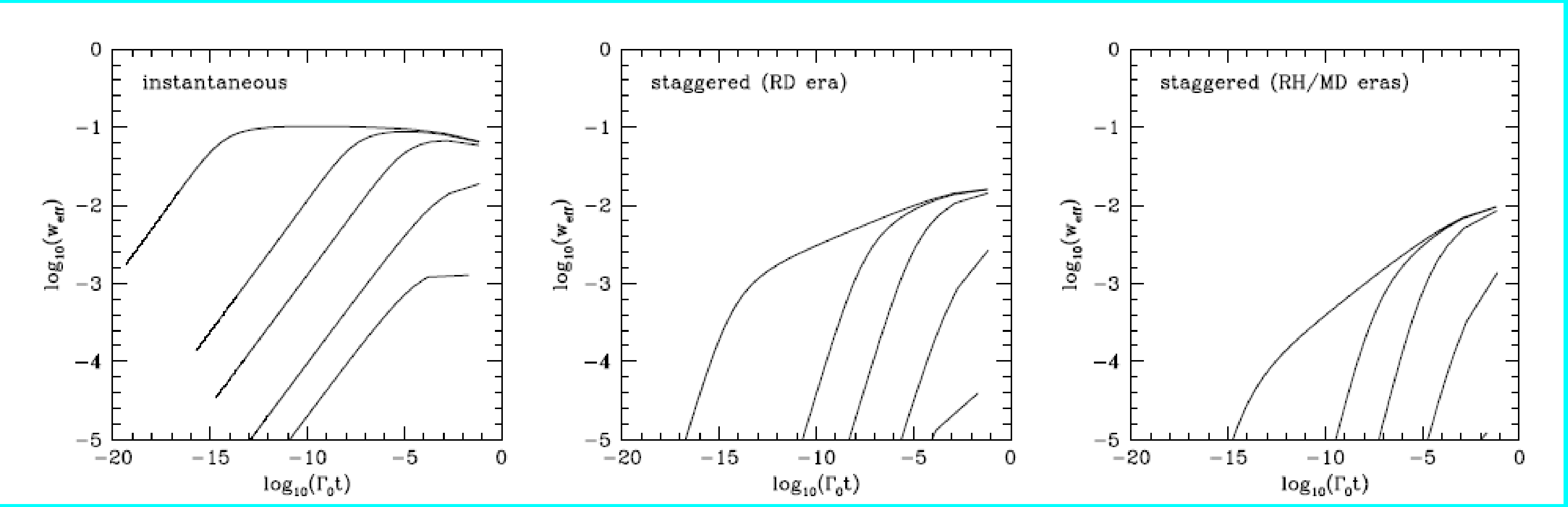


Tower fraction η



And finally...

Equation of state parameter $w_{eff}(t)$



Moreover, for a generic KK tower,
we find the following values of $x = \alpha + \beta$:

	large $\tilde{\lambda}$	small $\tilde{\lambda}$
instantaneous	$-4/3$	$-4/5$
staggered (RD era)	$-11/6$	$-11/10$
staggered (RH/MD eras)	-2	$-6/5$

TABLE I: Values of the equation-of-state parameter $x \equiv \alpha + \beta$ for different portions of a general KK tower with different "turn-on" phenomenologies. We observe that KK towers naturally give rise to values $x \lesssim -1$, which is precisely the range favored phenomenologically.

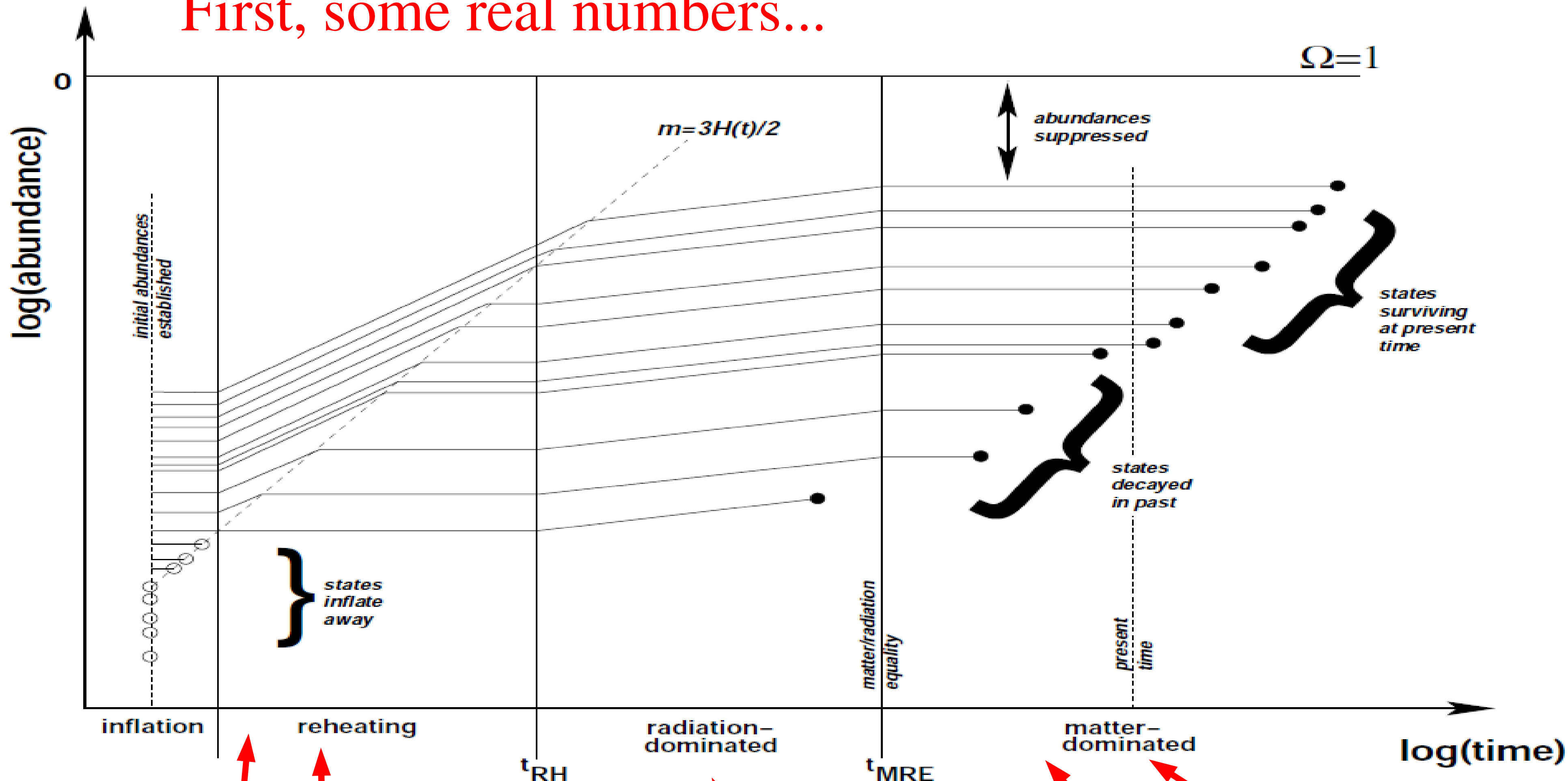
... precisely in the phenomenologically preferred range!!

OK, enough general formalism!

Let's now present a concrete example of this entire framework, along with real numbers and experimental bounds and constraints!

This will therefore serve as a "proof of concept" for the entire scenario.

First, some real numbers...



choose...

T_{max}~150 GeV
(H_{inf} ~1 MeV)

T_{QCD}~250 MeV

t_{RH}~10^{-1} sec
T_{RH}~5 MeV
(LTR cosmology!)

t_{BBN}~1 sec
T_{BBN}~MeV

t_{MRE}~10^{11} sec
T_{MRE}~keV

$t_{last\ scatt}$ ~10^{13} sec
$T_{last\ scatt}$ ~eV

t_{now}~10^{17} sec
T_{now}~3 K

Furthermore, let us consider the case where Φ = ***axion*** with decay constant f_X, corresponding to a general gauge group G with confinement scale Λ_G and coupling g_G

Such a choice is indeed gauge-neutral and well-motivated theoretically, both in field theory and in string theory.

Our analysis then follows exactly as before, with the specific values

$$\begin{cases} M & \to \quad 0 \\ m & \to \quad \dfrac{g_G \xi \Lambda_G^2}{4\sqrt{2}\pi \hat{f}_X} \end{cases}$$

brane mass comes from axion potential induced by instanton dynamics associated with group G at scale Λ_G

Likewise, couplings to brane fields take the form...

with $\mathcal{L}_{\rm int}$ given by...

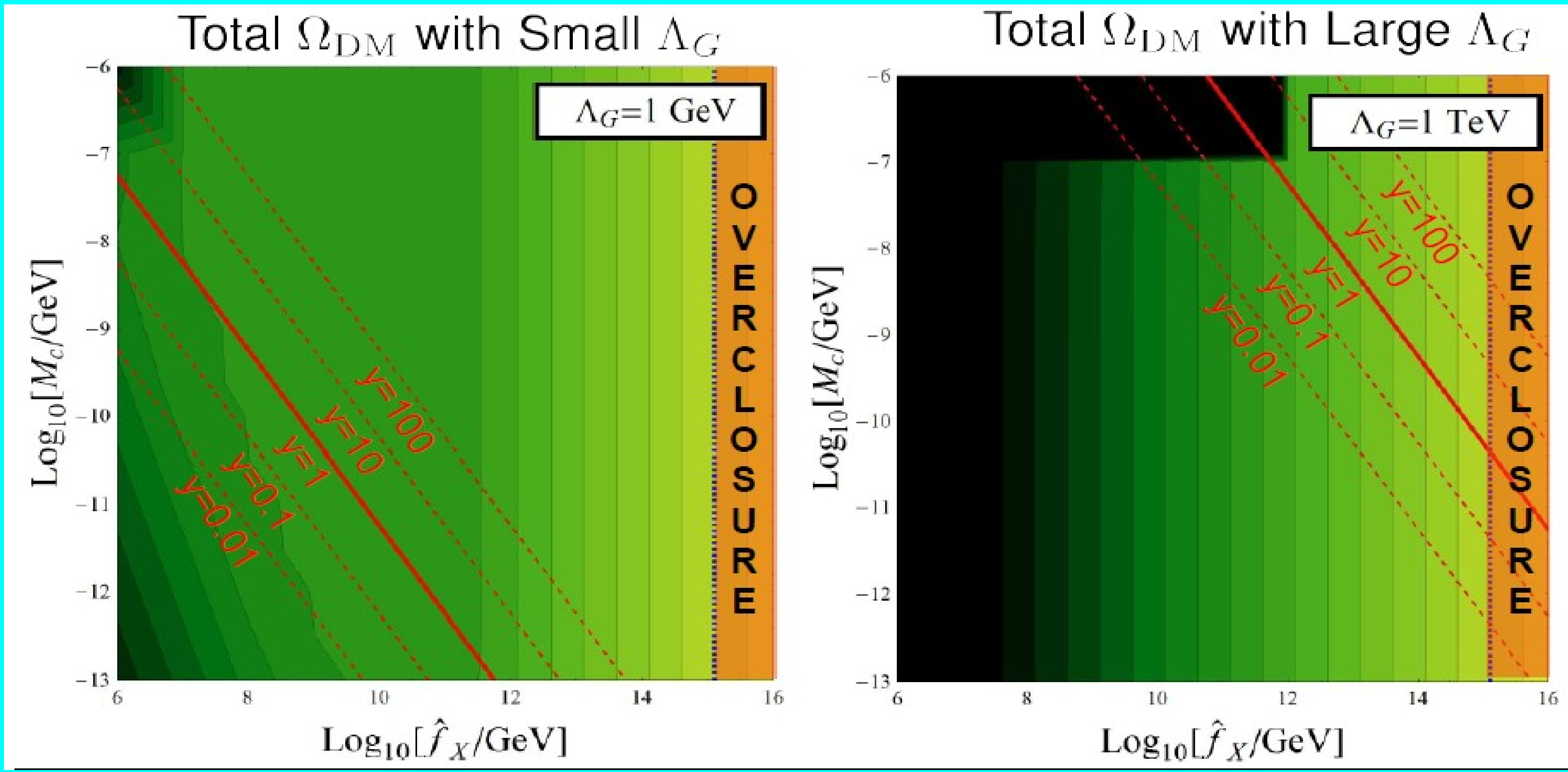
Interactions with G gauge fields

$$\mathcal{L}_{\rm int} = \frac{g_G^2 \xi}{32\pi^2 f_X^{3/2}} a \mathcal{G}^a_{\mu\nu} \tilde{\mathcal{G}}^{a\mu\nu} + \frac{g_s^2 c_g^2}{32\pi^2 f_X^{3/2}} a G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \sum_i \frac{c_i}{f_X^{3/2}} (\partial_\mu a) \overline{\psi}_i \gamma^\mu \gamma^5 \psi_i + \frac{e^2 c_\gamma}{32\pi^2 f_X^{3/2}} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Possible couplings to SM gauge and matter fields

We can then vary the free parameters (R, f_X, Λ_G) to survey different outcomes...

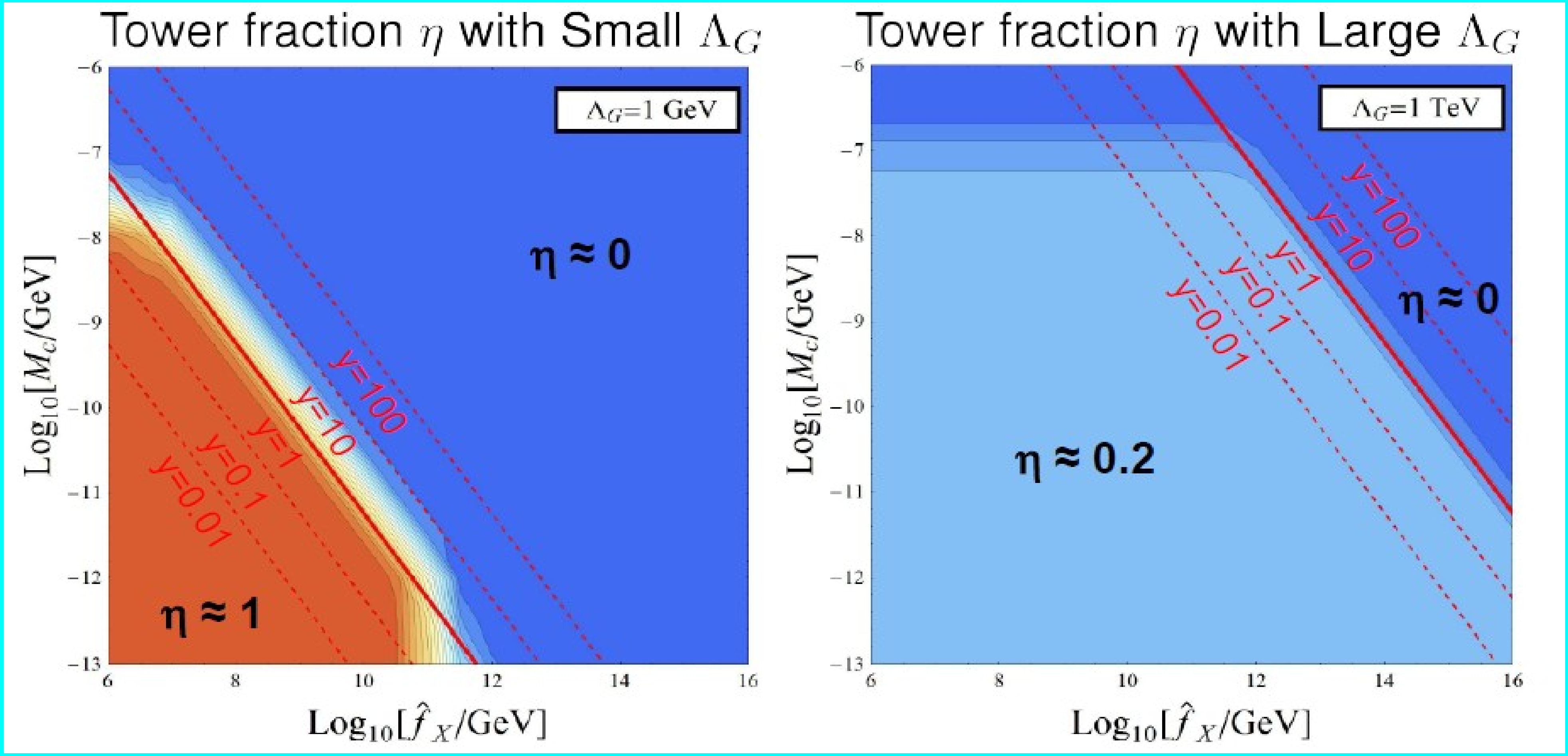
How does Ω_{tot} depend on f_X and $M_c = 1/R$?



simultaneous turn-on

staggered turn-on

How does η depend on f_X and $M_c = 1/R$?



simultaneous turn-on

staggered turn-on

What are the phenomenological constraints that govern such scenarios?

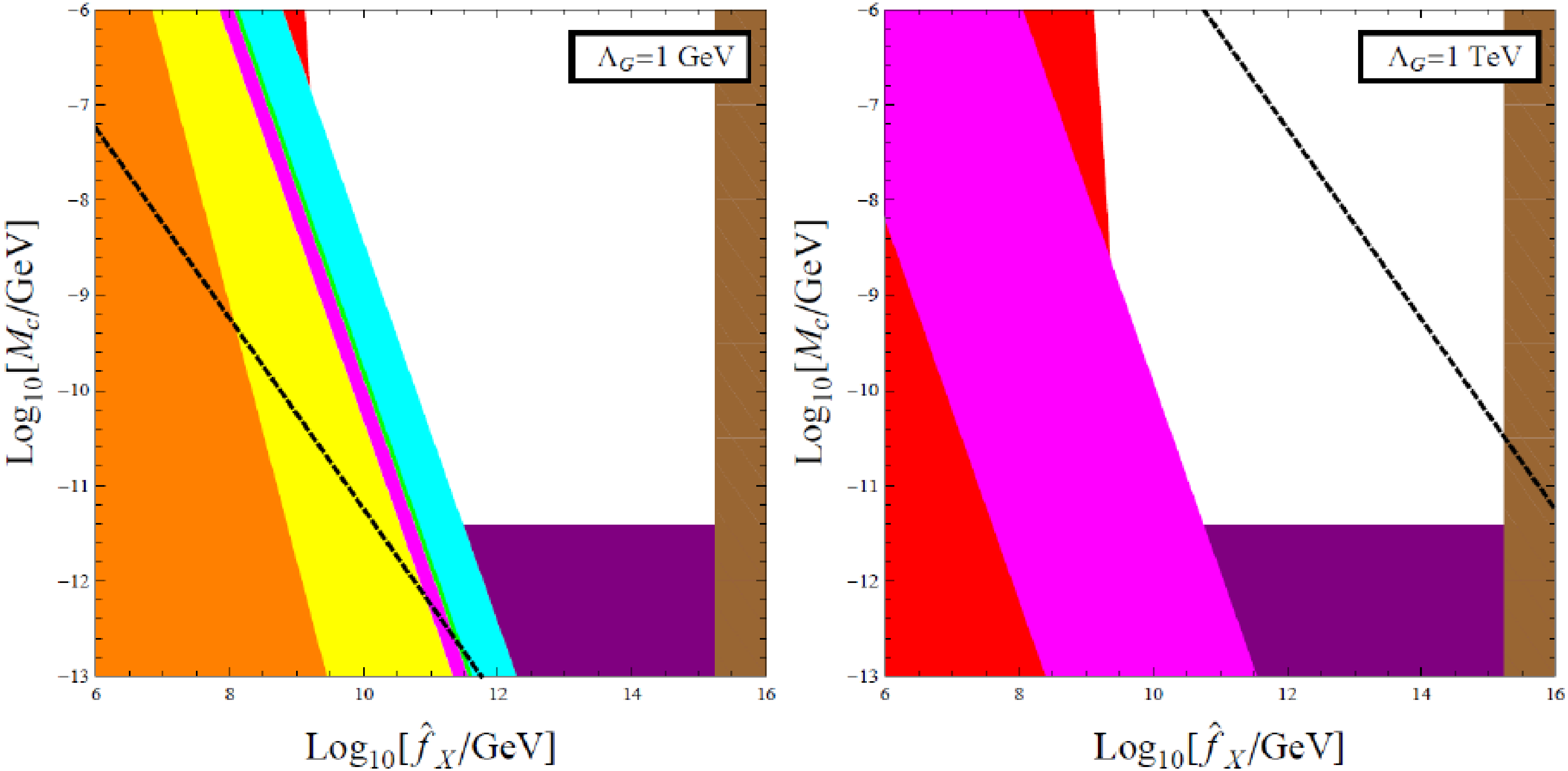
- GC (globular cluster) stars. Axions might carry away energy too efficiently, altering stellar lifetimes. GC stars give most stringent bound.
- SN1987a. Same --- axions would effect energy loss rate.
- Diffuse photon/X-ray backgrounds. Axion decays to photons would leave unobserved imprints.
- Eotvos. Cavenish-type "fifth force" experiments place bounds on sizes of extra spacetime dimensions.
- Helioscopes. Detectors on earth measure axion fluxes from sun.
- Collider limits. Constraints on missing energies, etc.
- Overclosure. Too great a DM abundance can overclose universe.
- Thermal production. Need to ensure that thermal production not contribute significantly to relic abundances (have assumed that misalignment production dominates).

Combined Limits on Dark Towers

Case I: "Photonic" Axion (couples only to photon field)

$(g_\gamma = 1, \xi = \theta = 1)$

- GC stars
- SN1987A
- Diffuse photon spectra
- Eötvös experiments
- Helioscopes (CAST)
- Collider limits
- DM overabundant
- Thermal production

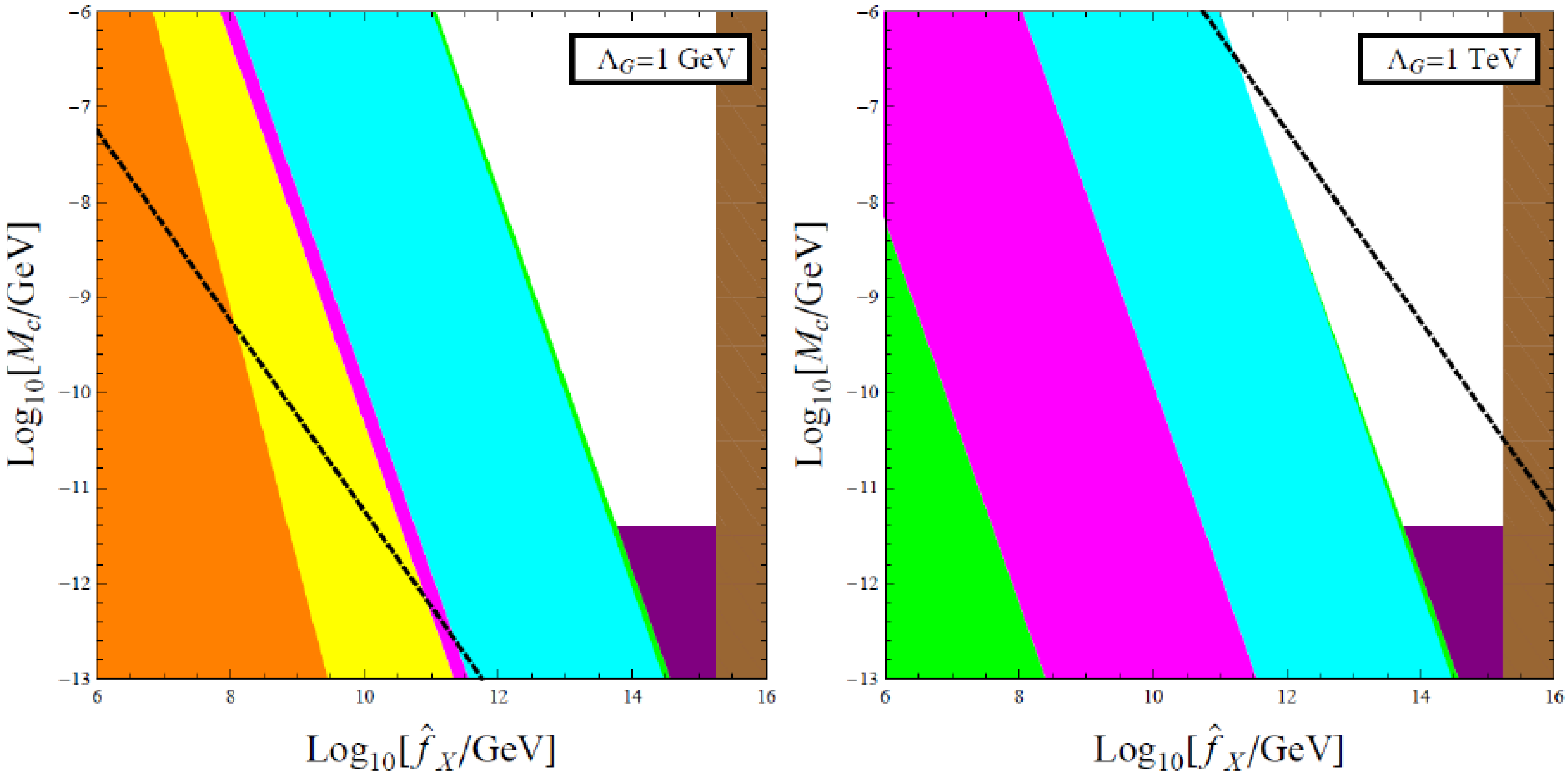


Combined Limits on Dark Towers

Case II: “Hadronic” Axion (couples to photon, gluon fields)

$$(g_\gamma = g_g = 1, \xi = \theta = 1)$$

- GC stars
- SN1987A
- Diffuse photon spectra
- Eötvös experiments
- Helioscopes (CAST)
- Collider limits
- DM overabundant
- Thermal production



Conclusions

"Dynamical dark matter": a new framework for dark-matter physics

- Stability is replaced by a delicate balancing between abundances and lifetimes across a vast collection of dark-matter components which collectively produce a time-varying Ω_{CDM}. Dark-matter decays occur throughout current epoch!
- This scenario is well-motivated in field theory and string theory, and can even be used to constrain the phenomenological and cosmological viability of certain limits of string theory.
- Specific examples of "dynamical dark matter" satisfy all known collider, astrophysical, and cosmological constraints, and potentially yield new signatures and features (e.g., decoherence) that transcend those usually associated with dark matter. Many extensions/generalizations are possible!

Dynamical dark matter is therefore a viable alternative to the standard paradigm of a single, stable, dark-matter particle, and must be considered alongside other approaches in future discussions of the dark-matter problem.