1. The group velocity in a homogeneous medium is $v_g = \nabla_{\mathbf{k}}\omega$, in magnitude and direction

This is a standard formula. To derive it, consider a gaussian wave packet

$$\psi(r;t) = \int \frac{d^3k}{(2\pi)^3} \exp(i k \cdot r - i \omega t) \exp\left[-(k - \bar{k})^2/2\sigma^2 \right].$$

Expanding $\omega(k) = \bar{\omega} + (k - \bar{k}) \cdot \mathbf{v_g}$, where v_g is $\nabla_{\mathbf{k}}\omega$ evaluated for $k = \bar{k}$, and putting $k = \bar{k} + k'$, one finds approximately

$$\begin{aligned} \psi(r;t) &= \exp(i\bar{k} \cdot r - i\bar{\omega}t) \int \frac{d^3k'}{(2\pi)^3} \exp\left(ik' \cdot (r - v_g t)\right) \exp\left[-k'^2/2\sigma^2 \right] \\ &= \frac{\sigma^3}{(2\pi)^{3/2}} \exp(i\bar{k} \cdot r - i\bar{\omega}t) \exp\left[-(\sigma^2/2)(r - v_g t)^2 \right] \end{aligned}$$

Manifestly, in this approximation ψ remains localized and moves with velocity v_g; the energy contained in ψ also remains localized and moves with velocity v_g. Thus v_g is both the group velocity (i.e. the velocity of a group of waves) and the velocity of energy propagation.

The exact ψ will disperse more or less depending on higher derivative of ω and the notion of group velocity (as derived here) is valid as long as this dispersion is small. See point 4 for a more general treatment.

2. If one considers the superposition of two waves with wave vectors k_1 and k_2 and corresponding frequencies ω_1 and ω_2, one sees a time-varying interference pattern (except that the pattern is time-independent if $\omega_1 = \omega_2$, as is the case when $|k_1| = |k_2|$ and ω depends only on $|k|$). For two waves of equal amplitude (which is the case considered in Greg Brown's notes), the interference pattern propagates with velocity $\hat{u}(\omega_1 - \omega_2)/|k_1 - k_2|$, where $\hat{u}$ is the unit vector in the direction of $k_1 - k_2$. For small $k_1 - k_2$ this interference pattern velocity is $\hat{u}(\nabla_{\mathbf{k}}\omega \cdot \hat{u})$, or, in words, it is equal to the projection of the group velocity in the direction of $k_1 - k_2$. This is equal to the group velocity $\nabla_{\mathbf{k}}\omega$ if k_1 and k_2 are collinear and ω depends only on $|k|$, but not in general. It is also worth stressing that the interference pattern velocity can be defined only if the two waves have the same amplitude. If the waves have different amplitudes, the interference pattern changes with time in a complicated manner and cannot be obtained at time t by rigidly shifting the pattern at $t = 0$ by an amount vt for any v.

3. To understand the meaning of the interference pattern velocity, consider two gaussian packets with average vectors $\bar{k}_1$ and $\bar{k}_2$ and the same spread σ. Suppose at $t = 0$ the two packets are both centered at the origin. The resulting wave packet displays an interference pattern similar to that discussed in 2, but confined to the width of the gaussian. At later times the interference pattern is visible until the two packets separate, and after that we have two separate gaussian packets moving with group velocities v_{g1} and v_{g2}. If we regard the two separate gaussian packets as a single packet, it is clear that the propagation

velocity of the center of this non-gaussian packet is $(v_{g1} + v_{g2})/2$. This is also the velocity of energy propagation and must be regarded as the group velocity of the non-gaussian packet, not only after the packets separate, but also when they overlap. It is clear that this group velocity (which is basically in the direction of $\vec{k}_1 + \vec{k}_2$) is different from the velocity of propagation of the interference pattern (which is in the direction of $\vec{k}_1 - \vec{k}_2$), unless $\vec{k}_1$ and $\vec{k}_2$.are collinear (as already discussed).

4. The example given in 3 is consistent with the general definition of the group velocity for any localized wave packet

$$\psi(\mathbf{r};t) = \int \frac{d^3k}{(2\pi)^3} \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)A(\mathbf{k})$$

which is as follows. The average position of the packet is

$$\bar{\mathbf{r}}(t) = \frac{\int \mathbf{r}\,|\psi|^2\, d^3r}{\int |\psi|^2\, d^3r}$$

assuming that the integrals exist (which they do if the packet is localized, by definition). Then the group velocity is

$$\mathbf{v}_g = \frac{d\bar{\mathbf{r}}}{dt}$$

It is left as an exercise to prove that

$$\mathbf{v}_g = \frac{\int \nabla_{\mathbf{k}}\omega\, |A|^2\, d^3k}{\int |A|^2\, d^3k}$$

or, in words, $\mathbf{v}_g$ is the average of $\nabla_{\mathbf{k}}\omega$, and obviously, if $A(\mathbf{k})$ is strongly peaked at $\vec{k}$, then $\mathbf{v}_g$ is $\nabla_{\mathbf{k}}\omega$ evaluated at $\vec{k}$. The proof of the "exercise" can be found in quantum mechanics books and hinges on the fact that

$$\mathbf{r}\exp(i\mathbf{k}\cdot\mathbf{r}) = -i\nabla_{\mathbf{k}}\exp(i\mathbf{k}\cdot\mathbf{r}).$$

It helps to note that $\int |\psi|^2\, d^3r$ is time-independent and equal to $\int |A|^2\, d^3k/(2\pi)^3$, so both can be taken to be 1.

5. In conclusion, the formula for the group velocity is quite general and Pendry does not pull it out of the air. The only reservation I have is that the argument given in 4 works for Schrodinger waves, and one should examine more carefully the case of Maxwell waves, where both $-i\omega t$ and $+i\omega t$ are present. I quit here.