THEOREM: If $\psi(\mathbf{r};t)$ is a wave packet,

$$\psi(\mathbf{r};t) = \int \frac{d^3k}{(2\pi)^3} A(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega(\mathbf{k})t) \qquad (1)$$

then, provided of course that the integrals exist,

$$\frac{\partial}{\partial t} \int d^3r\, \mathbf{r}\, |\psi(\mathbf{r};t)|^2 = \int \frac{d^3k}{(2\pi)^3} \nabla_{\mathbf{k}}\omega\, |A(\mathbf{k})|^2 \qquad (2)$$

(The restrictions on the integrals mean that the packet must be localized both in coordinate space and in momentum space)

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Start from

$$\mathbf{r}\,|\psi(\mathbf{r};t)|^2 = \psi^*(\mathbf{r};t)\,\mathbf{r}\,\psi(\mathbf{r};t) = \frac{1}{i}\int \frac{d^3k}{(2\pi)^3} \psi^*(\mathbf{r};t)\left[\nabla_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{r})\right] A(\mathbf{k}) \exp(-i\omega(\mathbf{k})t)$$

Integrating over $\mathbf{r}$ and using the complex conjugate of

$$\int d^3r\, \psi(\mathbf{r};t) \exp(-i\mathbf{k}\cdot\mathbf{r}) = A(\mathbf{k}) \exp(-i\omega(\mathbf{k})t),$$

which is the inverse of Eq. (1), obtain

$$\begin{aligned}
\int d^3r\, \mathbf{r}\, |\psi(\mathbf{r};t)|^2 &= \frac{1}{i}\int \frac{d^3k}{(2\pi)^3} \left[\nabla_{\mathbf{k}} A^*(\mathbf{k}) \exp(i\omega(\mathbf{k})t)\right] A(\mathbf{k}) \exp(-i\omega(\mathbf{k})t) \\
&= \frac{1}{i}\int \frac{d^3k}{(2\pi)^3} \left[\nabla_{\mathbf{k}} A^*(\mathbf{k})\right] A(\mathbf{k}) + t\int \frac{d^3k}{(2\pi)^3} \left[\nabla_{\mathbf{k}}\omega(\mathbf{k})\right] |A(\mathbf{k})|^2 .
\end{aligned}$$

The proof is completed by taking the time derivative.