Two kinds of charges: unlike charges attract, like charges repel.

Charge is conserved.

Charge is quantized.

**Fundamental amount of charge:** Charge of a single electron: \( e \)

Coulomb's Law

\[
F = k_e \frac{q_1 q_2}{r^2}
\]
Two identical small charged spheres, each having a mass of $3.0 \times 10^{-2}$ kg, hang in equilibrium. The length of each string is 0.15 m, and the angle is $5.0^\circ$. Find the magnitude of the charge on each sphere.
\[
\sin \theta = \frac{a}{L}
\]

\[
a = L \sin \theta = 0.013 \text{ m}
\]

\[
\sum F_x = T \sin \theta - F_e = 0
\]

\[
\sum F_y = T \cos \theta - mg = 0
\]

\[
\Rightarrow T = \frac{mg}{\cos \theta}
\]

\[
F_e = T \sin \theta = mg \sin \theta / \cos \theta
\]

\[
F_e = mg \tan \theta
\]

\[
F_e = k_e \frac{q^2}{r^2}
\]

\[
\Rightarrow |q| = \sqrt{\frac{F_e r^2}{k_e}}
\]
Electric Field
A Field:
• Has a well defined value at every point of space

Examples:
• Elevation: $H(x,y)$
• Temperature $T(x,y,z)$  Scaler field
• Wind Speed $W(x,y,z)$  Vector field

A Field:
• Has a well defined value at every point of space
• Field does not depend on how we measure it: ie: our measurement does not change the field
• The field is “out there” whether we measure it or not
Electric Field due to a charge $q$

\[ \mathbf{F}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}} \]

Electric field is the electric force acting on a unit positive charge

\[ \mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \]

\[ \mathbf{E} = \frac{\mathbf{F}_e}{q_o} \rightarrow \mathbf{F}_e = \mathbf{E} q_o \]

\[ \mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \]
For the dipole shown, find the electric field $\mathbf{E}$ at a point $P$ due to the charges, where $P$ is a distance $y \gg a$ from the origin.
The total field is \( \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \) where

\[
E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}
\]

\[
E = 2 \cdot E_1 \cos \theta
\]

\[
\cos \theta = a/r = a/(y^2 + a^2)^{1/2}
\]

\[
E = 2 \cdot E_1 \cos \theta = k_e \frac{q}{y^2 + a^2} \cdot \frac{a}{(y^2 + a^2)^{1/2}}
\]

\[
= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}
\]

\[
\approx k_e \frac{2qa}{y^3}
\]
The electric field at $P$ due to one element carrying charge $\Delta q$ is:

\[
E = k_e \frac{\Delta q}{r^2} \hat{r}
\]

\[
E = k_e \sum_i \frac{\Delta q_i}{r_{i}^2} \hat{r}_i
\]

\[
E = k_e \lim_{\Delta q \to 0} \sum_i \frac{\Delta q_i}{r_{i}^2} \hat{r}_i = k_e \int \frac{dq}{r^2} \hat{r}
\]
• Volume charge density $\rho$:

$$\rho = \frac{Q}{V}$$

• Area charge density $\sigma$:

$$\sigma = \frac{Q}{A}$$

• **Linear** charge density $\lambda$:

$$\lambda = \frac{Q}{l}$$
Example 23.7 from the text book: Electric field due to a charged Rod

The example worked out in detail in the text book is similar to the one we did in class (the location of the rod is different).