Assignment 9

Problem 4

(4 points) In a plane electromagnetic wave, what fraction of the energy is electric and what fraction is magnetic? A precise answer is expected, and should be derived using the simple plane wave described by Melissinos in eqs. (4.9) and (4.9'), but the answer is true more generally.

On pp. 118-119 Melissinos derives the wave equation from the Maxwell equations

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

and notes that $\mu_0 \epsilon_0 = 1/c^2$ where $c$ is the speed of light. He then considers a solution of the form

$$E_x = E_0 \cos(\omega t - kz),$$

where $\omega$ is the angular frequency, $k$ the wave number and $\omega/k = c$. From the Maxwell equations he obtains the corresponding magnetic field

$$B_y = \frac{E_0}{c} \cos(\omega t - kz).$$

Next note that the (instantaneous) electric field energy density is

$$u_E = \frac{\epsilon_0}{2} E^2,$$

while the magnetic field energy density is

$$u_B = \frac{1}{2\mu_0} B^2.$$

where $E^2 = E_x^2 + E_y^2 + E_z^2$. For the solution Melissinos considers, the fraction of magnetic field energy density to electric field energy density is

$$\frac{u_E}{u_B} = \frac{B_y^2}{\mu_0 \epsilon_0 E_x^2} = \frac{1}{c^2 \mu_0 \epsilon_0} = 1.$$

Therefore, the ratio of magnetic field energy to electric field energy is one; or in other words, the energy density is half electric and half magnetic.