Nuclear Experiment
Phys 318/317
Room 208

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NOTE: Some of the figures referred to in this document can be found in ORTEC Application Note 34. Copies of these notes are attached in the write-up handed to you in Lab.

EXPERIMENT 1
BASIC IDENTIFICATIONS IN ELECTRONIC MEASUREMENT SYSTEMS

PRELIMINARY DISCUSSION
PURPOSE

All the experiments described in this manual require that some combination of electronic instrument modules interconnected and adjusted to provide the desired information. The purpose of this experiment is to familiarize the student with the basic techniques to use in checking for proper system arrangement and responses.

ELECTRONIC CIRCUITS

First is an introduction to a pulse generator. This is an instrument that simulates the pulses that would originate in a nuclear radiation detector and furnishes the pulses with the known characteristics into the input of the system. The pulse generator is used for calibration, timing, and certain test operations. Used together, the oscilloscope and pulse generator can assure the student that the electronics have been set up according to the block diagrams that accompany each experiment so that he can complete the series of exercises as required.

The system that is used in this experiment includes a group of modules that are basic to many experiments: a preamplifier, an amplifier, a threshold discriminator, a single-channel analyzer, a scalar, and an oscilloscope. Each module provides a necessary function in the overall system, which, in this combination, is a simple counting system.

These electronic modules are divided into two general types, logic and linear. A more complete discussion of these devices is included in the Appendix, "Linear and Logic Signal Standards in ORTEC NIM Instruments." Briefly, a logic module is a device that generates an output pulse of fixed amplitude if its logic criteria are met. The simplest example of a logic device is a threshold discriminator, which gives an output pulse (always with the same amplitude) every time it receives an input pulse that has an amplitude greater than the threshold level. The single-channel analyzer is another good example of a logic module. A linear module is one in which the output linear signal contains information such as the energy of an incident particle that has been absorbed in a detector.

The procedure outlined in Experiment 1.1 tells the student how to operate the pulse generator to observe the direct and attenuated outputs. The procedures in Experiment 1.2 consist of three parts: (1) how to apply the output signal from the pulse generator
into the linear portion of the measurement system (amplifier) and observe the linear
shaping of these modules in the system; (2) how to determine whether the logic criteria
have been met by observing with the oscilloscope the logic output from a discriminator;
and (3) how to use a single-channel analyzer to replace the function of the
discriminator.
1.1 OBSERVING THE DIRECT AND ATTENUATED OUTPUTS OF THE PULSE GENERATOR (PULSER)

Equipment Needed:
Pulse Generator Ortec 480
Digital Oscilloscope Tektronix TDS 3012

INTRODUCTION
The pulse generator generates output pulses that are used to simulate pulses from nuclear radiation detectors. Normally the output from the pulse generator will be fed into a preamplifier to become the test pulse input. In our experiments it will be fed directly into the oscilloscope for testing or it will be fed into the amplifier.

The pulse generator has two output pulses that occur simultaneously: a direct output and an attenuated output. The direct output has a pulse height or full amplitude that is adjusted with a front panel dial and will usually be used to trigger the oscilloscope with a time = 0 reference. The attenuated output has a pulse height that is a selected fraction of the direct output and is the variable amplitude needed for energy calibration. The amplitude of these outputs can be varied continuously from 0 to about 3 V by the use of the pulse-height and calibration controls and the attenuator switches. The output polarity is selectable on the front panel.

PROCEDURE
1. Set the input impedance of channel 1 of the oscilloscope to 50 ohms. See the Lab instructor about operating the oscilloscope and pulse generator. Connect a 5-ft (or shorter) cable, type RG-58/U with BNC connectors, from the direct output of the pulse generator to the channel 1 input. This is known as receiving-end termination and in this case simulates the input impedance of a preamplifier.

2. Select a positive output polarity and set the Pulse-Height knob fully clockwise. Now turn the Calibrate control with a small screwdriver clockwise until the pulse is a maximum on the scope. The calibrate control will continue to turn even when the maximum is reached.

3. Trigger the oscilloscope by adjusting the Triggering Level Control. When the oscilloscope is operating properly, the output should appear approximately as that shown in Fig. 1.4 in the ORTEC Application Note 34. Note that in Fig. 1.4 the maximum pulse amplitude appears at the beginning and is a little greater than
3 Volts. Turn the Calibrate control counter clockwise until the peak voltage reads 3 Volts on the oscilloscope.

4. Now change the cable connection on the pulse generator to the **attenuated output**. Set all attenuation switches at X1. Leave the pulse-height control fully clockwise and calibration control at the previous setting. The peak amplitude on the oscilloscope should still be 3 Volts as in the previous setting. The pulse-height dial on the pulse generator is a 10-turn potentiometer with a duo-dial. There are 100 division marks for each turn of the knob for a total of 1000 divisions on the dial. The settings on the pulse-height dial can then be represented as a ratio. For example, 90% of full CW would be 900/1000, etc.

**Exercise c.** In Table 1.2, record from the oscilloscope the maximum voltage values observed for each of the following pulse-height settings of the pulse generator 1000, 800, 600, 400, and 200.

<table>
<thead>
<tr>
<th>Pulse Height Dial Settings</th>
<th>Voltage Amplitudes (Oscilloscope)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

**Exercise d.** Make a plot on linear graph paper of pulse height dial settings vs. oscilloscope voltage. Is this a straight line?

5. Return the pulse-height dial to 1000. Set the top attenuator switch at X2 on the pulse generator. The amplitude of the pulse should decrease by a factor of 2.

6. Try various combinations of these attenuator switches and observe the output. Return the attenuator switches to X1 and readjust the oscilloscope vertical sensitivity. Now slowly turn the calibrate control CCW while observing the output pulse. It should linearly attenuate the output voltage as does the pulse-height dial.
1.2 USING THE PULSER AS THE LINEAR INPUT TO A TYPICAL COUNTING SYSTEM

Equipment Needed:

Pulser Ortec 480
Amplifier Canberra 2022
Timing Single-Channel Analyzer (SCA) Canberra 2037A
Counter (or Scaler) Canberra 2071A
NIM Bin and Power Supply Ortec 401
Oscilloscope Tektronix TDS 3012

PROCEDURE

As is true for almost all NIM-type instrument modules, the Amplifier, Timing Single-Channel Analyzer, and Counter must be installed in a NIM Bin and the Power Supply must be turned on to provide the operating power requirements of the module. The Bin and Power Supply can accommodate any of these instrument modules in any configuration and supply the appropriate power to all the modules. Follow the instructions and connect the circuit up as shown below. Always turn off the power before inserting or removing any instrument module.

1. Make sure the Pulser, Amplifier, Timing Single-Channel Analyzer, and Counter are inserted into the NIM Bin.

2. Connect the attenuated output of the Pulse Generator to Ch. 1 of the Oscilloscope and to the input connector on the Amplifier (connections are always made with RG-58/U cable and BNC connectors unless otherwise specified). Use a Tee BNC connector to make this connection. See Figure below.
3. Set the Amplifier for a positive input.
4. Connect the unipolar output of Amplifier to the input of the Timing Single-Channel Analyzer,
5. Connect the positive output of the SCA to the input of the Counter.
6. Set the SCA mode switch at LE (leading edge timing). Set Lower Level at 50/1000. Set the Window Level full CW. This makes the SCA operate as a Threshold Discriminator.
7. Set the Start/Stop switch to stop.

Adjusting the Linear Portion of the System
1. Using the Pulse Generator attenuated output, set its parameters as follows: positive output, pulse height, 1000/1000.
2. Trigger the oscilloscope. See the instructor.
3. Adjust the attenuator switches and the calibrate control of the Pulse Generator so that the output pulses from the Pulser are about 0.1 V.
4. Now move the oscilloscope cable to the output of the Amplifier and adjust its gain controls until the output pulses have a 4 V amplitude. The correct output pulse should look like that in Fig. 1.6 in the ORTEC manual.
5. With the amplifier settings the same, switch to the Amplifier's bipolar output and move the SCA mode switch to CO (Crossover) timing. The correct output pulse should look like that in Fig. 1.7. (Retrigger oscilloscope if necessary.)
6. Reconnect the SCA to the Amplifier's Bipolar output.

Determining Logic Criteria
1. Connect the oscilloscope to the Positive output of the SCA.
2. Trigger the oscilloscope with the same parameters that were used for looking at the Amplifier output. A 2.5 V TTL logic pulse should be observed with the oscilloscope when the input impedance is 50 Ohms. If the input impedance is changed to 1M Ohm, then the output of the SCA will be the expected 5 V.
3. Raise the Counter Start/Stop switch to Start. The Counter should count the SCA output pulses.
4. Now, increase the setting of the SCA Lower-Level control until the scalar just stops counting. Record the control setting for the first line in Table 1.3.

5. On the Pulse Generator, decrease the Pulse-Height control to 800/1000.

**Exercise a.** Decrease the SCA Lower-Level control until the scalar just barely starts to count. Record this setting in Table 1.3, and continue for the other settings in Table 1.3.

**Exercise b.** Make a plot of the data in Table 1.3 on linear graph paper. This should produce a straight line.

<table>
<thead>
<tr>
<th>Pulse Height Dial Settings</th>
<th>Lower-Level on TSCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>
EXPERIMENT 1.3

1.3 USING A SINGLE-CHANNEL ANALYZER

EQUIPMENT NEEDED:
Pulser Ortec 480
Amplifier Canberra 2022
Timing Single-Channel Analyzer (SCA) Canberra 2037A
Counter (or Scaler) Canberra 2071A
NIM Bin and Power Supply Ortec 401
Oscilloscope Tektronix TDS 3012

PROCEDURE
1. Connect the positive output of the SCA to the Counter input.
2. Set the Lower-Level dial at 100/1000 and the Window-Level dial at 100/1000. This will allow the SCA to generate an output whenever the input signal is in between the Lower-Level and Lower-Level Plus the Window Level.
3. Decrease the Pulse Generator Pulse-Height control until the scalar starts to count. Record this setting in Table 1.5 as E Upper.

Exercise c. Continue to decrease the Pulse Generator Pulse-Height control until the scalar stops counting. Record this value as E Lower in Table 1.5.

Exercise d. Make a plot of the Window settings vs. (E upper - E Lower) on linear graph paper.

Table 1.5

<table>
<thead>
<tr>
<th>SCA Lower Level</th>
<th>SCA Window Level</th>
<th>Pulser E Upper</th>
<th>Pulser E Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Exercise e.** Repeat these measurements with the Lower Level set at 200/1000 as in Table 1.6.

<table>
<thead>
<tr>
<th>SCA Lower Level</th>
<th>SCA Window Level</th>
<th>Pulser E Upper</th>
<th>Pulser E Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>300</td>
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<td></td>
</tr>
<tr>
<td>200</td>
<td>600</td>
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<td></td>
</tr>
<tr>
<td>200</td>
<td>800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
EXPERIMENT 3
GAMMA-RAY SPECTROSCOPY USING NAI(Tl)

PRELIMINARY MATERIAL

PURPOSE

The purpose of this experiment is to acquaint the student with some of the basic techniques used for measuring gamma rays. It is based on the use of a sodium iodide (NaI) detector that is thallium-activated (Tl).

GAMMA EMISSION

Most isotopes that are used for gamma measurements also have betas in their decay schemes. The typical decay scheme for the isotope will include a beta decay to a particular level followed by gamma emission to the ground state of the final isotope. The beta particles will usually be absorbed in the surrounding material and not enter the scintillator at all. This absorption is normally assured with aluminum absorbers. For this experiment the betas offer no real problem, and so absorbers are not specified. There will be some beta absorption by the light shield over the phototube. The gammas, however, are quite penetrating and hence will pass easily through the aluminum light shield.

Generally there are two unknowns that we would like to investigate about a gamma source. One is the energy of the gammas from the source. The other is the number of gammas that leave the source per unit of time. In this experiment the student will become familiar with some of the basic NaI(Tl) measurements associated with gamma emitting unknowns.

References


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**Equipment Needed for Experiments 3.1 Through 3.7**

ORTEC 401 Bin and Power Supply  
Canberra NaI(Tl) Crystal, Phototube Assembly Model 802 – 2 x 2 and  
Canberra Photomultiplier Tube Base( Preamplifier Model 2007P)  
Hewlett Packard 6516A High Voltage Power Supply  
Canberra 2022 Amplifier  
137CS gamma source, 5 $\mu$Ci ±5%  
Source Kit (Located in locked draw)  
8 each lead plates (3 x 3 x 1/16 inch)  
Connecting cables  
Multichannel Analyzer (MCA)

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**3.1 ENERGY CALIBRATION**

**SETUP OF EQUIPMENT**

Set up the detector assembly, high voltage, UCS20, and computer as shown below in Fig. 3.1. There are three parameters that ultimately determine the overall gain of the system: the high voltage that is furnished to the phototube and the gain of the pre-amplifier and the amplifier in the UCS20 spectrometer.
Fig. 3.1 Electronic Block Diagram for Gamma-Ray Spectroscopy System with NaI(Tl) Detector.

The Detector Assembly consists of the crystal, PM tube, and pre-amplifier.
The Lab Instructor will assist you in connecting the signal cable, pre-amp cable, and high voltage cable. The signal cable is connected directly from the pre-amplifier to the Amp in connector of the UCS. The high voltage cable is connected to the high voltage power supply. The pre-amp cable is connected to the back of an amplifier module in the nuclear bin. The gain of the photomultiplier tube is quite dependent upon its high voltage. A rule of thumb for most phototubes is that a 10% change of the high voltage will change the gain by a factor of 2. The high-voltage is set at about the middle of the acceptable operating range (normally about +1000 V). If the voltage is not adequate or does not cover the range in the UCS, see Lab Instructor. Do not change it on your own. The UCS system consists of a computer, display, keyboard, and software to run the data acquisition program. Ask the Lab Instructor for help before using the system. To start the data acquisition system, follow the procedure below:

Start the computer by pressing the black button the rear of the computer box. (Usually this computer will already be on.)

Wait until the Windows operating system appears on the display.

Click on the UCS20 icon to start the UCS20 program.

An abbreviated manual of how to use UCS20 is on the lab bench. Do not remove it from the Lab. Ask the Lab Instructor to help interpret the panels on the display for you. A detailed manual for UCS20 is also available on the computer as a PDF file.

You may start acquiring data by clicking on GO. There is sufficient radiation in the room that the detectors will count even without the radioactive source in place. Click on stop to stop accumulating and display the spectrum.

PROCEDURE (You will need to ask the Lab instructor for assistance in getting started)

1. Request the $^{137}$Cs source from the Lab Instructor and place it about 2 cm in front of the NaI(Tl) crystal using the available holder. A the predominant gamma ray photopeak energy in the data is 0.662 MeV.

2. Set the voltage gain (Click on Settings and then on Amp/HV/ADC)) of the system such that the centroid of the 0.662-MeV photopeak for $^{137}$Cs is approximately in channel 500. Since the system is linear, this will correspond to about 0.0013 MeV per channel. This is different than in the illustrations shown in Figs. 3.1a and 3.2b where the gain of the system has been set so that 0.662 MeV falls at channels around 331.

3. Accumulate the $^{137}$Cs spectrum for a specified period time that is long enough to determine the centroid of the position of the most prominent peak. The spectrum will look similar to the one shown above. You may set the time on the computer so that it
accumulates data for a specified time. (Click on the clock and click on live time.) I would suggest about 300 sec.

4. Save the spectrum as a name.spu file on the computer in a folder with your name on it. You can read this file with Excel. All analysis will be done with Excel.

5. Determine the centroid (channel number) of the 0.662 photopeak and write it down in Table 3.1a. Do the same for the 0.032 MeV x-ray photopeak. Now calibrate the x-axis in KeV.

   Click on Settings
   Expand Energy Calibrate and click on Uncalibrate
   Click on Energy Calibrate again
   Click on 2 point
   Enter the energy in KeV and the Channel number of the two peaks. A linear fit will be made to calibrate the x-axis in KeV. Click on the peaks to be sure they read the correct energy in KeV. Your numbers should agree with the spectrum below in Fig 3.2a if you calibrated it correctly.

Figure 3.2a illustrates a typical $^{137}$Cs spectrum in 1024 channels that has been plotted on linear scale.
Fig 3.2b shown below shows the full 2048 channels using a logarithmic scale for the vertical axis. This scale emphasizes the channels with the fewest counts and shows background counts in channel 1000 and also up around 1800. These counts are not coming from the $^{137}$Cs source but are probable coming from background in the room.

Fig 3.2b shows the full 2048 channels plotted using a logarithmic vertical axis.
6. The next is to determine the background in the room that you also accumulated with the Cs spectrum. Click on the Eraser and delete the Cs data in the computer that is being displayed. Accumulate a spectrum with the $^{137}$Cs source completely removed from the detectors. It is necessary to do this to determine the cosmic ray and room background. Record it for the same amount of time as $^{137}$Cs. Record the live time again. Save this spectrum on the computer and follow the same procedure as above in order to plot it as a scatter plot in Excel. You may also use the Strip Background feature of the computer instead of Excel. Ask your instructor for assistance.

7. Multiply the number of counts in each channel in the background spectrum by the ratio of the live times. If the live times are the same, then this number is 1. Now subtract the background spectrum from the $^{137}$Cs spectrum. The subtracted $^{137}$Cs spectrum should look even closer to Fig 3.2 in the ORTEC Application Note 34.

8. Repeat steps 3, 4, and 5 with a $^{60}$Co, and $^{22}$Na source. Compare the $^{60}$Co to that in Fig. 3.3. They should be similar. What are the energies of any prominent peaks in the background. Estimate the centroids and determine the energies.

Exercise a. Plot the $^{137}$Cs, $^{60}$Co, and $^{22}$Na spectra and fill in items 1, 2, 3, 4, and 5 in Table 3.1A. How do your measured energies of the peaks compare with known values. Explain why there are differences? What is the error in determining the centroid of a photopeak? How does this error propagate to error in energy in your calibration?

Exercise b. From items 1, 2, 3, 4, and 5 in Table 3.1A make a plot of energy of the photopeaks vs. channel number. Figure 3.4 shows this calibration for the data taken from Figs. 3.2 and 3.3. Using the methods in Ref 9 to perform a linear least squares fit of the data to a straight line so that the y variable predicts the energy as a function of the x variable which is the channel number. Determine the slope and y-intercept of the straight line that best fits the data in Table 3.1A and the error on the slope and y-intercept. Determine the overall Chi squared and comment on your goodness of fit. You may use the computer program WinCurveFit on the desktop to do the least squares fit. Ask you instructor for assistance.
Exercise c. What feature in the $^{137}\text{Cs}$ spectrum is equal to the energy of the backscattered peak plus the Compton edge? This is true for any gamma ray spectrum.
3.3 SPECTRUM ANALYSIS OF $^{60}$CO AND $^{137}$CS

PURPOSE

The purpose of this experiment is to explain some of the features other than the photopeaks, that are usually present in a pulse-height spectrum. These are the Compton edge, backscatter peak, and x-rays.

The Compton interaction is a pure billiard-ball type collision between a gamma photon and what might be termed a free electron in the NaI(Tl) crystal. By this process the incident gamma gives up only part of its energy to the electron. The amount given to the recoil electron (and hence the intensity of the light flash) depends on whether the collision is head on or glancing. For a head-on collision the gamma imparts the maximum allowable energy to the free electron. The energy of the scattered gamma can be determined by solving the energy and momentum equations for this billiard-ball collision. The solution for these equations in terms of the scattered gamma ray energy can be written as

$$E' = \frac{1}{1 + \frac{1 - \cos \theta}{E \cdot m_e c^2}}$$

(1)

where

$E'_\gamma$ = the energy of the scattered gamma-ray in MeV,
$\theta$ = the scattering angle of the gamma ray,
$E_\gamma$ = energy of the incident gamma-ray in MeV.

$m_e c^2$ = 0.511 MeV the rest mass energy of the electron.

This formula can be written approximately as

$$E' = \frac{E_\gamma}{1 + 1.96 E_\gamma (1 - \cos \theta)}$$

(2)
Compton Edge
If $\theta = 180$ degrees due to a head-on collision in which the gamma ray is scattered directly back, Eq. (2) becomes

$$E'_\gamma = \frac{E_\gamma}{1 + 3.96E_\gamma}$$

(3)

As an example, we will calculate $E'_\gamma$ for an incident gamma energy of 1 MeV:

$$E'_\gamma = \frac{1}{1 + 3.96} = 0.202 \text{ MeV}$$

By conservation of energy, the energy of the recoil electron ($E_e$) is given by

$$E_e = E'_\gamma - E_\gamma$$

(4)

In the above example the energy given to the recoil electron is 0.80 MeV and consequently the Compton edge is located at 0.80 MeV. Hence the position of the Compton edge, which is the maximum energy that can be imparted to an electron by the Compton interaction, is given by Eq. (4).

Backscatter Peak
Consider a 1.0 MeV gamma ray emitted from a radioactive source in opposite direction than that of the detector. It now undergoes a 180 degree scattering from another material such as a lead shield and goes back into the detector with a gamma ray energy of 0.202 MeV. It gets totally absorb producing a photopeak at 0.20 MeV called the backscatter peak.
**Exercise a.** Calculate the energy of the Compton edge for the 0.662-MeV gammas from $^{137}\text{Cs}$. Enter this value in Table 3.1. From your plot and calibration curve, does this calculation agree with your measured value? Explain.

**Exercise b.** Figure 3.5 was taken from Reference 8 and is a good illustration of the various events that can take place using a typical radioactive source with a NaI(Tl) detector and a lead shield arrangement. Backscattered gammas from these interactions make photoelectric interactions in the NaI(Tl) when they enter the crystal. Calculate the energy of the Compton edge and backscattered peaks using Eqs. (2), (3), and (4) and fill in Table 3.1B. Using your energy calibration based on the photopeak data, how do your measured energies compare with the calculated energies from Eq. (2)? If the backscatter peak is not very pronounced in your spectrum, it can be improved by accumulating a spectrum with a sheet of lead placed closed to, but on the far side of the source. You will see another small peak above the Barium x-ray. Where does it come from? Explain.

**Table 3.1B**

<table>
<thead>
<tr>
<th>Items</th>
<th>Event</th>
<th>Known Energy (MeV)</th>
<th>Measured Channel Number</th>
<th>Predicted Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Compton Edge $^{137}\text{Cs}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Backscatter $^{137}\text{Cs}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Backscatter $^{60}\text{Co}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Barium x ray</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Lead x ray</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.4 ENERGY RESOLUTION

PURPOSE

The resolution of a spectrometer, $R$, is a measure of its ability to resolve two peaks that are fairly close together in energy. Figure 3.2 shows the gamma spectrum that was plotted for the $^{137}$Cs source. The resolution of the photopeak is found by using the following equation:

$$R = \frac{\delta E}{E} \times 100$$

where

- $R$ = the resolution in percent,
- $\delta E$ = the full width of the peak at half of the maximum count level (FWHM), measured in number of channels,
- $E$ = the channel number at the centroid of the photopeak.

As an example the photopeak in Fig. 3.2 is in channel 280 and its FWHM = 32 channels. From Eq. (5) the resolution is calculated to be 11.5%.

**Exercise a.** Calculate the resolution of your system from the $^{137}$Cs spectrum. Record this value. What is the error on your measurement?
3.6 ACTIVITY OF A GAMMA EMITTER (ABSOLUTE METHOD)

PURPOSE

The activity of the standard used in Experiment 3.5 can be determined by the absolute method. The purpose of this experiment is to measure the absolute activity of $^{137}$Cs and determine the error on your measurement.

PROCEDURE

1. Place the U1 source ($^{137}$Cs) about 10.0 cm away from the face of the detector.

2. Accumulate a spectrum and note the live time that is used.

3. Read out the multichannel analyzer and determine the number of events under the photopeak, which we designate as $\Sigma_1$. Then erase the spectrum, remove the source, and accumulate background for the same live time and determine the number of events in the same range of channel number, which we designate as $\Sigma_b$.

4. Use the following formula to calculate the activity of U1 in disintegrations per sec. Convert it to microcuries and compare with the value given on the source itself.

$$
\text{Activity of U1} = \left( \frac{\Sigma_{U1} - \Sigma_b}{t} \right) \frac{1}{G\varepsilon_p f}
$$

(7)

where

$t$ = live time in seconds

$\varepsilon_p$ = intrinsic peak efficiency for the gamma energy and detector size used (see Fig. 3.6 or ref. 8),
f = the decay fraction of the unknown activity, which is the fraction of the total disintegrations in which the measured gamma is emitted (see refs. 5 and 6 and Table 3.2),

\[
G = \frac{\text{Area of detector face in cm}^2}{4\pi s^2}.
\]

s = source-to-detector distance in cm.

### Table 3.2. Gamma Decay Fraction f for Some Common Isotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Gamma Energy (MeV)</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{137}\text{Cs})</td>
<td>0.662</td>
<td>0.92</td>
</tr>
<tr>
<td>(^{51}\text{Cr})</td>
<td>0.323</td>
<td>0.09</td>
</tr>
<tr>
<td>(^{60}\text{Co})</td>
<td>1.17</td>
<td>0.99</td>
</tr>
<tr>
<td>(^{60}\text{Co})</td>
<td>1.33</td>
<td>0.99</td>
</tr>
<tr>
<td>(^{22}\text{Na})</td>
<td>1.276</td>
<td>0.99</td>
</tr>
<tr>
<td>(^{22}\text{Na})</td>
<td>0.511</td>
<td>0.99</td>
</tr>
<tr>
<td>(^{54}\text{Mn})</td>
<td>0.842</td>
<td>1.00</td>
</tr>
<tr>
<td>(^{65}\text{Zn})</td>
<td>1.14</td>
<td>0.44</td>
</tr>
</tbody>
</table>

**Exercise a.** Estimate the error on your measurement. Use simple error propagation theory. Explain why your value is much lower and probably outside the bounds of your error?

**Exercise b.** Name two possible sources of error and discuss how much in percent do you think they affected your experimental value.
3.7 MASS ABSORPTION COEFFICIENT

PURPOSE

The purpose of the experiment is to measure experimentally the mass absorption coefficient in lead for 662-keV gamma rays and determine the error on your measurement. The source of the gamma rays is $^{137}$Cs.

References 2, 3, and 4 point out that gammas interact in matter primarily by photoelectric, Compton, or pair production interactions. The total-mass absorption coefficient can be measured easily with a gamma-ray spectrometer. In this experiment we will measure the number of gammas that are removed from the photopeak by photoelectric or Compton interactions that occur -- in a lead absorber placed between the source and the phototube.

From Lambert's law the decrease of intensity of radiation as it passes through an absorber is given by

$$I = I_0 e^{-\mu x}$$  \hspace{1cm} (8)

where

$I$= intensity after the absorber,
$I_0$ = intensity before the absorber,
$\mu$ = total-mass absorption coefficient in cm$^2$/g,
$x$ = density thickness in g/cm$^2$.

The density thickness is the product of the density in g/cm$^3$ times the thickness in cm.

The half-value layer (HVL) is defined as the density thickness of the absorbing material that will reduce the original intensity by one-half. From Eq. (8):

$$\ln I/I_0 = -\mu x \text{ or}$$
$$\ln I = \ln I_0 - \mu x$$  \hspace{1cm} (9)
If \( l/l_0 = 0.5 \) and \( x = \text{HVL} \), \( \ln 0.5 = -\mu \) (HVL) and hence \( \text{HVL} = 0.693/\mu \). \( (10) \)

In this experiment we will measure \( \mu \) in lead for the 0.662 MeV gammas from \(^{137}\text{Cs}\) and determine the error on \( \mu \). The accepted value is 0.105 cm\(^2\)/g. Values for other materials can be found in ref. 6.

**PROCEDURE**

1. Place the \(^{137}\text{Cs}\) source about 1.5 in. from the NaI(Tl) detector. Place the lead brick with a hole in it in front of the detector so that only parallel paths of gamma rays enter the detector. This will lower the count rate but guarantees you only detect gamma rays that traverse the lead sheets normal to the surface. Accumulate the spectrum long enough for the sum under the 0.662-MeV peak \((\SigmaCs - \Sigma_b)\) to be of the order of 4000 to 5000 counts. Record the live time, Read out the multichannel analyzer, and determine \((\SigmaCs - \Sigma_b)\).

2. Erase the spectrum in the multichannel analyzer and insert the first piece of lead (the thickness should be 1/16 in. or less) between the source and the detector. Accumulate the spectrum for the same period of live time as in step 1 above. Read out the multichannel analyzer and determine \((\SigmaCs - \Sigma_b)\).

3. Erase the multichannel analyzer and insert another piece of lead, making the total thickness about 1/8 in. Determine \((\SigmaCs - \Sigma_b)\). Repeat with additional thicknesses of lead until the count-sum is less than 1000. Fill in the data in Table 3.3.
### Table 3.3. Data for Mass Absorption Coefficient

<table>
<thead>
<tr>
<th>Absorber</th>
<th>Absorber Thickness (g/cm²)</th>
<th>(ΣCs − Σb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>7</td>
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</tbody>
</table>

**Exercise a.** Using Excel to plot $I$ vs. absorber thickness in g/cm², where $I = (ΣCs − Σb)/$ Live time. Determine the HVL from this curve and calculate $\mu$ from Eq (10) and estimate the error on $\mu$. How does your value compare with the accepted value of 0.105 cm²/g? Now determine $\mu$ and its error from a linear least squares fit of eq. 9 to the data. Let $y = \ln I$ and $b = \ln I_0$ and $\mu = \text{slope}$. Compare the two values. Which do you think is a better determination? Explain.

**Exercise b.** Name two possible sources of error and discuss how much you think they affected your experimental value.
EXPERIMENT 4
TERRESTIAL RADIATION AND COSMIC RAYS

PURPOSE

You noticed when you were taking spectra with the radioactive sources that you had a significant count rate in the detector even when you had no source present. This is due to background radiation or simply other sources of radiation. The purpose of this experiment is to determine the origin of the background radiation in the room. Sources of natural radiation are found in building materials, air in the room, and cosmic rays, which continually bombard the Earth. The most abundant naturally occurring radioactive elements are $^{40}\text{K}$, $^{238}\text{U}$ and $^{232}\text{Th}$. The most important single source is the radioactive gas radon, which traces back to the decay of Uranium. Radon is colorless and odorless and is present nearly everywhere. Usually, it is in dosages too small to cause damage to your health. Nevertheless, it is second to cigarettes in causing lung cancer. Let us first investigate the air in the room.

4.1 OPEN AIR SPECTRUM

In order to gather sufficient statistics of the radiation in the open air, you will accumulate a spectrum with the NaI detector overnight for a 24-hour period. Connect up the NaI detector as in Fig 3.1. Place the detector on the table top as shown below.
Make sure there are no sources of radiation on the desktop and use a lead brick to shield any sources in the draw where the sources are stored. Place the detector out in the open on the desktop. The High Voltage power is set to +1000V and switched to on. The NIM Bin which supplies electrical power to the preamp detector, is switched to on. Both are allowed to warm up for one hour to ensure that any voltage fluctuations or voltage drifts are now stable. The MCA program is started and set for 24 hours. Make sure the computer is plugged into the circuit protector with battery. When you return in 24 hours the data is exported and saved in the MCA data folder and then converted to a .txt file and opened in Excel, where a graph of channel vs. counts is created.

**Exercise a.** Using your energy calibration, convert the spectrum to counts versus energy and measure the energy of as many peaks as you can in the spectrum. Ignore peaks below 0.150 MeV. Your instructor will help you in getting started. You may also look in the textbook “Radiation Detection and Measurement”, Second Edition, G.F. Knoll, pg 716. Below is a sample air spectrum that was recently taken in Room 218. The following table indicates the energy of various peaks in the spectrum and the possible source. The energy resolution will not be good enough to identify the weak and broad peaks and to associate them with a particular nucleus. It will be necessary to use the pure Germanium detector for that purpose in Exp. 4.3.

**Exercise b.** What nuclear isotopes do peaks 9 and 12 come from? What kind of material can they found in? Explain.
### Channel 1

**Peak** | 1 | 2 | 3 | 4 | 5 | 6  
---|---|---|---|---|---|---  
**Channel** | 59 | 159 | 201 | 240 | 411 | 456  
**Energy** | 0.094816 | 0.239816 | 0.300716 | 0.357266 | 0.605216 | 0.670466  
**Source** | \( {214\text{ Pb}} \) | \( {214\text{ Pb}} \) | \( {214\text{ Pb}} \) | \( {214\text{ Bi}} \) | 137 Cs  
**Source E** | 0.241 | 0.295 | 0.352 | 0.609 | 0.662  
**Error** | 0.491286 | -1.93763 | -1.49602 | 0.621346 | -1.27885  
**Difference** | 0.001184 | -0.00572 | -0.00527 | 0.003784 | -0.00847

### Channel 2

**Peak** | 7 | 8 | 9 | 10 | 11 | 12  
---|---|---|---|---|---|---  
**Channel** | 639 | 774 | 1000 | 1201 | 1454 | 1768  
**Energy** | 0.935816 | 1.131566 | 1.459266 | 1.750716 | 2.117566 | 2.572866  
**Source** | \( {214\text{ Bi}} \) | \( {214\text{ Bi}} \) | \( {40\text{ K}} \) | \( {214\text{ Bi}} \) | \( {214\text{ Bi}} \) | 208 Tl  
**Source E** | 0.934 | 1.12 | 1.461 | 1.764 | 2.204 | 2.614  
**Error** | -0.19443 | -1.03268 | 0.118686 | 0.753061 | 3.921688 | 1.573604  
**Difference** | -0.00182 | -0.01157 | 0.001734 | 0.013284 | 0.086434 | 0.041134
4.2 COSMIC RAY SPECTRUM

Purpose
In addition to the peaks that are still present in the spectrum, there is also an underlying continuum background due to cosmic rays. Cosmic rays are very high energy charged particles entering our atmosphere from outer space. Many of these charged particles start out as protons with energies up to $10^{20}$ eV of energy traveling nearly the speed of light. These same particles interact with our atmosphere producing pions and neutrons. The neutrons interact with $^{14}$N to produce $^{14}$C used for radioactive carbon dating. Highly energetic pions and also muons from pion decay are also produced. The high-energy pions and muons reach the Earth and pass through the NaI crystal, leaving roughly a constant amount of energy per unit thickness of material of the order of 2 MeV-cm$^2$/g. For the NaI detector the density is 3.67 g/cm$^3$. This gives the amount of energy lost by a charged particle as it traverses the NaI detector equal to 7.34 MeV/cm. The NaI crystal is cylindrical in shape with a diameter of 2.0 in and length of 2.0 in. The maximum amount of energy deposited is roughly equal to the length of the detector times the energy loss per unit length, which is $2 \times 2.54 \times 7.34 = 37.3$ MeV. In order to see the maximum amount of energy deposited in the spectrum, reduce the high voltage of the phototube to about 900 volts. Also place a 6dB attenuator on the input of UCS20, the multi-channel analyzer (computer). Accumulate data for 24 hours following the procedure in 4.1. Your calibration is no longer valid since the high voltage has been reduced. Repeat your energy calibration using the same radioactive isotopes.

Exercise a. Explain the shape of the energy spectrum. Why does the spectrum have a sharp spike at very low channel numbers? Why does it have a flat plateau and then fall off above channel 500? To answer the latter question, look up the Bethe-Bloch formula which describes the energy loss of a high energy charged particle traveling though matter. Also see the suggested references in Knoll or Leo.
4.3 OPEN AIR SPECTRUM USING THE HIGH RESOLUTION HIGH PURITY GE DETECTOR

Purpose
In order to obtain better energy resolution of the gamma radiation peaks in the open air geometry, you will accumulate a spectrum with the high resolution, high purity Ge detector. The detector is a small disk with a diameter of 48 mm and length 36 mm. The energy resolution (FWHM) of the detector is 1.85 keV at 1.33 MeV with a photo-peak efficiency of 12.4 % relative to NaI. The recommended bias voltage is +3200 volts with a leakage current of 0.01 nano-amperes. The detector must be operated at 78 degrees Kelvin. For details on how the detector works see Chapter 12 in “Radiation Detection and Measurement”, Second Edition, G.F. Knoll, The purpose of this experiment is to identify gamma ray peaks with high accuracy so that you can associate them with radioactive nuclei in the decay chain of Radon 222 and, therefore, identify the presence of radon. The gamma rays of interest are 0.352, 0.295, 0.241 and 0.609 MeV. The first three gamma rays are from the gamma ray decay of the first three energy levels in $^{214}$Bi. The 0.609 MeV gamma ray is from the decay of the first excited state of $^{214}$Po. For more information, see the attached radioactive chains and gamma ray decay schemes.

Procedure
Place the detector out in the open near the desktop. The instructor will show you how to connect up the electronics to the Ge detector and fill the liquid nitrogen Dewar. After the electronics are connected and the detector is cooled down to 78 degrees Kelvin and stabilized for an 1 hour, you can start taking data. First take an energy calibration using your known radioactive sources and accumulate data for a sufficient time to identify the peaks. Now make sure all sources are put away in the draw and shielded. Accumulate date for 48 hours and follow the same procedure in 4.1. I would recommend running this for 48 hours if possible because the efficiency of the detection is much less than the NaI, although the energy resolution is much better.

Exercise a. Convert the spectrum to counts versus energy and measure the energy of as many peaks as you can in the spectrum. As a check you should easily identify peaks 9 and 12 in the NaI spectrum.

Exercise b. Compare your measured peak energies with gamma rays expected from $^{214}$Bi and $^{214}$Po if radon is present. Can you conclude that you identified the presence of $^{222}$Rn? Explain. Use the radioactive decay chain below to identify the heaviest nucleus that the radon ultimately originates from. Where do you think this nucleus or element can be found in the room?
EXPERIMENT 9  TIME COINCIDENCE TECHNIQUES

This is another exercise designed to familiarize you with the electronics with emphasis on timing.

9.1 SIMPLE FAST-COINCIDENCE EXPERIMENT

Equipment Needed

Pulser Ortec 480
Two Amplifier Canberra 2022
Two Timing Single-Channel Analyzer (SCA) Canberra 2037A
Canberra 2040 Universal Coincidence Counter (or Scaler) Canberra 2071A
NIM Bin and Power Supply Ortec 401
Oscilloscope Tektronix TDS 3012

PURPOSE

This experiment will utilize some of the basic instrument configurations for time coincidence studies.

PROCEDURE

1. Connect up the circuit shown in Fig. 9.6, and make the following module settings:
   Amplifiers: Positive Input, Bipolar Output. Timing Single-Channel Analyzers: Lower Level = 50/1000, Delay 0.5 µs, Window full CW, CO.
   Coincidence: Inputs A and B switch up and Coinc, switch C and D down. Resolving Time maximum (1 µs).
   Pulser: Positive output, power on, attenuated output approximately 0.5 V (measured with an oscilloscope).

2. Adjust the gain of each Amplifier so that its output pulse is about 5 V.

3. Set the Counter/ Timer for a long time (100 s). Vary the delay on either SCA until the maximum counting rate is observed on the Counter. The two branches are now approximately coincident.

4. Clear the scalar. Set the timer for 10 s and count. If the maximum counting rate was set properly in step 2, the scalar should read about 600 (60 Hz for 10 s). Change the delay in either SCA by 50 ns (50 dial divisions) and repeat the 10-s count.

Exercise a. Continue changing the delay for enough readings to plot a time coincidence curve similar to that shown in Fig. 9.7. You may have to narrow the delay
near the beginning of the window and near the end of the window in order to see exactly where it begins and ends.

**Exercise b.** Narrow the resolving time of the Coincidence module to 0.5 s and plot the coincidence curve, which is similar to Fig. 9.7 (note: take readings every 20 ns). You may have to narrow the delay near the beginning of the window and near the end of the window again in order to see exactly where it begins and ends.

**Exercise c.** Narrow the resolving time of the Coincidence module to 100 ns and measure the coincidence curve (take readings every 10 ns). You may have to narrow the delay near the beginning of the window and near the end of the window again in order to see exactly where it begins and ends.

The student should now begin to understand the concept of simple fast time coincidence spectroscopy.

Fig. 9.6. Electronics Block Diagram for Experiment 9.1.
EXPERIMENT 13 GAMMA-GAMMA COINCIDENCE

13.1 OVERLAP COINCIDENCE METHOD FOR MEASURING GAMMA-GAMMA COINCIDENCE OF $^{22}\text{Na}$

REFERENCES


EQUIPMENT NEEDED

Two Canberra NaI(Tl) Crystal, Phototube Assembly Model 802 – 2 x 2 and Canberra Photomultiplier Tube Base( Preamplifier Model 2007P)
Two Hewlett Packard 6516A High Voltage Power Supply
Two Amplifier Canberra 2022
Two Timing Single-Channel Analyzer (SCA) Canberra 2037A
Canberra 2040 Universal Coincidence Counter (or Scaler) Canberra 2071A
NIM Bin and Power Supply Ortec 401
Oscilloscope Tektronix TDS 3012
$^{22}\text{Na}$ source
ORTEC 306 Gamma-Gamma Angular Correlation Table with rotating detector and shields
PURPOSE

Two annihilation quanta are radiated from a $^{22}$Na source in coincidence with each other for each radiation event that will be measured in this experiment. The purpose of the experiment will be to verify that these quanta emanate from the source with an angular separation of $180^\circ$.

INTRODUCTION

Sodium-22 is an excellent source for a simple gamma-gamma coincidence experiment. The decay scheme for this isotope is shown in Fig. 13.1. From the decay scheme it can be seen that 99.95% of the time the $^{22}$Na decay occurs by positron emission and electron capture through the 1.274 MeV state of $^{22}$Ne. Ninety percent of these decay events occur with positron emission, which then annihilate and produce a pair of 0.511-MeV gamma rays that can be seen in the gamma spectrum.

Figure 13.2 shows a typical gamma spectrum for $^{22}$Na that was obtained with a NaI(Tl) detector. The 0.511-MeV peak will usually be quite a bit more intense than the 1.274-MeV peak, primarily because of the detector efficiency differences at the two energy levels and the annihilation process.

The $^{22}$Na source is usually covered with a thin absorber such as a thin (-1/16-in.) piece of metal or plastic. Positrons from the source will lose energy in the absorber by $de/dx$ and will be annihilated in the absorber. The NaI(Tl) detectors will see an approximate point source of radiation. When the positrons are annihilated, two 0.511-MeV gammas will leave the source with an angular separation of $180^\circ$. Experimentally the pair of gamma rays are detected and measured with one detector that is fixed and another detector that can rotate about the source.

The above photograph shows the goiniometer used for the experiment. The $^{22}$Na source is placed in the holder in the center.
A typical electronics diagram for measuring a gamma-gamma coincidence is shown below in Fig 3.3. The pulser and detector are alternately connected to the AMP inputs and are not connected simultaneously.

**Fig 13.3 Electronics Block Diagram**
PROCEDURE

1. Set up the electronics as shown in Fig. 13.3. The instructor will assist you in arranging the two detectors on the goniometer. The radioactive $^{22}\text{Na}$ is placed in the holder at the center of the goniometer.

2. Set the Amplifiers for positive input and Bipolar output. Adjust the gain of both amplifiers so that the 0.511-MeV line of $^{22}\text{Na}$ results in approximately 3-V pulses at the outputs. Use the oscilloscope for this measurement. Ask the Lab Instructor for assistance. This is hard to observe.

3. Set the Timing Single-Channel Analyzers to CO. Set the Delay controls at minimum. Set the Lower-Level dial at 40 and the Window dial to full CW.

4. Connect the SCA Out from one of the Timing Single-Channel Analyzers to the A input of the Coincident module and connect the output from the other SCA to the B input of the Coincident module. Set the Resolving Time to 1 $\mu$S. Set the A and B switches up and C and D toggle switches down. Disconnect the NaI detector and connect the pulser into the circuit. The pulser should indicate coincidence for the two signal paths. Ask your Lab Instructor to illustrate this for you. You may use either the oscilloscope or counter/timer. The oscilloscope is preferable. Disconnect the pulser and connect the NaI detector back into the circuit.

5. Set the Counter/Timer for a long timing period, such as 8 minutes, and permit the Counter to operate while the movable detector is rotated in steps of 0, 1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 20, and 24 degrees to both sides of 0 degrees. The counting rate should be maximum at 0 degrees.

6. Set the Counter/Timer for a long enough accumulation period to provide reasonable statistics at the points of interest and fill in the values in Table 13.1.

**Exercise a.** Plot the data in Table 13.1 on linear graph paper. For each counting rate (N) the statistical variation $\pm \sqrt{N}$ should be included as an error bar on the graph. Figure 13.5 shows a typical set of experimental data for this experiment. Using the detector geometry and activity rate of $^{22}\text{Na}$, estimate the expected coincidences and compare with data. Does it make sense? Follow the inverse procedure in Exp 3.6.

Exercise b. What causes the finite width at FWHM of the peak in you data? Explain. Estimate the width of the peak from your geometry.

Exercise c. What causes the few counts or background on the far sides of the peak at very large angles? Why don't these counts go to 0 even when you can no longer connect a straight line between them? Explain.
<table>
<thead>
<tr>
<th>$\Theta$ (Deg) Positive</th>
<th>Counts (N)</th>
<th>Statistical Variation</th>
<th>$\Theta$ (Deg) Negative</th>
<th>Counts (N)</th>
<th>Statistical Variation</th>
</tr>
</thead>
<tbody>
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