Physics 751: Final Review

First, it is important to review all the homework questions. I’ll ask at least one homework question on the final. The final will have six questions, of which you should attempt five.

Most of the exam will be on material covered before the second midterm: we’ve only had six lectures since that, and I will not examine on the last two lectures, since that material has not been assigned for homework. So, nothing on the hydrogen atom.

That leaves two major topics: orbital angular momentum and spin. For orbital angular momentum, I do not expect you to memorize the mathematical derivations, or the exact forms of the eigenstates. However, you should be able to work with \( l = 1 \) and \( l = 2 \) eigenstates, both in terms of \( \theta, \phi \) and \( x, y, z \). You should also read through and understand the general derivations, and know the general pattern of the eigenstates as functions of angle. For spin, read my notes carefully, and be sure you understand everything covered on the last homework – the answers are available in the library. I feel free to ask questions of this type in the final.

This exam is on the whole semester: here are the review sheets posted previously, they’re still valid:

Midterm I Review
I will ask no questions on the historical stuff at the beginning, except for the purely physics content. For example, you should have a good understanding of the Bohr atom, including the classical limit of a very large atom and how that fixes the quantum of angular momentum, and know how the Uncertainty Principle can be used to estimate the size of bound states, such as the atom on the table.

You should know Schrödinger’s equation by heart, the probability interpretation of the wavefunction, the conservation of probability current, and the expectation value of an operator. Be able to find the time-dependent probability and current distribution for a superposition of states of different energies.

Understand the connection between particle speed and wave and group velocity.

Know the definition of the delta function as a limit of a Gaussian wavepacket, and be able to Fourier transform a Gaussian wavepacket. (Know the integral you need by heart.) Also, know how to do Fourier series and transforms for simple functions, and be able to discuss the limitations of Fourier series for discontinuous functions.

You must know the bra and ket notation, be able to construct the first few elements of an orthogonal basis, the definitions of adjoint, Hermitian and unitary, and be able to diagonalize a Hermitian or a unitary matrix. Know the definition of determinant and trace, and how these relate to eigenvalues.
You should also feel comfortable going back and forth between the Schrödinger wavefunction notation and the Dirac bra ket notation.

Know the normalization conventions for the eigenkets of the momentum operator, and those of the position operator. Know the identity in terms of these kets, and be able to use it to express a Schrödinger wave function as an integral over $|x\rangle$ or $|k\rangle$ kets. Be able to express $|x\rangle$ in terms of $|k\rangle$’s.

*I expect you to be able to solve simple boundary-matching problems for square barriers, steps, square wells, and delta function potentials.*

Be able to derive the Generalized Uncertainty Principle: know the definition of expectation value and of the root mean square deviation in a set of measurements on identically prepared systems.

**Midterm II Review**

**Simple Harmonic Oscillator:** know

$$x = \sqrt{\hbar/2m\omega}(a^\dagger + a), \quad p = \sqrt{m\omega\hbar/2}(a^\dagger - a), \quad \langle n+1 | a^\dagger | n \rangle = \sqrt{n+1}, \quad \langle n-1 | a | n \rangle = \sqrt{n}.$$

Know the definition of the **propagator**, and be able to derive the free particle propagator.

Be familiar with the **Heisenberg representation**, know how to find the equation of motion for an operator in that representation, and know the connection with Ehrenfest’s theorem.

Memorize the result $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$, know for what operators it is valid, and be able to use it for normalizing coherent states, etc.

Know what the **P-basis** is, and when it’s useful.

**Angular Momentum**

*From class notes:* be able to show, if the operator $\hat{J}$ is defined by $|\psi\rangle \rightarrow e^{\frac{-\hat{J}^2}{\hbar}} |\psi\rangle$ under a rotation defined by $\hat{\theta}$, then from our knowledge of *classical* rotations, $[\hat{J}_i, \hat{J}_j] = i\hbar \varepsilon_{ijk} J_k$, and know this formula by heart! Be able to derive from this the commutation relations among $J^2, J_x, J_z$. Be able to derive the matrix elements of these operators for the common eigenkets of $\hat{J}^2, J_z$, and know how to prove that $2j$ is an integer. In fact, you should **memorize**:

$$J_z |j, m\rangle = \hbar \sqrt{j(j+1) - m(m\pm 1)} |j, m\pm 1\rangle.$$

**Orbital angular momentum:** know the formulas for the components $L_i$ in Cartesian coordinates.