Reading Quiz

To correct nearsightedness, one uses

1) a converging lens

2) a diverging lens
The Eye

The Eye: A complex yet simple object with basically (in order of appearance): The cornea, Aqueous humor, Lens+ciliary muscles, vitrous humor, retina.

Basically, light travels say, in air, with an index of refraction of 1 gets refracted at the cornea with
an index of refraction of 1.38. Some further (small) refraction is done by the Aqueous humor ($n = 1.33$), the lens ($n = 1.40$) and the vitrous humor ($n = 1.34$).

The ciliary muscles can change the radius of curvature of the lens so that the image can be properly focused on the retina.

**Object at “infinity”:** This is when the ciliary muscles are relaxed ⇒
the focal point is situated farthest from the lens.

**Near object**: This is when the ciliary muscles are tense ⇒ the focal point is situated closer to the lens.

**Near Point**: Nearest point to the eye for which a near object is still focused, and beyond (i.e. as one gets closer and closer) which objects appear fuzzy.
Far point: The farthest point from the eye for which an object is still focused. In general, it is practically infinity.

In the example below we will treat the eye as if it were a single thin lens.

Example 27-1:

The near-point distance of a given eye is $N = 25 \text{ cm}$. Treating the
eye as if it were a single thin lens a distance 2.5 cm from the retina, find the focal length of the lens when it is focused on an object (a) at the near point; (b) at infinity.

Solution:

(a) See the drawing in the book.

\[ p = 25 \text{ cm}, \quad q = 2.5 \text{ cm}. \]
\[
\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{25\, cm} + \frac{1}{2.5\, cm} = 0.44\, cm^{-1} \Rightarrow f = 2.3\, cm.
\]

(b) \( p = \infty, \ q = 2.5\, cm. \)

\[
\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} + \frac{1}{2.5\, cm} = \frac{1}{2.5\, cm} \\
\Rightarrow f = 2.5\, cm.
\]

The focal length for the “relaxed” state (b) is larger than that for a “tense” state.
The camera

Same principle with one exception: The lens is fixed and the focus is changed by moving the lens.

Example:

A simple camera uses a thin lens with a focal length of 50.0 mm.
What are the image distances for an object situated at 40.0 m and 3.00 m away respectively?

Solution:

From the thin lens equation, one gets

(a) For \( p = 40.0 \, m \), one gets

\[ q = 5.006 \, cm. \]
(b) For $p = 3.0 \, m$, one gets

$q = 5.08 \, cm$.

Difference in image distance: $0.074 \, cm$.

Suppose the distance between the lens and the film is fixed and equal to one focal length, an object at $3 \, m$ away will have an image behind the film. The object at $p = 40.0 \, m$ will be, to a good approximation, an image focused on the film.
Conceptual Question 1

Suppose the lens is at a position such that the focus is made for the distance $40.0 \, m$. In order to focus on an object at $3.0 \, m$ away, one should move the lens a distance $0.074 \, cm$

1) away from the film.

2) toward the film.
Conceptual Question 1

Suppose the lens is at a position such that the focus is made for the distance 40.0 m. In order to focus on an object at 3.0 m away, one should move the lens a distance 0.074 cm

1) away from the film.**

2) toward the film.

Draw a picture to convince yourself.
f-number:  

\[ f - number = \frac{f}{D} \]  (1)

\( D \): Diameter of the lens.

For a fixed \( f \), the intensity of light going into the camera is found to be \( I \propto D^2 \). Large diameter \( \Rightarrow \) large intensity.

f-number: typically 1.4, 2, 2.8, 4, 5.6, ...  
From one f-number to the next
higher one, the area of the aperture is decreased by half.
Lenses in combination

A series of two or more lenses can be very useful for a number of circumstances: eyesight corrections, microscopes, telescopes, etc....

One simple example: Two converging lenses
Two converging lenses, separated by a distance of 40.0 cm are used in combination. The focal lengths are \( f_1 = +10.0 \text{ cm} \) and \( f_2 = +12.0 \text{ cm} \). An object, 4.00 cm high, is placed 15.0 cm in front of the first lens. (see the picture in class.) Find the intermediate and final image distances, the total transverse magnification, and the height of the final image.

Solution:
1) Use the thin lens equation.

2) Let \( s = 40.0 \, cm \) be the distance between the two lenses then:

\[
\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}
\]

\[
\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}
\]

\( p_2 = s - q_1 \).

3) \( p_1 = 15.0 \, cm \).
\[ q_1 = +30.0\ cm \]

\[ p_2 = 40.0\ cm - 30.0\ cm = 10.0\ cm \]

\[ q_2 = -60.0\ cm \]

4) Image 1 is to the right of lens 1 and between the two lenses. Image 2 is to the left of lens 2 and since the two lenses are separated by 40.0 cm, it is 20.0 cm to the left of lens 1.
5) Magnification for a single lens:

\[ M = -\frac{q}{p}. \]

The total magnification for two lenses is:

\[ M = M_1 \times M_2 = \left(-\frac{q_1}{p_1}\right) \times \left(-\frac{q_2}{p_2}\right) = \left(-\frac{30.0 \text{ cm}}{15.0 \text{ cm}}\right) \times \left(-\frac{60.0 \text{ cm}}{10.0 \text{ cm}}\right) = -12.0. \]

The final image is inverted and twelve times larger!
Correcting eyesights

- **Myopia** (Nearsightedness): Image of distant object is formed in front of retina ⇒ blurred vision. The focal length for this case is too short.

How do we correct it? Use a diverging lens.
Light rays \textit{diverge} when incident on the diverging lens in front of the eye. The diverging rays get \textit{refocused} on the retina if the proper focal lengths are chosen. See figure.

Actually, as can be seen from the figure, the diverging lens forms a \textit{virtual image} at the \textit{far point} of a nearsighted eye.

Example:
A nearsighted person has a far point located only 521 cm from the eye. Assuming that eye-glasses are to be worn 2 cm in front of the eye, find the focal length needed for the diverging lenses of the glasses so the person can see distant objects.

Solution:

1) Virtual image at the far point to the left of the lens:
\( q = -519 \text{ cm} \) because the glasses are 2 cm in front of the eye.

Here, \( p = \infty \).

2) \( \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} + \frac{1}{-519 \text{ cm}} \)
\[ \Rightarrow f = -519 \text{ cm} < 0. \] It is negative as one would expect for a diverging lens.

- **Hyperopia** (farsightedness): Image of nearby object is formed behind the retina.
By *nearby*, one means an object which is located between the eye and the *near point*.

Put a *converging lens* in front of the eye to bring the image back to the retina. This converging lens acts in such a way as to create a *virtual image* at the *near point*. See picture.

Example:
A farsighted person has a near point located at 210 cm from the eye. Obtain the focal length of a converging contact lens that can be used to read a book held at 25.0 cm from the eye.

Solution:

1) Contact lens \( p = 25.0 \text{ cm} \)

2) Virtual image at near point \( q = -210 \text{ cm} \)
\[
\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{25.0\, cm} + \frac{1}{-210\, cm}
\]

\[\Rightarrow f = 28.4\, cm.\]

Refractive power of the lens:

\[
Refractive\, power = \frac{1}{f\,(meters)}.
\]

Unit: Diopter

For the nearsightedness example, refractive power = \(1/\text{\(-5.19\, m = \(-0.193\, diopter.\)}}\)
For the farsightedness example, refractive power $= \frac{1}{0.284\, m} = 3.52\, \text{diopter}$.
Magnifying glass

The eye sees blurry objects if they are located at distances less than the near point $N$. At $N$, the angle subtended is $\theta \approx h_0/N$.

Put a convex lens of focal length $f < N$ in front of the eye. As the object approaches the focal
point, the image which is upright and virtual approaches infinity. This can be focused on. At $F$, the angle subtended is $\theta' \approx h_0/f > \theta$.

Angular magnification:

$$M = \frac{\theta'}{\theta} = \frac{N}{f}$$
Compound microscope

Two lenses: **Objective** and **eye-piece**.

Magnification:

\[ M = - \frac{qN}{f_{\text{objective}} f_{\text{eyepiece}}} \]
Telescope

Telescope: Objective and eyepiece

Magnification:

\[ M = \frac{\theta'}{\theta} = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} \]

Example:
A telescope has the following specifications: \( f_o = 985 \, mm \) and \( f_e = 5.00 \, mm \). Find (a) the angular magnification, and (b) the approximate length of the telescope.

Solution:

1) \( M = \frac{f_o}{f_e} = \frac{985 \, mm}{5.00 \, mm} = 197 \)

2) \( L \approx f_o + f_e = 990 \, mm \)