Physics 201

Professor P. Q. Hung

311B, Physics Building
Summary of last lecture

- Electrostatic force between 2 point charges:
  \[ \vec{F}_{21} = k \frac{q_1 q_2}{r^2} \hat{r} \]
Electric Charges, Forces and Fields

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- Electric field of a point charge:
  \[ \vec{E}(r) = k \frac{q}{r^2} \hat{r} \]
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- **Electrostatic force between 2 point charges:**
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- **Electric field of a point charge:**
  \[ \vec{E}(r) = k \frac{q}{r^2} \hat{r} \]

- **Force on a point charge** \( q_0 \) **in the presence of an electric field:**
  \[ \vec{F} = q_0 \vec{E}(r) \]
Question

Determine the electric field at a distance $r$ from a line of charge of total charge $Q$.

A line of charge is **not** a point charge. How do we calculate the electric field then?
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\[ \Rightarrow \text{Very complicated!!} \]
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- More powerful concept using symmetry to solve this. Later...
Electric Field Lines

The electric field is a vector field:

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- It points radially outward (from the point charge) if $q > 0$ and inward if $q < 0$. 
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- It points radially outward (from the point charge) if \( q > 0 \) and inward if \( q < 0 \).
- It becomes stronger as one gets closer to the point charge.
- It depends directly on the magnitude of the point charge.
Electric field of point charge: \[ \vec{E}(r) = k \frac{q}{r^2} \hat{r} \]
Electric Field Lines

Electric field lines point away from positive charges and point into negative charges.
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- The **density of lines** (number of lines per unit area) is **higher** at places where the electric field is **stronger**.
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- The higher the charge is, the higher the number of lines
Electric Field Lines

- Electric field lines point **away** from **positive** charges and point **into** **negative** charges,
- The **density of lines** (number of lines per unit area) is **higher** at places where the electric field is **stronger**.
- The higher the charge is, the higher the number of lines.
- No two field lines can cross each other.
Electric Field Lines for a system of charges
Electric Field Lines for a charged plate

Why is the electric field uniform?
Electric Field Lines for a parallel-plate capacitor

Why is the electric field uniform between the plates and zero outside?
Electric Charges, Forces and Fields

Electric Field inside a Conductor: Concepts

A conductor contains a number of conducting electrons which are free to move about.
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- A conductor in electrostatic equilibrium has zero electric field inside it otherwise the conducting electrons will be free to move which brings it out of electrostatic equilibrium.
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- A conductor in electrostatic equilibrium has zero electric field inside it otherwise the conducting electrons will be free to move which brings it out of electrostatic equilibrium.

- In electrostatic equilibrium, any excess charge on a conductor will have to reside on its surface because the electric field is zero inside. We will prove this after discussing Gauss’s law.
Conductor in electrostatic equilibrium with an excess charge $\Rightarrow$ the electric field is always perpendicular to the surface. If not, there will be a component of the electric field which is parallel to the surface $\Rightarrow$ loss of electrostatic equilibrium.
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- Conductor in electrostatic equilibrium with an excess charge $\Rightarrow$ the electric field is always perpendicular to the surface. If not, there will be a component of the electric field which is parallel to the surface $\Rightarrow$ loss of electrostatic equilibrium.

- An external electric field incident upon a conductor cannot penetrate it $\Rightarrow \vec{E} = 0$ inside. Example: Inside a hollow conductor $\Rightarrow$ Shielding. A car is one of such example during a thunderstorm.
Electric Field inside a Conductor: Concepts

(a) The electric field $E$ vanishes inside a conductor

(b) $E$ field lines meet a conducting surface at right angles
Electric Field inside a Conductor: Concepts
Electric Flux for uniform electric field:

\[ \Phi = EA \cos \theta \]
Gauss’s Law

The total electric flux through a surface enclosing a charge $Q$ is

$$\Phi = \frac{Q}{\epsilon_0}$$

Gauss’s Law for any closed arbitrary surface surrounding the point charge $\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} C^2/N.m^2$ is called the permittivity of free space.

Under special circumstances, can one use Gauss’s Law to calculate the electric field?
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How?
Gauss’s Law: How to get the electric field of a point charge

Take a point charge. Draw a (imaginary) sphere of radius \( r \) (Gaussian surface) with the charge at the center.
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- Take a point charge. Draw a (imaginary) sphere of radius $r$ (Gaussian surface) with the charge at the center.
- Let the value of the electric field at a point on the surface of the sphere be $E(r)$. 
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Take a point charge. Draw a (imaginary) sphere of radius \( r \) (Gaussian surface) with the charge at the center.

Let the value of the electric field at a point on the surface of the sphere be \( E(r) \).

By spherical symmetry, the value of the electric field is the same at any other point on the surface of that sphere.
Gauss’s Law: How to get the electric field of a point charge
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Electric flux through the surface $A = 4\pi r^2$ of the sphere of radius $r$:

$$\Phi = EA = E(r)(4\pi r^2) = \frac{Q}{\varepsilon_0}$$ (Last equality is Gauss’s Law)
Gauss’s Law: How to get the electric field of a point charge

1. Electric flux through the surface $A = 4\pi r^2$ of the sphere of radius $r$:
   \[
   \Phi = EA = E(r)(4\pi r^2) = \frac{Q}{\varepsilon_0} \quad \text{(Last equality is Gauss’s Law)}
   \]

2. \[
   E(r) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} = k \frac{Q}{r^2} \quad \text{Same as Coulomb’s Law.}
   \]
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$

Find $E$ at a distance $r$ from the line of charge.

Problem 19.70.
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$

- Find $E$ at a distance $r$ from the line of charge. **Problem 19.70.**
- Calculate the electric flux through a Gaussian surface of radius $r$. 
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$

- Find $E$ at a distance $r$ from the line of charge. Problem 19.70.
- Calculate the electric flux through a Gaussian surface of radius $r$.
- What kind of Gaussian surface?
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$

- Find $E$ at a distance $r$ from the line of charge. Problem 19.70.
- Calculate the electric flux through a Gaussian surface of radius $r$.
- What kind of Gaussian surface?
- What symmetry gives a constant electric field on the Gaussian surface?
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$
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The Gaussian surface would be a cylinder of radius $r$ and length $l$. The area of the cylinder (minus the 2 ends) is $A = 2\pi rl$
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$

- The Gaussian surface would be a cylinder of radius $r$ and length $l$. The area of the cylinder (minus the 2 ends) is $A = 2\pi rl$.

- Why? Because the problem has cylindrical symmetry. The value of the electric field would be the same at any point on the surface of that imaginary cylinder.
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- Why? Because the problem has cylindrical symmetry. The value of the electric field would be the same at any point on the surface of that imaginary cylinder.

- Gauss’s Law: $\Phi = EA = E(2\pi rl) = \frac{Q}{\epsilon_0}$
Gauss’s Law: How to get the electric field of a line of charge with total charge $Q$

$$E = \frac{Q}{2\pi rl\varepsilon_0} = \frac{\lambda}{2\pi r\varepsilon_0}$$

where

$\lambda = \frac{Q}{l}$ is the linear charge density.
What is the most important thing that we learned in this lecture? Gauss’s Law

The total electric flux through a surface enclosing a charge \( Q \) is

\[
\Phi = \frac{Q}{\varepsilon_0}
\]

Gauss’s Law for any closed arbitrary surface surrounding the point charge