Reading Quiz

Three points A, B, and C are along a line. The value of the potential is decreasing from A to C. A positive test charge is released from B. Where does it go to?

1) A

2) C
Electric Potential Energy

Recall that the gravitational force is

\[ F_g = -G \frac{m_1 m_2}{r^2} \] (1)

and it is a conservative force \( \Rightarrow \)

A potential energy is associated with it.
From the 1st lecture, the electrostatic force between 2 charges is

$$F_e = k\frac{q_1q_2}{r^2}$$  \hspace{1cm} (2)

It is also a **conservative force** ⇒
There should be a **potential energy** associated with it.

What is it?

Let us recall a few facts.
• The work done by gravity when an object moves from point A to point B is (positive direction pointing upward):

\[
W_{AB} = U^A_g - U^B_g = mgh_A - mgh_B
\]  

(3)

• Let us take as an example, the motion of a particle of charge \( +q \) in a uniform electric field
\( \vec{E} \) assumed to be pointing vertically downward.

- Force on \( +q \): \( \vec{F} = q \vec{E} \).

- Work done by this force in moving the particle from A to B with \( h_A > h_B \):

\[
W_{AB} = q \, E \, h_A - q \, E \, h_B = U_e^A - U_e^B
\]

(4)
• (3) and (4) are very similar. The electrostatic potential energy is defined in (4) by the symbol $U_e$.

Electric potential:

$$V = \frac{U_e}{q}$$ (5)

Unit: $V = \text{joule/coulomb}$. 

Notice that $V$ is not a vector. It is a scalar.
From (4) and (5), the potential difference between B and A is:

\[
V_B - V_A = \frac{U_e^B}{q} - \frac{U_e^A}{q} = -\frac{W_{AB}}{q} = -E(h_A - h_B) < 0 \quad (6)
\]

(6) tells us that \( V_B < V_A \).

(6) is only valid for a uniform electric field. It can also be written as

\[
V_B - V_A = -\vec{E} \cdot \hat{s} \quad (7)
\]
where \( \vec{s} \) is the displacement vector pointing from A to B.

\[ \Rightarrow \text{The positive charge which moves in the direction of the electric field, goes from high to low potential.} \]

This also implies that the electric field points in the direction of decreasing potential.
IMPORTANT POINT: As with the gravitational case, only the potential energy and potential difference makes physical sense.
Examples

• An electric force does work $W_{AB} = +5.0 \times 10^{-5} J$ in moving a charge $q_0 = +2.0 \mu C$ from A to B. 1) Find the potential energy difference; 2) Find the potential difference between B and A.

1) $U_e^B - U_e^A = -W_{AB} = -5.0 \times 10^{-5} J < 0$. The potential energy is higher at A than at B.
2) \( V^B - V^A = -\frac{W_{AB}}{q} = -\frac{5.0 \times 10^{-5}}{+2.0 \mu C} \)
\(-25 \text{ V}.\)

So \( V_B < V_A.\)

- A 12-V battery powers a 60.0 W headlight for one hour. How many electrons have passed through the terminals of the battery during that time?

1) Energy consumed by the headlight in one hour:
Energy = Power x time

\[ E = 60.0\,W \times 3600\,s = 2.2 \times 10^5\,J. \]

2) That energy should be equal to the change in potential energy of the total number of electrons which pass through the terminals:

\[ E = Q\Delta V = Q12V = 2.2 \times 10^5\,J \]

\[ \Rightarrow Q = 1.8 \times 10^4\,C. \]
3) Each electron carries a charge of magnitude $1.6 \times 10^{-19} \, C$. The total number of electrons is

$$N = \frac{1.8 \times 10^4 \, C}{1.6 \times 10^{-19} \, C} = 1.1 \times 10^{23}.$$
Conceptual Question 1

A negative test charge will accelerate from

1) a region of higher potential to a region of lower potential;

2) a region of lower potential to a region of higher potential.
Conceptual Question 1

A negative test charge will accelerate from

1) a region of higher potential to a region of lower potential;

2) a region of lower potential to a region of higher potential.**

(2) is because a negative charge will accelerate in the direction opposite to that of the electric field and the electric field points in the direction of decreasing potential.
Conservation of Energy

Point B has an electric potential that is 25 V greater than that of point A. A particle of mass $1.8 \times 10^{-5}$ kg and a charge whose magnitude is $3.0 \times 10^{-5} C$. We will neglect gravity and friction. (a) If the particle has a positive charge and is released from rest at B, what speed $v_A$ does the
particle have when it arrives at A? (b) If the particle has a negative charge and is released from rest at A, what speed $v_B$ does the particle have when it arrives at B? (c) What if the negatively charged particle is released from rest from B?

Key point: Energy is conserved. Therefore

$$KE_A + U_e^A = KE_B + U_e^B.$$
(a)  \( \frac{1}{2} m v_A^2 + qV_A = \frac{1}{2} m v_B^2 + qV_B. \)

\[ \Rightarrow \frac{1}{2} m v_A^2 = q(V_B - V_A) \]

\[ \Rightarrow v_A = \sqrt{\frac{2q(V_B-V_A)}{m}} = \text{9.1 m/s}. \]

(b) A negatively charged particle accelerating from rest from A is the same as a positively charged particle accelerating from rest from B \( \Rightarrow v_B = \text{9.1 m/s}. \)

(c) It will never reach B.
Another unit of energy: $eV$ or electronvolt. One $eV$ is the change in potential energy of an electron moving a potential difference of one volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}.$$
Potential due to a point charge

Recall the electric field due to a point charge is

\[ \vec{E}(r) = k \frac{q}{r^2} \hat{r} \]

It is not a constant so Eq. (7) cannot be used. Instead one should use calculus. So the potential difference between point B and
point A due to a point charge is now written as

\[ V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s} \]  \hspace{1cm} (8)

Using \( \vec{E} \) above one obtains

\[ V_B - V_A = k \frac{q}{r_B} - k \frac{q}{r_A} \]  \hspace{1cm} (9)

We will impose that \( V_A = 0 \) when \( r_A \) is infinite: choice of boundary condition.
$V = k \frac{q}{r}$ (10)
Example

A charge \( q_1 = 2.00 \mu C \) is located at the origin and a charge \( q_2 = -6.00 \mu C \) is located at \((0, 3.00 \, m)\). Find the total electric potential at point \( P \) situated at \((4.00 \, m, 0)\). See the figure drawn in class.

Solution:
1) The individual potentials add like **scalars**.

2) The distance between $q_1$ and $P$ is $4.00\, m$. The distance between $q_2$ and $P$ is $5.00\, m$.

3) $V_P = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$

$\Rightarrow V_P = 8.99 \times 10^9 \frac{N.m^2}{C} \left( \frac{2.00 \times 10^{-6} C}{4.00m} + \frac{-6.00 \times 10^{-6} C}{5.00m} \right) = -6.29 \times 10^3 \, V$. 