Reading Quiz

two capacitors are connected in series. If a third capacitor is connected in series with the other two, does the equivalent capacitance

1) increase?

2) decrease?

3) remain the same?
Kirchhoff’s Rules

- When a current comes to a junction that has many branches, what happens?

Answer: We know that electric charge is conserved. The number of electrons that flow into a junction is shared among
the branches. None is destroyed. 
⇒ Current conservation

Kirchhoff’s Junction Rule:

Total current going into a junction is equal to the total current going out of the junction.

• Knowing that the current is in the direction of a positive test charge, how do I deal with all these resistors along the way
if I want to look at the potentials?

Answer:

1) Current crossing a resistor: loss in potential energy due to resistance \( \Rightarrow \) Potential drop: \(-IR\).

2) Since energy is conserved, when you start from a given point and come back to that point, the energy is unchanged.
Start the repositive test charge from the negative terminal and take it to the positive terminal. It gains a potential $\mathcal{E}$. (Like the top of the slope.) Every time it crosses a resistor with resistance $R_i$, it loses potential by an amount $-IR_i$. (Like loss of potential energy due to friction.) Since it comes back to the same point, the total change is zero. It means that the total change of the
potential in a closed loop is zero.

Kirchhoff’s Loop Rule:

\[ \sum \Delta V = 0 \]

Around a closed loop: Sum of potential drop = Sum of potential rise

3) Draw + and - at the 2 ends of a resistor with the direction of the current from + to -.
Examples

1) Example of loop rule:

Two batteries, 24 V and 6 V, are connected in series with two resistors, 8 Ω and 12 Ω as shown in class. Find the current in the circuit.

Solution:
a) First step is to draw the current. Clockwise or counterclockwise? It doesn’t matter! The correct sign will tell the true direction.

So let’s draw it clockwise.

b) Label by + and - the ends of the resistors with the direction of the current from + to -.
c) Start from the positive terminal of the 24 V battery. As the signs show, one always goes from $+$ to $-$. 

d) Sum of rise = Sum of drops

$24V = I(12\Omega) + 6V + I(8\Omega)$.

$\Rightarrow I = 0.9A$. Positive: correct initial choice of direction.
2) Example of both Junction and Loop rules:

Two batteries, 12 V and 14 V, are connected to three resistors, 1.2 Ω, 0.1 Ω, 0.01 Ω, as shown in class. The circuit is now composed of two loops. Determine the three currents shown in the picture.

Solution:
a) The + and - are labeled in the figure with the current directions arbitrarily chosen.

b) Junction rule at point B:

\[ I_1 + I_2 = I_3. \]

c) Loop BCDE: (potential rise on the left of the equation and potential drop on the right)

\[ 12V = I_3(1.2\Omega) + I_2(0.01\Omega) \]
d) Loop ABEF: (clockwise from point B)

\[14V + I_2(0.01\,\Omega) = 12V + I_1(0.1\,\Omega).\]

3 equations with 3 unknowns \(\Rightarrow\)

\[I_1 = 19.1\,A;\ I_2 = -9.0\,A;\ I_3 = 10.1\,A.\]

The - sign for \(I_2\) says that it goes in the opposite direction to the one in the drawing.
Capacitors in parallel and in series

- What do I get when I connect two capacitors in parallel?

Answer:

1) In parallel means that the same potential difference runs across the two capacitors.
2) If the capacitances of the 2 capacitors are $C_1$ and $C_2$, the charges stored on each will be

$$q_1 = C_1V; \quad q_2 = C_2V$$

3) One can draw an equivalent capacitor with the same voltage but now with

$$q = q_1 + q_2 = C_1V + C_2V = C_{eq}V$$

$\Rightarrow$
\[ C_{eq} = C_1 + C_2 \] (1)

Eq. (1) can be extended to any number of capacitors in parallel.

- What about when they are connected in series?

Answer:

1) When they are connected in series and to a battery, \( C_1 \)
will have $+q$ on the 1st plate and $-q$ on the 2nd plate. Same thing for $C_2$ because $+q$ has been pushed out of the 2nd plate of the 1st capacitor, etc.. So the voltages across are different with $V = V_1 + V_2$.

2) Replace it with an equivalent capacitor with

$$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = \frac{q}{C_{eq}}$$

$\Rightarrow$
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \] (2)

Eq. (2) can be extended to any number of capacitors in series.

- Notice that Eqs (1), (2) are just the reverse of what one obtained for resistors.
Conceptual Question 1

Two capacitors, $C$ and $2C$, are connected to a battery. Which capacitor stores more energy when they are connected to the battery in parallel?

1) $C$

2) $2C$
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Two capacitors, $C$ and $2C$, are connected to a battery. Which capacitor stores more energy when they are connected to the battery in parallel?

1) $C$

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In parallel means that they have the same voltage across. Since $U = \frac{1}{2}CV^2$, (2) is correct.
**RC circuit**

I) Charging up a capacitor:

- Connect a battery to a capacitor and a resistor. What do I get when I close a switch to make a closed circuit?

Answer:
1) Kirchoff’ Loop law tells me that

$$\mathcal{E} - \frac{q}{C} - IR = 0$$

2) Just after closing the switch at $t = 0$, the charge in the capacitor is zero $\Rightarrow$

$$I_0 = \frac{\mathcal{E}}{R}$$ maximum current

3) After the capacitor is fully charged, there is no current flowing anymore $\Rightarrow$
\[ Q = C \varepsilon \text{ maximum charge} \]

4) Since \( I = \frac{\Delta q}{\Delta t} \), one can solve for \( q \) \Rightarrow

\[ q(t) = Q(1 - e^{-t/RC}) \quad (3) \]

5) From (3) one also finds

\[ I(t) = \frac{\varepsilon}{R} e^{-t/RC} \quad (4) \]

6) \( \tau = RC \): time constant of the circuit
II) Discharging a capacitor:

- With a fully charged capacitor connected to a resistor, and no battery, what happens when one closes the switch?

Answer:

1) Now $\mathcal{E} = 0$.

2) Kirchoff’’ Loop law tells me that
\[-\frac{q}{C} - IR = 0\]

3) Maximum charge \( Q \) at time \( t = 0 \)

\[ q(t) = Qe^{-t/RC} \quad (5) \]

4) Maximum current at \( t = 0 \)

\[ I(t) = -\frac{Q}{RC}e^{-t/RC} \quad (6) \]

Example:
An uncharged capacitor and a resistor are connected in series to a battery. If \( \mathcal{E} = 12V \), \( C = 5.00\mu F \), and \( R = 8.00 \times 10^5\Omega \), find the charge and current as a function of time.

Solution:

1) \( Q = C\mathcal{E} = (5.00\mu F)(12.0V) = 60.0\mu C \).

2) \( I_0 = \frac{\mathcal{E}}{R} = 15.0\mu A \).
3) \( \tau = RC = 4.00\text{s} \).

4) \( q(t) = (60.0\mu C)(1 - e^{-t/4.00\text{s}}) \)

5) \( I(t) = (15.0\mu A)e^{-t/4.00\text{s}} \).