The Standard Model and Beyond

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The status of particle physics

1. The **Standard Model (SM)** is the **best theory** of describing the nature of particle physics, which is in excellent agreement with **almost** of all current experiments.

2. However, there are some **theoretical problems** & **recent experimental results**, which strongly suggest **New Physics beyond the SM**.

3. Many well-motivated **New Physics models** have been proposed.

4. Many planned and on-going (collider) experiments may reveal **New Physics in the near future**.
Lecture 1

Status of the Standard Model and its problems
1. Definition of the Standard Model

What is the **Standard Model (SM)**?

General theoretical framework: **Gauge Field Theory**

- Quantum field theory → relativity + quantum mechanics
- Gauge principle → invariance under Gauge Transformation

General definition of a model of gauge field theory

1. Define **Gauge Group** (U(1), SU(N) etc.)
2. Fix **Particle Contents**
The Standard Model

Fermions & Gauge Bosons
The Standard Model

Gauge group: \( SU(3)_c \times SU(2)_L \times U(1)_Y \)

QCD int.              Electroweak int.

Gauge fields:           gluon    \( W, Z, \gamma \)

Particle contents: leptons & quarks & Higgs doublet

\[
\begin{align*}
(\ell_L)_i &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L : (1, 2, -\frac{1}{2}) \\
(q_L)_i &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L : (3, 2, \frac{1}{6}) \\
(\ell_R)_i &= e_R, \mu_R, \tau_R : (1, 1, -1) \\
(u_R)_i &= u_R, c_R, t_R : (3, 1, \frac{2}{3}) \\
(d_R)_i &= d_R, s_R, b_R : (3, 1, -\frac{2}{3})
\end{align*}
\]
Electroweak Symmetry Breaking & Higgs mechanism

Higgs doublet scalar \( H : (2, 1/2) \) under \( SU(2)_L \times U(1)_Y \)

Higgs potential: \( V = -m^2 H^\dagger H + \frac{1}{2} \lambda (H^\dagger H)^2 \)

\[ \langle \phi \rangle = \frac{v}{\sqrt{2}} = 174 \text{GeV} \]

Electroweak symmetry breaking: \( SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{EM}} \)

Higgs mechanism: Massive weak gauge bosons & Higgs boson

Fermion masses are also generated
W & Z bosons get masses through gauge coupling

\[ M_W^2 = \frac{1}{4} g^2 v^2 \]

\[ M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2 \]

Fermions get masses through Yukawa coupling

Ex) top quark mass

\[ m_t = \frac{Y_t}{\sqrt{2}} v \]

Higgs boson mass

\[ \phi = \frac{1}{\sqrt{2}} (v + h) \]

\[ m_h^2 = \lambda v^2 \]
Interactions between fermions and gauge bosons

Charged current interaction

\[
\mathcal{L}_{CC} = g (J_\mu^1 A_\mu^1 + J_\mu^2 A_\mu^2) = \frac{g}{\sqrt{2}} (J_\mu^- W^- \mu + J_\mu^+ W^+ \mu)
\]

\[
J_\mu^+ = J_\mu^1 + i J_\mu^2 = \bar{L}' \gamma_\mu \tau^+ L' = \bar{\nu}_e L \gamma_\mu e_L = \frac{1}{2} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e,
\]

\[
J_\mu^- = J_\mu^1 - i J_\mu^2 = \bar{L}' \gamma_\mu \tau^- L' = \bar{e}_L \gamma_\mu \nu e_L = \frac{1}{2} \bar{e}_\gamma_\mu (1 - \gamma_5) \nu_e.
\]
Neutral current interaction

\[ \mathcal{L}_{int} = \frac{e}{s_w c_w} J^\mu_Z Z_\mu + e J_{em} A_\mu \]

\[ J^\mu_Z = \bar{f} \gamma^\mu \left( T^3 P_L - Q s_w^2 \right) f \]

\[ J^\mu_{em} = Q \bar{f} \gamma^\mu f \]

\[ T^3 = \begin{cases} + \frac{1}{2} & : \text{up-type fermion} \\ - \frac{1}{2} & : \text{down-type fermion} \end{cases} \]
2. Experimental status on the Standard Model

2-1. Search for weak gauge bosons

Implicit: Muon weak decay

\[ \Gamma(\mu \rightarrow e + \bar{\nu}_e + \nu_\mu) = \frac{m_\mu^5 G_F}{192 \pi^3} \]

\[ G_F = \frac{1}{\sqrt{2}v^2} = 1.166 \times 10^{-5} \text{GeV}^{-2} \]

\[ m_\mu = 1.0566 \text{MeV} \]

Investigation of Electroweak theory began
2-2. Direct evidence of weak gauge boson

Weak gauge bosons were discovered at CERN in 1983

\[ p\bar{p} \text{ collider with } \sqrt{s} = 540 \text{GeV} \]

\[ u, d \quad \rightarrow \quad - - - \quad W^\pm \quad \rightarrow \quad - - - \quad Z \]

\[ u, d \quad \rightarrow \quad - - - \quad W^\pm \quad \rightarrow \quad - - - \quad Z \]

Exp.) \[ M_W = 80.33 \pm 0.15 \text{GeV} \]

\[ \Gamma(W^+ \rightarrow e^+\nu) = 224 \pm 15 \text{MeV} \]

\[ \Gamma(W^+ \rightarrow \mu^+\nu) = 215 \pm 19 \text{MeV} \]

\[ \Gamma(W^+ \rightarrow \tau^+\nu) = 226 \pm 127 \text{MeV} \]

\[ \Gamma(W^+ \rightarrow e^+\nu) = \Gamma(W^- \rightarrow e\bar{\nu}) \]

Theory) \[ \Gamma(W^+ \rightarrow e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} = 226 \text{MeV} \]

Numerically consistent, Coupling universality
More precise measurements

**LEP Experiment**

Z-boson production at Z-pole

\[ e^+ e^- \rightarrow \text{hadrons} \]

- Huge number of Z bosons
- \( \sqrt{s} = m_Z \)

\[ e^- \quad Z \quad f \]

\[ e^+ \quad Z \quad f \]

\[ \Rightarrow \text{Very precise measurements} \]
**LEP $e^+e^-$ collider**

**Exp.**) \[ M_Z = 91.187 \pm 0.007 \text{GeV} \]

\[
\begin{align*}
\Gamma(Z \to e^+e^-) &= 83.82 \pm 0.30 \text{MeV} \\
\Gamma(Z \to \mu^+\mu^-) &= 83.83 \pm 0.39 \text{MeV} \\
\Gamma(Z \to \tau^+\tau^-) &= 83.67 \pm 0.44 \text{MeV} \\
\Gamma(\text{total}) &= 2490 \pm 7 \text{MeV}
\end{align*}
\]

**Th.**) \[
\Gamma(Z \to e^+e^-) = \frac{G_FM_Z^3}{12\sqrt{2}\pi}(1 - 2\sin^2\theta_w)^2 + 4\sin^4\theta_w
\]

\[= 83.4 \text{MeV}\]

**for** $\sin^2\theta_w = 0.2315$

**Numerically consistent, Coupling universality**
Measurements of couplings between leptons and weak bosons
Number of neutrinos

Exp.) $\Gamma(\text{invisible}) = 498.3 \pm 4.2\text{MeV}$

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{3\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\mu\mu}}{(E - M_Z)^2 + \Gamma_t^2/4}$$

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{3\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{had}}}{(E - M_Z)^2 + \Gamma_t^2/4}$$

$\Gamma(\text{invisible}) = \Gamma_t - 3\Gamma_{ee} - \Gamma_{\text{had}}$

i) $Z$ mass & total decay width from the position & width of the peak

ii) Measure cross sections at $Z$-pole

Th.) $\Gamma(Z \rightarrow \nu_e\bar{\nu}_e) = \Gamma(Z \rightarrow \nu_\mu\bar{\nu}_\mu) = \Gamma(Z \rightarrow \nu_\tau\bar{\nu}_\tau)$

$$= \frac{G_F m_Z^3}{12\sqrt{2}\pi} = 165.9\text{MeV}$$

$\Gamma(\text{invisible}) = 497.6\text{MeV}$

Number of neutrinos = 3
Left-right production cross section asymmetry & lepton decay asymmetry of the Z-boson

SLC @ Stanford $e^-$ polarized beam

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{2(1 - 4\sin^2 \theta_w)}{1 + (1 - 4\sin^2 \theta_w)^2}$$

Exp.) $A_{LR} = 0.1628 \pm 0.0099$

$\rightarrow \sin^2 \theta_w = 0.2292 \pm 0.0013$

Independent of $\frac{m_W^2}{m_Z^2} = \cos^2 \theta_w$
Hadronic decays of the Z and W bosons

Exp.) \( \Gamma(\text{hadron}) = 1.741 \pm 0.006 \text{GeV} \)

Th.) \( \Gamma(d_k \bar{d}_k) = \frac{G_F m_Z^3}{4 \sqrt{2} \pi} \left( 1 - \frac{4}{3} \sin^2 \theta_w + \frac{8}{9} \sin^4 \theta_w \right) = 0.3677 \text{GeV} \)

\( \Gamma(u_k \bar{u}_k) = \frac{G_F m_Z^3}{4 \sqrt{2} \pi} \left( 1 - \frac{8}{3} \sin^2 \theta_w + 32 \frac{9}{9} \sin^4 \theta_w \right) = 0.2583 \text{GeV} \)

\( \Gamma(\text{hadron}) = 1.6737 \text{GeV} \)

Consider QCD corrections:

\[ f = 1 + \frac{\alpha_s}{\pi} + 1.411 \left( \frac{\alpha_s}{\pi} \right)^2 - 12.8 \left( \frac{\alpha_s}{\pi} \right)^3 \approx 1.038 \]

\( \Gamma(\text{hadron}) \rightarrow f \Gamma(\text{hadron}) = 1.737 \text{GeV} \) \( \leftarrow \text{more consistent} \)
Triple & Quartic Gauge couplings at LEP2

**LEP → LEP2: E = m_Z → 200 GeV**

i) Test of non-Abelian nature of SM gauge sector

ii) Probe for new physics ← anomalous coupling ← not exists in SM

<table>
<thead>
<tr>
<th>3-gauge couplings: SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+W^-\gamma$, $W^+W^-Z$</td>
</tr>
<tr>
<td>anomalous $Z\gamma\gamma$, $ZZ\gamma$, $ZZZ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4-gauge couplings: SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+W^-\gamma\gamma$, $W^+W^-\gamma Z$, $W^+W^-ZZ$, $W^+W^-W^+W^-$</td>
</tr>
<tr>
<td>anomalous $ZZ\gamma\gamma$</td>
</tr>
</tbody>
</table>
$e^+ e^- \rightarrow W^+ W^-$

$188.6 \text{ GeV}$

$\sigma = 15.98 \pm 0.23 \text{ pb}$

$e^+ e^- \rightarrow W e \nu_e$

$188.6 \text{ GeV}$

$\sigma = 0.60 \pm 0.09 \text{ pb}$
Consistent with the SM with 5%-10% accuracy
Neutral triple gauge couplings $\not\equiv$ not exist in the SM

Search

\[ e^+ \rightarrow \gamma^*/Z^* \rightarrow e^- + \gamma \]

SM background

\[ e^+ \rightarrow e^- + \gamma \]

Consistent with the SM

\[ h_1^{Z'} \]

LEP Preliminary

\[ h_1^{Z'} \]
Quartic gauge couplings

\[ e^+ e^- \rightarrow \nu\nu\gamma\gamma, q\bar{q}\gamma\gamma \]

- Anomalous couplings

- Main diagrams

\[ e^+ e^- \rightarrow W^+ W^- \gamma \]

\[ a_0^Z, a_\epsilon^Z \]

\[ a_0^W, a_\epsilon^W \]

\[ a_0^W, a_\epsilon^W, a_\gamma \]

No anomalous coupling
Beyond Tree level

LEP precision measurements → beyond % level

→ level of quantum corrections

Mass ratio:

\[ m_Z^2 = \left( g^2 + g'^2 \right) \frac{v^2}{4} + \Pi_{ZZ}(m_Z^2) \approx 91.1876 ± 0.0021 \text{GeV} \]

\[ m_W^2 = g^2 \frac{v^2}{\alpha} + \Pi_{WW}(m_W) \approx 80.454 ± 0.059 \text{GeV} \]
\[
\rho - 1 = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} - 1 = \frac{e^2}{s_w^2 c_w m_Z^2} [\Pi_{11} - \Pi_{33}]
\]

\[
= \frac{3G_F}{8\sqrt{2}\pi^2} \left[ m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2} \ln \left( \frac{m_t^2}{m_b^2} \right) + m_W^2 \ln \left( \frac{m_H^2}{m_W^2} \right) - m_Z^2 \ln \left( \frac{m_H^2}{m_Z^2} \right) \right]
\]
Strong constraints on New Physics Models

→ **Loop corrections by New Particles should be < 1% contribution**

Ex) 4\textsuperscript{th} generation leptons

\[ \Delta \rho = \alpha T \]

\[ \Delta T_{new} = \frac{1}{16\pi s_w^2 c_w^2 m_Z^2} \left[ m_N^2 + m_L^2 - \frac{2m_N^2 m_L^2}{m_N^2 - m_L^2} \ln \left( \frac{m_N^2}{m_L^2} \right) \right] \]

\[ \approx \frac{1}{12\pi s_w^2 c_w^2} \frac{(\Delta m)^2}{m_Z^2} \]

\[ \Delta m = |m_N - m_L| \ll m_N, m_L \]

**Mass splitting should be small**
Two main structures of the Standard Model

**Gauge invariance**

→ very precisely checked by experiments

**Matters**

→ some representations under SM gauge group

  all matters have been observed

**Higgs mechanism**

→ Higgs has not yet been observed

  Origin of symmetry breaking (Higgs potential)?

  Mass generation mechanism is not yet confirmed

→ should be checked in future experiments
Higgs boson search at LHC

Higgs boson production dominantly by gluon fusion, followed by weak boson fusion
Branching fractions

![Graph showing branching fractions as a function of mass](image)
Discovery potential
Direct Higgs mass bound by LEP & Tevatron
Fitting of Electroweak precision measurements

Higgs mass < 160 GeV seems favorable
Most recent data at LHC  

http://blog.vixra.org/2011/07/22/big-day-for-higgs-boson/

Window for Higgs boson mass

\[ 114 \text{ GeV} < m_H < 137 \text{ GeV} \]
\[ 205 \text{ GeV} < m_H < 295 \text{ GeV} \]  \hspace{1cm} \text{Disfavored by EWPM}
3. Problems in the Standard Model

(A) Theoretical (conceptual) problem

Quantum corrections of Higgs mass $\rightarrow$ quadratic divergence

$$\Delta M_{H}^{2} = Y_{t}^{2} \Lambda^{2}$$

$$M_{\text{Phys}}^{2} = M_{0}^{2} + \Delta M_{H}^{2} \rightarrow \mathcal{O}(M_{W}^{2})$$

If $\Lambda_{\text{New}} \gg M_{W}$ we need to explain the reason

$\rightarrow$ hierarchy problem

(fine-tuning problem: Big # - Big# $\rightarrow$ small #)

No such a problem $\rightarrow$ New Physics around TeV
Experimental observations which the SM cannot explain

Wilkinson Microwave Anisotropy Probe (WMAP) satellite has established the energy budget in the present Universe with a great accuracy

(1) Dark Matter

$$0.096 \leq \Omega_{DM} h^2 \leq 0.122$$

Massive, charge neutral, stable

Suitable candidate: weakly interacting massive particle

(WIMP) $\rightarrow$ No Candidate in the SM $\rightarrow$ Need New Physics

$$\Omega h^2 = \frac{1.07 \times 10^9 x_f \text{GeV}^{-1}}{\sqrt{g_*} M_P |\langle \sigma v \rangle|} \sim 0.1 \rightarrow \langle \sigma v \rangle \sim \alpha^2 \left( \frac{1}{1 \text{ TeV}} \right)^2$$

TeV scale New Physics is relevant to DM physics!
Neutrinos are massless in the Standard Model

(2) Neutrino Oscillation Data

→ Evidence of New Physics beyond the SM

neutrino non-zero mass & flavor mixings

Oscillation data

\[ 7.2 \times 10^{-5} < \Delta m_{12}^2 (\text{eV}^2) < 9.2 \times 10^{-5} \]
\[ 1.4 \times 10^{-3} < \Delta m_{23}^2 (\text{eV}^2) < 3.3 \times 10^{-3} \]
\[ 0.25 < \sin^2 \theta_{12} < 0.39 \]
\[ \sin^2 2\theta_{23} > 0.9 \]
\[ |U_{e3}| < 0.22. \]

Tiny mass scale & large mixing angles

http://hitoshi.berkeley.edu/neutrino
Measurement of theta_13 at T2K

\[ \Delta m^2_{23} > 0 \]

\[ \Delta m^2_{23} < 0 \]

T2K
1.43 \times 10^{20} \text{ p.o.t.}
We focus on TeV scale New Physics

(1) motivated by the hierarchy problem
(2) suitable for Dark Matter Physics

TeV scale

→ Accessible at future collider experiments!

Large Hadron Collider (LHC)

International Linear Collider (ILC)
In next lectures, we will discuss

**Supersymmetry (SUSY)**

Examples of New Physics models

- **Supersymmetric models**: MSSM, GUT, SUSY breaking,
- **Extra-dimension models**: large Xdim, Warped Xdim, Universal Xdm, gauge-Higgs model, Higgs-less model,....
- **non-SUSY models in 4D**: Technicolor, Little Higgs models,....
- **Unexpected**: unparticle, hidden valley, quirk models,....