Physics 751 Midterm II

16 November 2007

This is a pledged midterm: you must not use stored information, paper or electronic, or communicate with anyone concerning the questions during the exam.

Answer only three of the questions. If you attempt all four, make clear which one should not be graded, otherwise only the first three attempted will be graded.

The **partial credit** assigned to parts of questions is shown thus: (5).

Possibly Useful Info:

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{p}{m\omega}\right), \quad a^\dagger \,|\, n\rangle = \sqrt{n+1}\,|\, n+1\rangle$$

$$[J_i, J_j] = i\hbar\varepsilon_{ijk}J_k, \quad J^2 \,|\, jm\rangle = \hbar^2 j(j+1)\,|\, jm\rangle$$

$$J_\pm = J_x \pm iJ_y, \quad J_\pm \,|\, jm\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}\,|\, j, m\pm 1\rangle$$

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1. (5) (a) A simple harmonic oscillator is in the ground state $|0\rangle$. What is the expectation value $\langle x^4\rangle$?

(5) (b) A simple harmonic oscillator is in the state $\left(1/\sqrt{2}\right)\left(|0\rangle + |1\rangle\right)$. Find the expectation value of $\langle x^3\rangle$, including time dependence.

2. (5) (a) Assuming p, x obey canonical commutation relations, find the commutation relations for the simple harmonic oscillator raising and lowering operators. Generalize your result to find $[a, (a^\dagger)^n]$. (First take n = 2, then n = 3, look for the pattern, and find the general result by induction.)

(5) (b) Prove that $e^{\lambda a^\dagger}\,|\,0\rangle$ is an eigenstate of the (non-Hermitian) annihilation operator a. Find its eigenvalue, and normalize it. Derive any commutation relation you use.

3. (2) (a) A nonrelativistic particle in zero potential moving in one dimension goes from the origin to the point x in a time t. What is the action in classical mechanics?

(5) (b) Write down Feynman's formal expression for the quantum mechanical propagator as a sum over paths, and show that for the quantum mechanical equivalent of part (a) the propagator is determined by requiring the $t \to 0$ limit to be a δ-function. Evaluate the expression for the propagator.

(3) (c) The propagator as evaluated cannot by itself be directly interpreted as a wave function of a particle. Explain briefly why not, and why it is still useful.

4. (6) (a) Using the matrix elements of J_+, J_- in the usual j, m representation, find the matrix form of J_y for $j = 1$. Find the eigenkets, and normalize them.

(4) (b) A stream of $j = 1$ particles goes through a Stern-Gerlach apparatus with the magnetic field in the z-direction, which splits it into three streams. The $m_z = 0$ stream is then sent through a second Stern-Gerlach apparatus with the field in the y-direction. What fraction of the ingoing $m_z = 0$ stream emerges with $m_y = 0$?