Physics 751 Homework #9

1.

$$\left|z\right\rangle = e^{-|z|^2/2}\left(\left|0\right\rangle + z\left|1\right\rangle + \frac{z^2}{\sqrt{2!}}\left|2\right\rangle + \frac{z^3}{\sqrt{3!}}\left|3\right\rangle + \dots\right).$$

Exercise: Check that this state is correctly normalized, and is an eigenstate of $\hat{a}$.

2. Prove using an algebraic identity that $e^{\left(z\hat{a}^\dagger - z^*\hat{a}\right)}\left|0\right\rangle$ is an eigenstate of $\hat{a}$. Is it also an eigenstate of $\hat{a}^\dagger$? Prove your assertion.

2. Prove that if $\left|z\right\rangle = e^{-|z|^2/2}\left(\left|0\right\rangle + z\left|1\right\rangle + \frac{z^2}{\sqrt{2!}}\left|2\right\rangle + \frac{z^3}{\sqrt{3!}}\left|3\right\rangle + \dots\right)$,

the unit operator $I = \iint \frac{dxdy}{\pi}\left|z\right\rangle\left\langle z\right|$

3. Prove that $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$ is correct up to terms A^3 and B^3 by expanding the exponentials on both sides and comparing.

4. How does a (position) translation operator affect a wave function expressed in momentum space, $\psi(p)$? What is the operator that shifts the momentum space wave function $\psi(p)$ to $\psi(p-p_0)$? How does *that* operator change $\psi(x)$?

5. Prove:

$$f(x) = e^{x\hat{A}}\hat{B}e^{-x\hat{A}} = \hat{B} + x\left[\hat{A},\hat{B}\right] + \frac{x^2}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] + \dots$$

by writing the Taylor series for $f(x)$ and finding the successive derivatives at the origin.
A unitary squeeze operator is defined by:

$$U(\theta) = e^{(\theta/2)\left(\hat{a}\hat{a} - \hat{a}^\dagger\hat{a}^\dagger\right)}.$$

Use the result for $f(x)$ above to prove that:

$$U^\dagger(\theta)\hat{a}U(\theta) = \hat{a}\cosh\theta - \hat{a}^\dagger\sinh\theta,$$
$$U^\dagger(\theta)\hat{a}^\dagger U(\theta) = -\hat{a}\sinh\theta + \hat{a}^\dagger\cosh\theta.$$

Deduce that

$$U^\dagger(\theta)\hat{x}U(\theta)=e^{-\theta}\hat{x},\quad U^\dagger(\theta)\hat{p}U(\theta)=e^{\theta}\hat{p},$$

so for positive θ, the wave function is scaled down—squeezed—in x space, but simultaneously expanded in p space, as it must be, since it was a minimum uncertainty packet.

Is it still a minimum uncertainty packet? Is it still an eigenstate of the annihilation operator? If not, what is it an eigenstate of? How do you think it develops in time?