# Rare allowed pion and muon decays

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# Outline

Introduction and overview of $\pi$, $\mu$ decays

The $\pi^+ \to e^+\nu_e$ ($\pi_{e2}$), electronic decay

The $\pi^+ \to e^+\nu_e\gamma$ ($\pi_{e2\gamma}$), radiative decay

The $\pi^+ \to \pi^0 e^+\nu$ ($\pi_{e3}$), pion beta decay

Radiative muon decay

Summary

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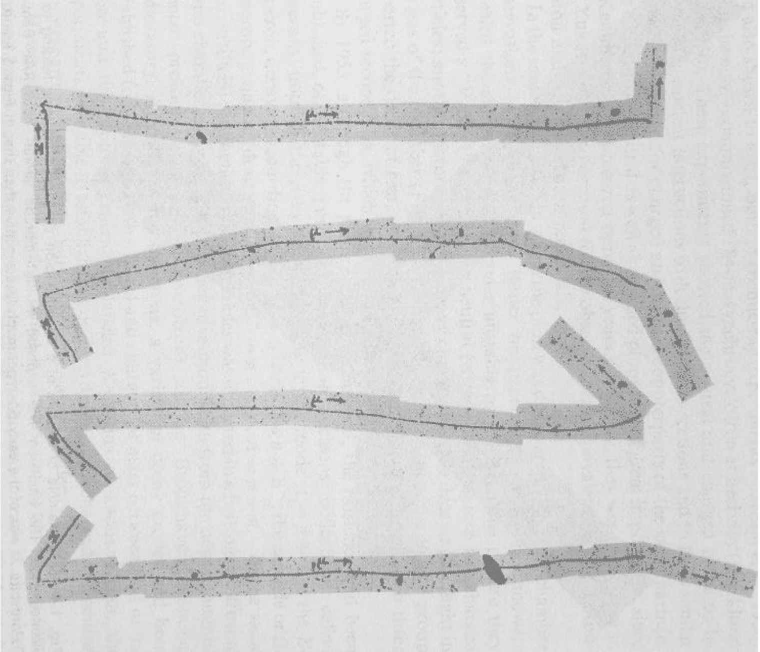
Radiative muon decay

Summary

# Discovery of the pion: Cecil Powell's emulsion tracks 1947



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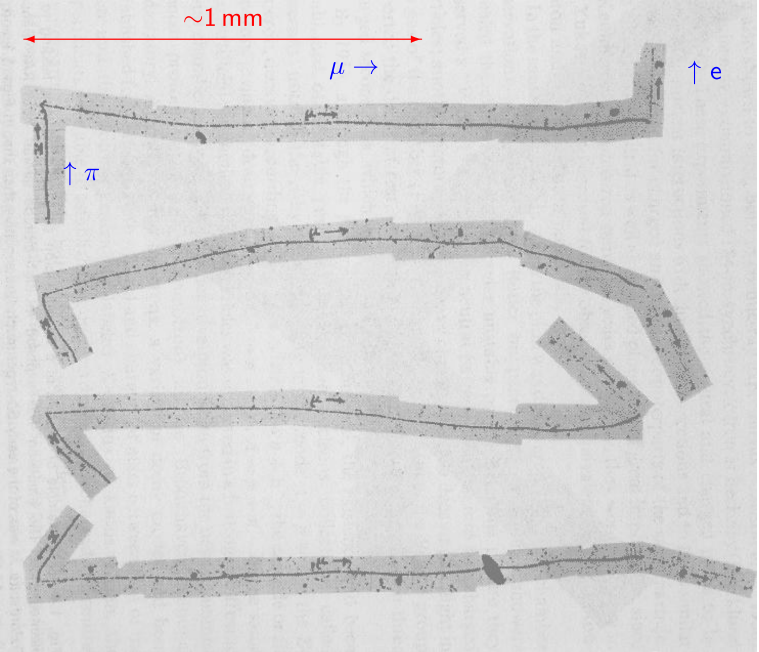

# Pions, muons, symmetries and conservation laws

- Why isn't $\pi \to \mathbf{e}$ the dominant decay mode?
  Deep link to $\mathbf{V-A}$ nature of the **weak interaction** $\Longleftrightarrow$ **PV**;
- Pion triplet ($\pi^+$, $\pi^0$, $\pi^-$), are the **Goldstone bosons** in the spontaneous breaking of **Chiral Symmetry**;
- Explicit breaking of $\chi$**S**: $\mathbf{m_\pi^2 \propto m_q}$ (Gell-Mann–Oakes–Renner rel.);
- **Conserved Vector Current**: $\Longleftrightarrow$ $\mathbf{SU(2)_V}$;
- Why is the beta energy spectrum in $\mu \to \mathbf{e}$ decay continuous?
  and **many other questions** . . . $\Rightarrow$ Lepton Flavor Conservation, . . .

# Known and measured pion and muon decays

| | Decay | BR | | |
|---|---|---|---|---|
| $\pi^+ \rightarrow$ | $\mu^+\nu$ | $0.9998770\,(4)$ | $(\pi_{\mu 2})$ | |
| | $\mu^+\nu\gamma$ | $2.00\,(25) \times 10^{-4}$ | $(\pi_{\mu 2\gamma})$ | |
| | $e^+\nu$ | $1.230\,(4) \times 10^{-4}$ | $(\pi_{e2})$ | ✓ |
| | $e^+\nu\gamma$ | $7.39(5) \times 10^{-7}$ | $(\pi_{e2\gamma})$ | ✓ |
| | $\pi^0 e^+\nu$ | $1.036\,(6) \times 10^{-8}$ | $(\pi_{e3},\ \pi_\beta)$ | ✓ |
| | $e^+\nu e^+e^-$ | $3.2\,(5) \times 10^{-9}$ | $(\pi_{e2ee})$ | |
| $\pi^0 \rightarrow$ | $\gamma\gamma$ | $0.98798\,(32)$ ✓ | | |
| | $e^+e^-\gamma$ | $1.198\,(32) \times 10^{-2}$ | (Dalitz) | |
| | $e^+e^-e^+e^-$ | $3.14\,(30) \times 10^{-5}$ | | |
| | $e^+e^-$ | $6.2\,(5) \times 10^{-8}$ | | |
| $\mu^+ \rightarrow$ | $e^+\nu\bar{\nu}$ | $\sim 1.0$ ✓ | (Michel) | |
| | $e^+\nu\bar{\nu}\gamma$ | $0.014\,(4)$ ✓ | (RMD) | |
| | $e^+\nu\bar{\nu}e^+e^-$ | $3.4\,(4) \times 10^{-5}$ | | |

# The electronic ($\pi_{e2}$) decay:

$$\pi^+ \rightarrow e^+\nu$$

$$\mathbf{BR \sim 10^{-4}}$$

# $\pi_{e2}$ decay: SM calculations, lepton universality

- Early evidence for $V - A$ nature of weak interaction.

$$R^{\pi}_{e/\mu} = \frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))} = \frac{g_e^2}{g_\mu^2} \frac{m_e^2}{m_\mu^2} \frac{(1 - m_e^2/m_\mu^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta R_{e/\mu}\right)$$

- Modern SM calculations: $R^{\pi}_{e/\mu} = \frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))}_{\text{CALC}} =$

$$\begin{cases} 1.2352\,(5) \times 10^{-4} & \text{Marciano and Sirlin, [PRL \textbf{71} (1993) 3629]} \\ 1.2354\,(2) \times 10^{-4} & \text{Finkemeier, [PL B \textbf{387} (1996) 391]} \\ 1.2352\,(1) \times 10^{-4} & \text{Cirigliano and Rosell, [PRL \textbf{99} (2007) 231801]} \end{cases}$$

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- $\mathbf{R^{\pi}_{e/\mu}}$ tests lepton universality: in SM **e**, $\boldsymbol{\mu}$, $\boldsymbol{\tau}$ differ by Higgs couplings only; there could also be new S or PS bosons with non-universal couplings (New Physics).

# $\pi_{e2}$ decay: experiments

Experimental world average is **40×** less accurate than SM calculations:

$$\frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))}_{\text{EXP}} = 1.230(4) \times 10^{-4}; \quad \left[1.2352(1) \times 10^{-4}\right]_{\text{SM calc}}$$

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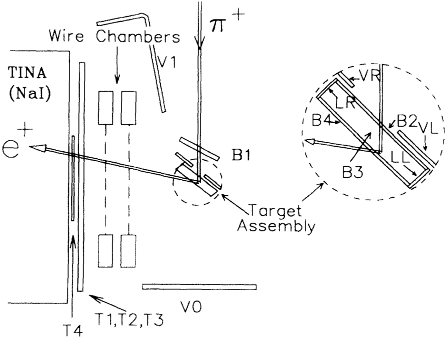
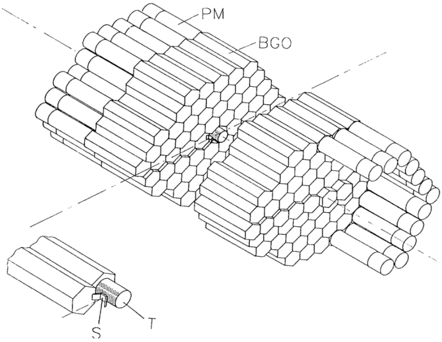
Most recent and relevant experiments for world average:

TRIUMF – TINA (NaI det)




Britton *et al*, PRL **68** (1992) 3000

Bern – PSI (BGO array)




Czapek *et al*, 1993 PRL **70** (1993) 17

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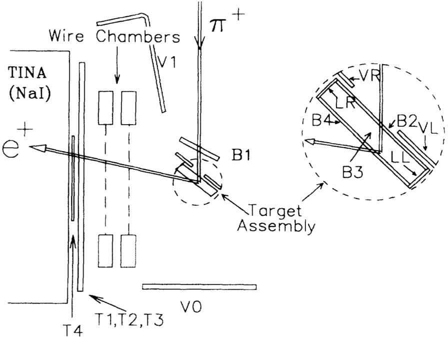


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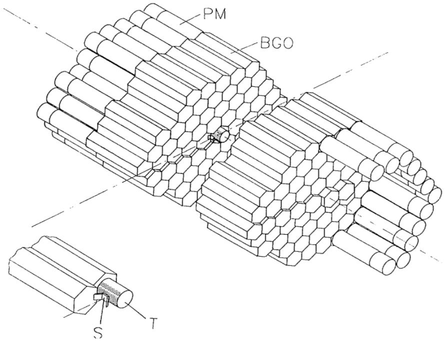


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Why should we improve the current 0.4% result?

# Reach of $\pi_{e2}$ decay beyond the SM (New Physics)

$$\mathcal{L}_{\text{NP}} = \left[ \pm \frac{\pi}{2\Lambda_V^2} \bar{u}\gamma_\alpha d \pm \frac{\pi}{2\Lambda_A^2} \bar{u}\gamma_\alpha\gamma_5 d \right] \bar{e}\gamma^\alpha(1-\gamma_5)\nu$$

$$+ \left[ \pm \frac{\pi}{2\Lambda_S^2} \bar{u} d \pm \frac{\pi}{2\Lambda_P^2} \bar{u}\gamma_5 d \right] \bar{e}(1-\gamma_5)\nu\,, \quad (\Lambda_i \text{ ... scale of NP})$$

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$$\Lambda_V \geq 20\,\text{TeV}, \qquad \text{and} \qquad \Lambda_S \geq 10\,\text{TeV}\,.$$

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At $\Delta R^\pi_{e/\mu}/R^\pi_{e/\mu} = 10^{-3}$, $\pi_{e2}$ decay is directly sensitive to:

$$\boxed{\Lambda_P \leq 1000\,\text{TeV}} \qquad \text{and} \qquad \boxed{\Lambda_A \leq 20\,\text{TeV}}\,,$$

and indirectly, through loop effects to $\boxed{\Lambda_S \leq 60\,\text{TeV}}$.

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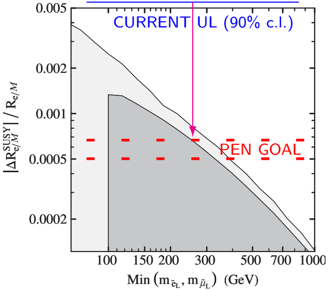
In general multi-Higgs models with charged-Higgs couplings

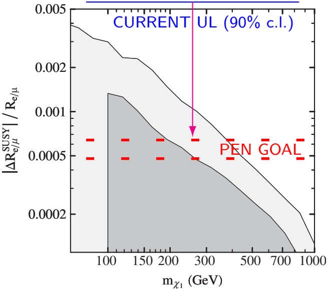
$\lambda_{e\nu} \approx \lambda_{\mu\nu} \approx \lambda_{\tau\nu}$, at $0.1\,\%$ precision, $R^\pi_{e\mu}$ probes $\boxed{m_{H^\pm} \leq 400\,\text{GeV}}$.

# MSSM calculations (R parity cons.) [Ramsey-Musolf et al., PR D**76** (2007) 095017]

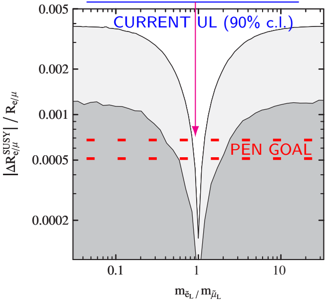
minimal selectron, smuon masses:



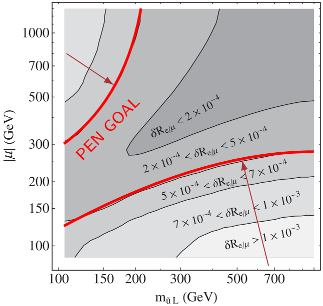

lowest mass chargino:




slepton mass degeneracy:




Higgsino mass param's. $\mu$, $m_{\tilde{u}_L}$:




(R parity violating scenario constraints also discussed.)

# Lepton universality (and neutrinos)

$$R_{e/\mu} = \frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))} = \frac{g_e^2}{g_\mu^2} \frac{m_e^2}{m_\mu^2} \frac{(1 - m_e^2/m_\mu^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta R_{e/\mu}\right)$$

$$R_{\tau/\pi} = \frac{\Gamma(\tau \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))} = \frac{g_\tau^2}{g_\mu^2} \frac{m_\tau^3}{2m_\mu^2 m_\pi} \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta R_{\tau/\pi}\right)$$

lead to these limits:

$$\left(\frac{g_e}{g_\mu}\right)_\pi = 1.0021 \pm 0.0016 \quad \text{and} \quad \left(\frac{g_\tau}{g_\mu}\right)_{\pi\tau} = 1.0030 \pm 0.0034\,.$$

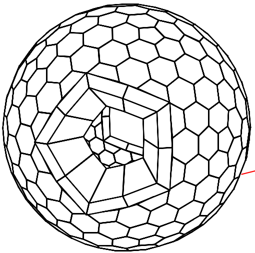
For comparison

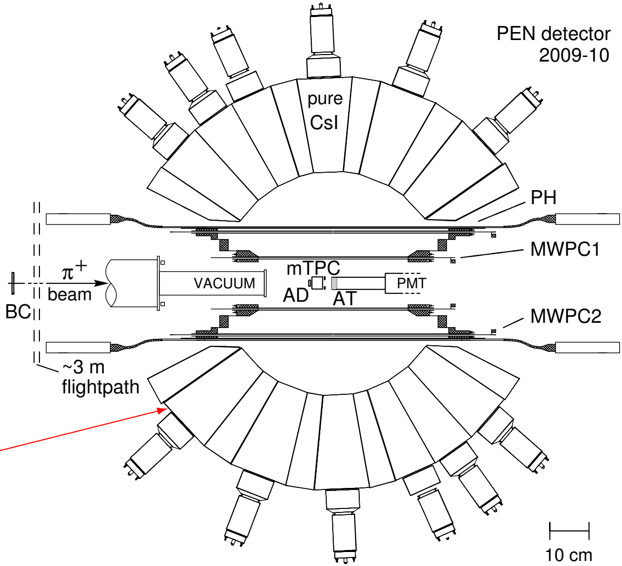
$$\left(\frac{g_e}{g_\mu}\right)_W = 0.999 \pm 0.011 \quad \text{and} \quad \left(\frac{g_\tau}{g_e}\right)_W = 1.029 \pm 0.014\,.$$

[Presently allowed level of LUV could account for "NuTeV anomaly."]

# The PEN/PIBETA apparatus

- stopped $\pi^+$ beam
- active target counter
- 240-detector, spherical pure CsI calorimeter
- central tracking
- beam tracking
- digitized waveforms
- stable temp./humidity





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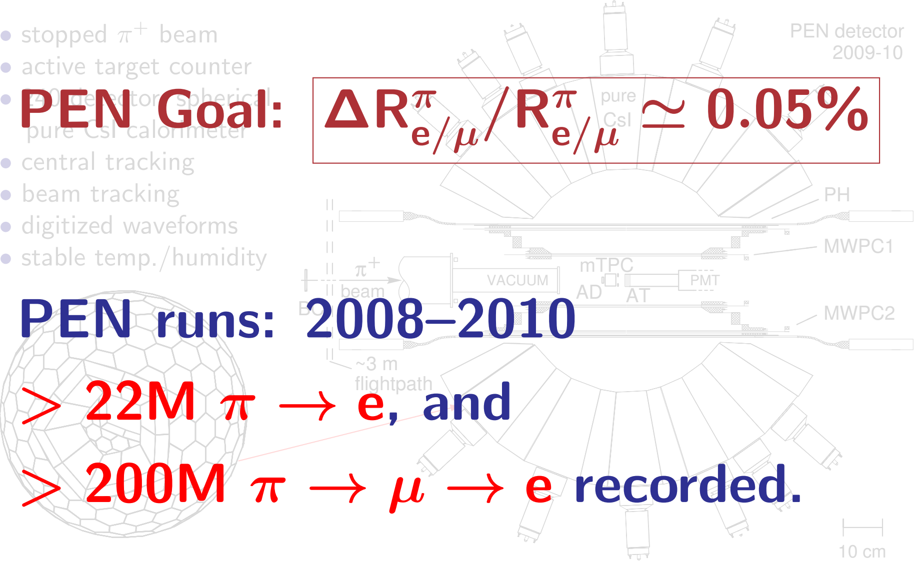



**PEN Goal:** $\Delta R^{\pi}_{e/\mu}/R^{\pi}_{e/\mu} \simeq 0.05\%$

**PEN runs: 2008–2010**

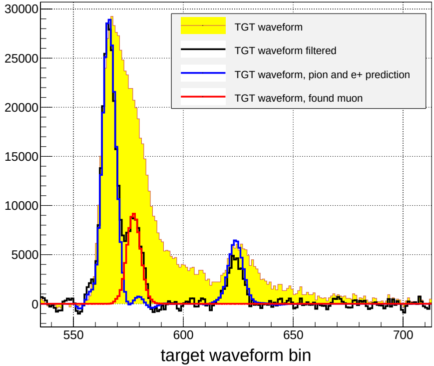
**$>$ 22M $\pi \to$ e, and**

**$>$ 200M $\pi \to \mu \to$ e recorded.**

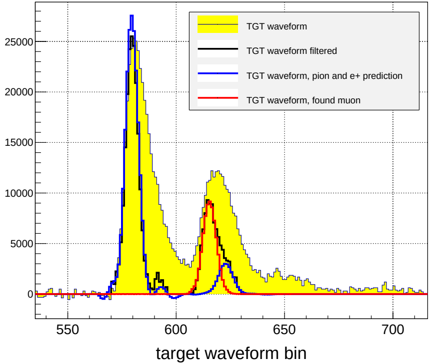
# Highlights and challenges of PEN analysis (under way)

## Active target waveforms: separating the decay particle pulses!

Early pion decay (extremely common)

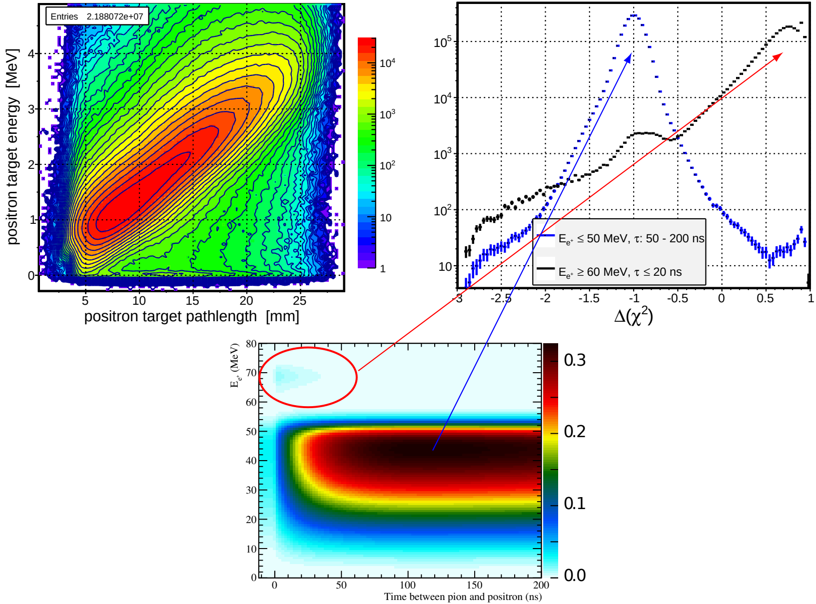



Early muon decay (still annoying)

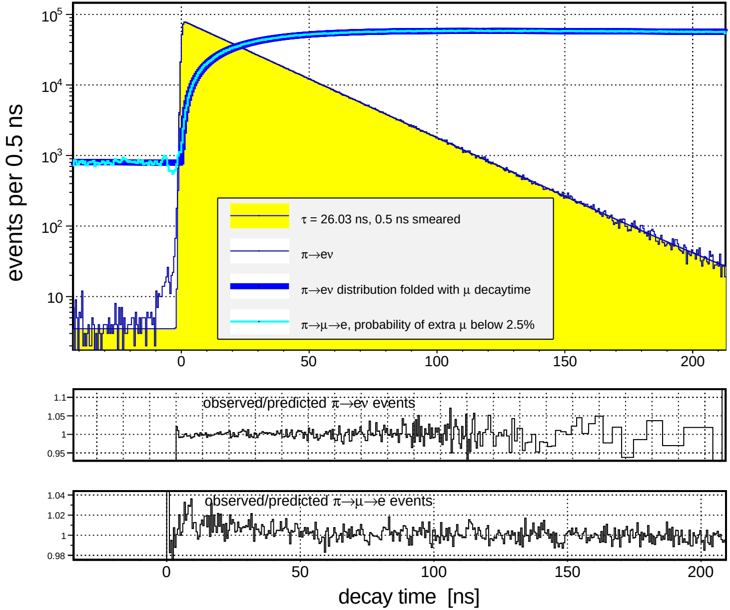



- $\pi$ and $e^+$ pulse time and amplitude predicted from other detector systems (mTPC, MWPCs, PH)!
- Waveform system functions evaluated based on prompt hadronic events.
- Hypotheses with/without a $\mu$ pulse evaluated.

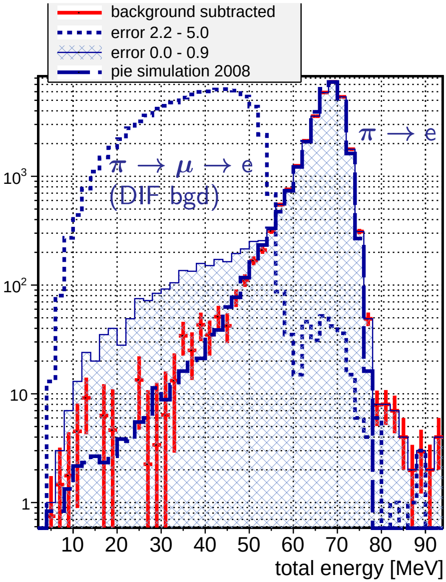
# PEN separating $\pi \to e$ and $\pi \to \mu \to e$ events

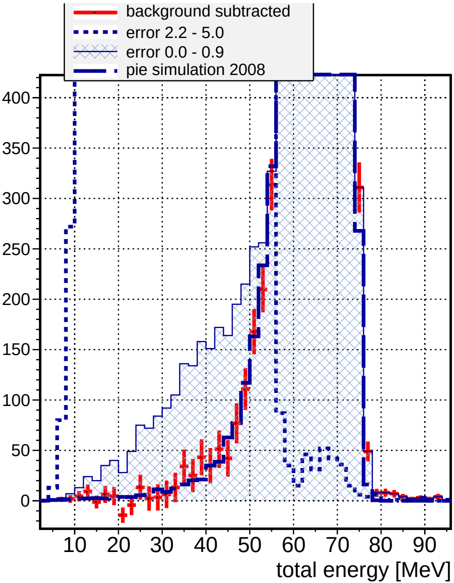


# PEN: agreement with predictions (2010 data analysis)

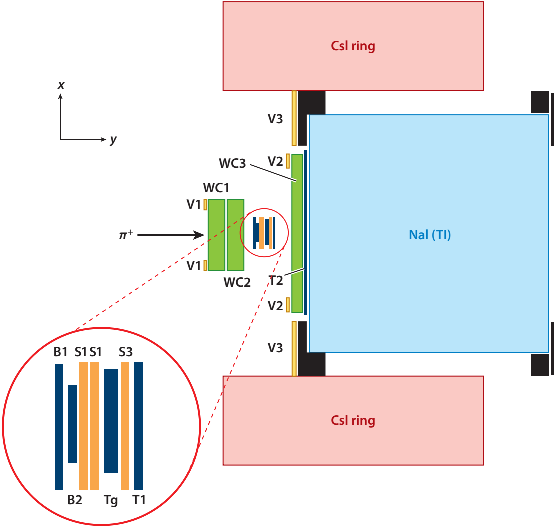


# Key PEN systematic: low $E$ "tail" response



# PiENu experiment at TRIUMF




- Goal: $\Delta B/B \simeq 0.001$
- Excellent $E$ resolution
- Very precise tracking with Si-strip detectors and MWPCs
- Data taking completed in 2012
- $\mathcal{O}(10^7)$ $\pi_{e2}$ events collected
- analysis under way

# Radiative electronic ($\pi_{e2\gamma}$) decay:

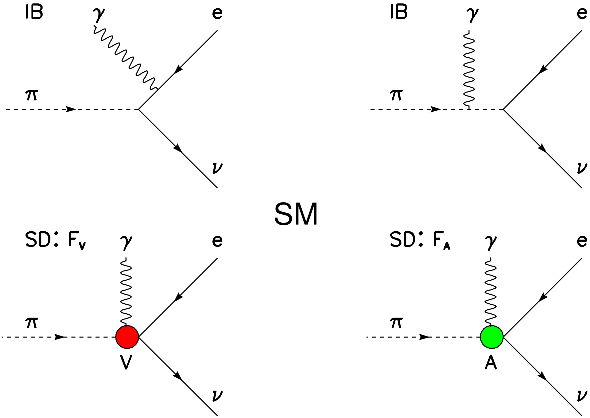
$$\pi^+ \to e^+\nu_e\gamma$$

$$\mathbf{BR}_{\text{non-IB}} \sim \mathbf{10^{-7}}$$

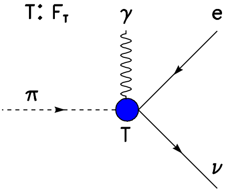
## Physics of $\pi^+ \to e^+\nu\gamma$ (RPD):

QED IB terms:

and SD $V$, $A$ terms:




A tensor interaction, too?




Exchange of S=0 leptoquarks

P Herczeg, PRD 49 (1994) 247

# The $\pi \to e\nu\gamma$ amplitude and FF's

The IB amplitude (QED **uninteresting!**):

$$M_{\text{IB}} = -i\frac{eG_F V_{ud}}{\sqrt{2}} f_\pi m_e \epsilon^{\mu *} \bar{e} \left( \frac{k_\mu}{kq} - \frac{p_\mu}{pq} + \frac{\sigma_{\mu\nu} q^\nu}{2kq} \right) \times (1-\gamma_5)\,\nu\,.$$

The structure-dependent amplitude (**interesting!**):

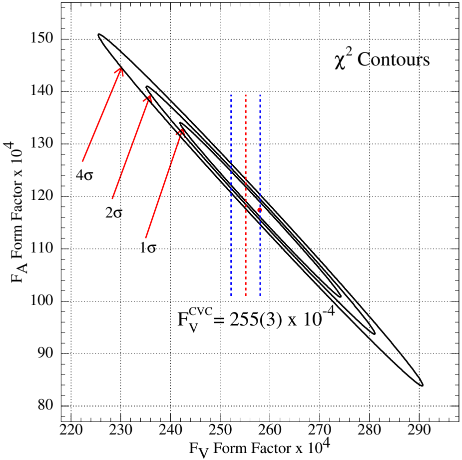
$$M_{\text{SD}} = \frac{eG_F V_{ud}}{m_\pi \sqrt{2}} \epsilon^{\nu *} \bar{e} \gamma^\mu (1-\gamma_5)\nu \times \left[ F_V \epsilon_{\mu\nu\sigma\tau} p^\sigma q^\tau + i F_A (g_{\mu\nu} pq - p_\nu q_\mu) \right] .$$

The SM branching ratio ($x = 2E_\gamma/m_\pi$; $y = 2E_e/m_\pi$),

$$\frac{\mathrm{d}\Gamma_{\pi e2\gamma}}{\mathrm{d}x\,\mathrm{d}y} = \frac{\alpha}{2\pi}\Gamma_{\pi e2}\Big\{ IB(x,y) + \left(\frac{m_\pi^2}{2f_\pi m_e}\right)^2 \times \left[ (F_V+F_A)^2\, \mathbf{SD}^+(x,y) + (F_V-F_A)^2\, SD^-(x,y) \right] + \frac{m_\pi}{f_\pi}\left[ (F_V+F_A)\, S^+_{\text{int}}(x,y) + (F_V-F_A)\, S^-_{\text{int}}(x,y) \right] \Big\} .$$

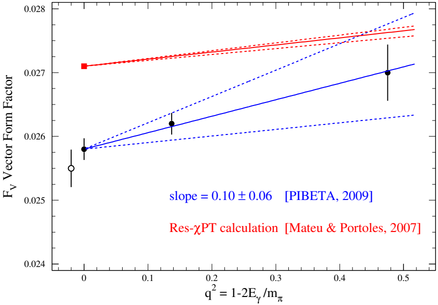
# PIBETA results for $\pi \to e\nu\gamma$

## Best values of pion Form Factor Parameters:




Combined analysis of 1999-01 and 2004 data sets

[Bychkov et al., PRL **103**, 051802 (2009)]



# Summary of PIBETA results on $\pi \to e\nu\gamma$ [PRL **103**, 051802 (2009)]

| | |
|---|---|
| $\mathbf{F_V = 0.0258 \pm 0.0017}$ | $(\mathbf{8}\times)$ |
| $\mathbf{F_A = 0.0119 \pm 0.0001^{exp}_{(F_V^{CVC})}}$ | $(\mathbf{16}\times)$ |
| $\mathbf{a = 0.10 \pm 0.06}$ ($\mathbf{q^2}$ dep of $\mathbf{F_V}$) | $(\infty)$ |
| $\mathbf{-5.2 \times 10^{-4} < F_T < 4.0 \times 10^{-4}}$ | 90 % C.L. |

$\mathbf{B_{\pi_{e2\gamma}}(E_\gamma > 10\,\text{MeV}, \theta_{e\gamma} > 40^\circ) = 73.86(54) \times 10^{-8}}$ $(\mathbf{17}\times)$

# Summary of PIBETA results on $\pi \to e\nu\gamma$ [PRL **103**, 051802 (2009)]

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At L.O. $(l_9 + l_{10})$, $F_A$, $F_V$ are related to pion polarizability and $\pi^0$ lifetime

$$\alpha_E^{\text{LO}} = -\beta_M^{\text{LO}} = (2.783 \pm 0.023_{\text{exp}}) \times 10^{-4}\ \text{fm}^3$$

$$\tau_{\pi^0} = (8.5 \pm 1.1) \times 10^{-17}\,\text{s}$$

- current PDG avg: 8.52 (12)
- PrimEx PRL '10: 8.32 (23)

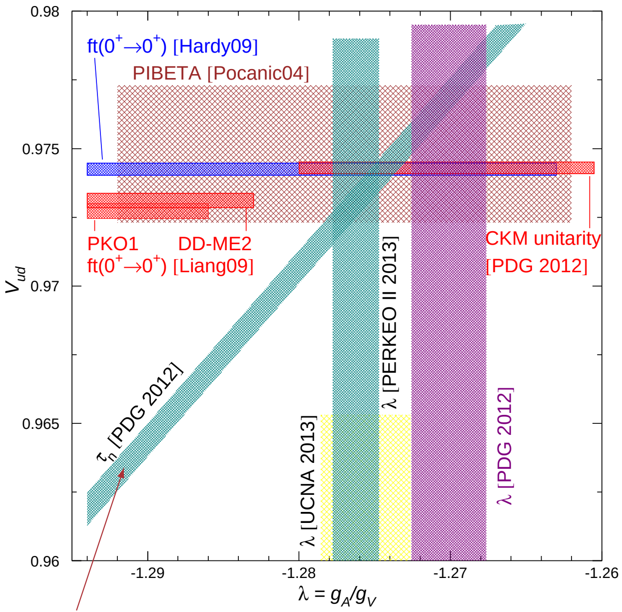
# Pion beta ($\pi_{e3}$) decay:

$$\pi^+ \rightarrow \pi^0 e^+ \nu_e$$

$$\mathbf{BR} \sim 10^{-8}$$

Theoretically the cleanest path to access CKM $V_{ud}$

# Current status of $V_{ud}$:




$$\tau_n^{-1} \propto |V_{ud}|^2 |g_V|^2 (1 + 3|\lambda|^2)$$

Neutron $\beta$ decay results need to be sorted out before returning to $\pi_{e3}$.

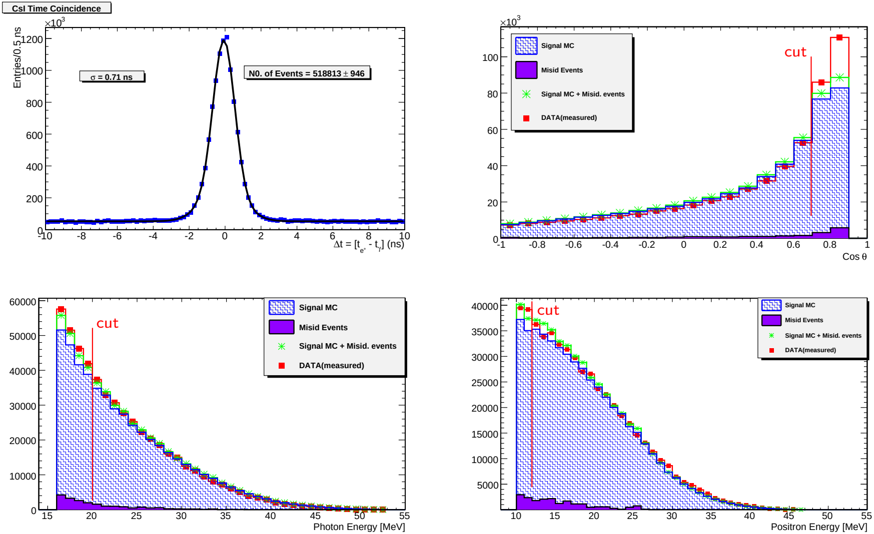
# Radiative muon decay:

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu \gamma$$

**BR** $\sim$ **$10^{-3}$** for energetic $\gamma$'s

- Sensitive to admixtures beyond $V - A$
- Limiting factor in $\mu \to e\gamma$ LFV searches

# Radiative muon decay, $\mu^+ \to e^+ \nu \bar{\nu} \gamma$, (new analysis of 2004 data)




"Split clumps" very well accounted for!

# RMD preliminary results, cont'd.

Preliminary result for RMD branching ratio (thesis E. Munyangabe):

$$B_{\mathrm{exp}} = 4.365\,(9)_{\mathrm{stat.}}\,(42)_{\mathrm{syst.}} \times 10^{-3}\,, \qquad \boxed{\mathbf{29}\times}$$

$$B_{\mathrm{SM}} = 4.342\,(5)_{\mathrm{stat\text{-}MC}} \times 10^{-3} \qquad (\text{for } E_\gamma > 10\,\mathrm{MeV},\ \theta_{e\gamma} > 30^\circ)$$

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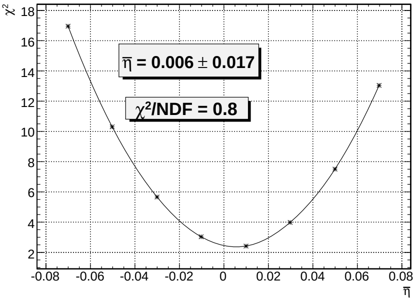


NB: preliminary results!

Analysis of PS subset:
$13\,\text{MeV} < E_\gamma < 45\,\text{MeV}$, and
$10\,\text{MeV} < E_{e^+} < 43\,\text{MeV}$, yields

$$\bar{\eta} = 0.006\,(17)_{\text{stat.}}\,(18)_{\text{syst.}}\,,$$ or

$$\bar{\eta} < 0.028 \quad (68\%\text{CL})\,.$$

**∼ 4×** better than best previous experiment (Eichenberger et al, 84).

# Study of allowed $\pi$ and $\mu$ decays in PEN

- A significant experimental effort is under way (in PEN and in other experiments) to make use of the unparalleled theoretical precision in the weak interactions of the lightest particles.
- Information obtained is complementary to expected collider results, and valuable for their proper interpretation.
- Improvements in precision for
  - $\pi \to e\nu$,
  - $\pi \to e\nu\gamma$ ($F_V$, $F_T^{\mathrm{ul}}$), and
  - $\mu \to e\nu\bar{\nu}\gamma$.

  to be achieved in the near future.

# Current and former PIBETA and PEN collaborators


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Home pages: http://pibeta.phys.virginia.edu
http://pen.phys.virginia.edu

# Backup slides

# Available data on pion form factors

$$|F_V| \stackrel{\text{CVC}}{=} \frac{1}{\alpha}\sqrt{\frac{2\hbar}{\pi\tau_{\pi^0} m_\pi}} = 0.0255(3) \,.$$

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| $106 \pm 60$ | Bolotov et al. (1990) |
| $135 \pm 16$ | Bay et al. (1986) |
| $60 \pm 30$ | Piilonen et al. (1986) |
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We fix $V_{ud}$ to the extraordinarily precise PDG 2013 recommended value

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and adjust $R^{\pi}_{e/\mu}$ until the extracted value of $V^{\pi\beta}_{ud}$ agrees. This exercise yields

$$\boxed{(R^{\pi}_{e/\mu})^{\text{PIBETA}} = (1.2366 \pm 0.0064) \times 10^{4}},$$

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[recall $(R^{\pi}_{e/\mu})^{\text{SM}} = 1.2352(1) \times 10^{-4}$]

Adding the new value to the world data set would move the average slightly to

$$(R^{\pi}_{e/\mu})^{\text{new avg}} = (1.2317 \pm 0.0031) \times 10^{-4}\,.$$

# What to do with Michel parameters?

For $\mu \to e\nu_\mu\bar{\nu}_e\gamma$: $\left(x = \frac{E_e}{E_{max}} \text{ and } y = \frac{E_\gamma}{E_{max}}\right)$

$$\frac{d^3B(x,y,\theta)}{dx\,dy\,2\pi\,d(\cos\theta)} = f_1(x,y,\theta) + \bar{\eta} f_2(x,y,\theta) + (1 - \frac{4}{3}\rho) f_3(x,y,\theta)$$

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$$\rho = \frac{3}{4} - \frac{3}{4}\left[|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{RL}g^{T*}_{RL} + g^S_{LR}g^{T*}_{LR})\right] \stackrel{SM}{\equiv} \mathbf{\frac{3}{4}},$$

$$\bar{\eta} = \left(|g^V_{RL}|^2 + |g^V_{LR}|^2\right) + \frac{1}{8}\left(|g^S_{LR} + 2g^T_{LR}|^2 + |g^S_{RL} + 2g^T_{RL}|^2\right) + 2\left(|g^T_{LR}|^2 + |g^T_{RL}|^2\right) \stackrel{SM}{\equiv} \mathbf{0}.$$

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Combined with $\eta$, $\delta$, $\rho$, parameters of OMD (TWIST), a global fit will yield model-independent limits on non–$(\mathbf{V}-\mathbf{A})$ couplings.

# $\pi_{e3}$ decay: quark-lepton (Cabibbo) universality

The basic weak-interaction **V-A** form (e.g., $\mu$ decay):

$$\mathcal{M} \propto \langle e|l^\alpha|\nu_e\rangle \rightarrow \bar{u}_e\gamma^\alpha(1-\gamma_5)u_\nu$$

is replicated in hadronic weak decays

$$\mathcal{M} \propto \langle p|h^\alpha|n\rangle \rightarrow \bar{u}_p\gamma^\alpha(G_V - G_A\gamma_5)u_n \quad \text{with} \quad G_{V,A} \simeq 1 \,.$$

Departure from $G_V = 1$ (**CVC**) comes from weak quark (Cabibbo) mixing:
$G_V = G_\mu \cos\theta_C (= G_\mu V_{ud}) \quad \cos\theta_C \simeq 0.97$

3 **q** generations lead to the Cabibbo-Kobayashi-Maskawa (CKM) matrix (1973): $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

CKM unitarity cond.: $\Delta V^2 = 1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) \stackrel{?}{=} 0$,
stringently tests the SM. Until 2004 appeared violated by $\sim 3\sigma$!

# $\pi_{e3}$ decay rate in the SM

$$\Gamma = \Gamma_0(1+\delta_\pi) = \frac{G_F^2 |V_{ud}|^2 \Delta^5}{30\pi^3} f(\epsilon, \Delta) \left(1 - \frac{\Delta}{2m_+}\right)^3 (1+\delta_\pi)\,,$$

where

$$\Delta = m_+ - m_0 = 4.5936(5)\,\text{MeV} \qquad \text{and} \qquad \epsilon = \left(\frac{m_e}{\Delta}\right)^2 \simeq \frac{1}{81}$$

while

$$f(\epsilon,\Delta) = \sqrt{1-\epsilon}\left(1 - \frac{9}{2}\epsilon - 4\epsilon^2\right) + \frac{\epsilon^2}{4}\ln\left(\frac{1-\sqrt{1-\epsilon}}{\sqrt{\epsilon}}\right) - \frac{3}{7}\frac{\Delta^2}{(m_+ + m_0)^2} \simeq 0.941$$

and $\delta_\pi \sim 0.035$ is the sum of radiative/loop corrections with $\sim 0.05\%$ uncertainty.

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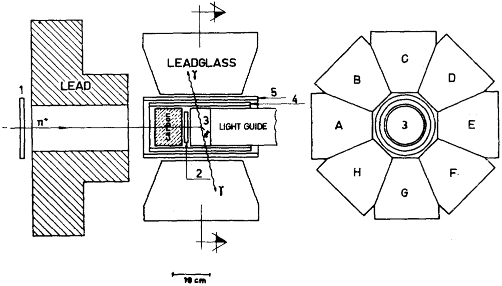
Pion beta decay provides the theoretically cleanest access to $V_{ud}$.

# $\pi_{e3}$ decay: experimental studies

1967 CERN: $\Delta B/B \simeq 10\%$

[Depommier etal NP B4 (68) 189]

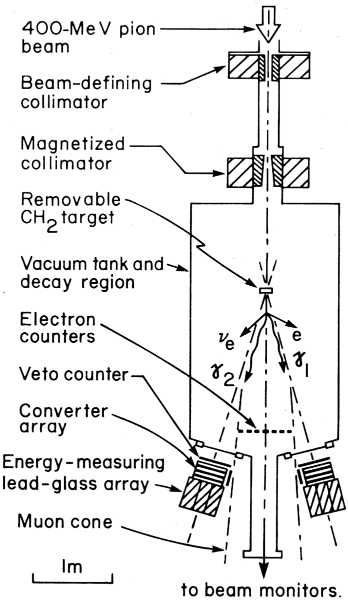
$\Downarrow$



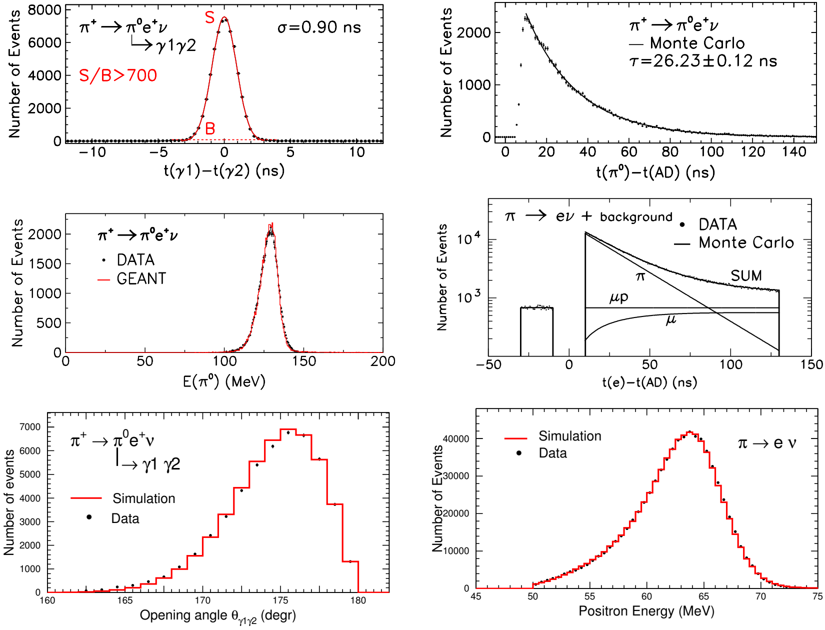

1984 LAMPF: $\Delta B/B \simeq 4\%$

[McFarlane etal PRD 32 (85) 547] $\Rightarrow$

1999-2001 PIBETA at PSI (below)



# PIBETA results; 1999-2001 runs



# PIBETA result for $\pi^+ \to \pi^0 e^+ \nu$ ($\pi_\beta$) decay [PRL **93**, 181803 (2004)]

Pion beta decay yield normalized to measured $\pi \to e\nu$ events:

$$B_{\pi\beta}^{\text{exp-t}} = [1.040 \pm 0.004\,(\text{stat}) \pm 0.004\,(\text{syst})] \times 10^{-8}\,,$$

$$B_{\pi\beta}^{\text{exp-e}} = [1.036 \pm 0.004\,(\text{stat}) \pm 0.004\,(\text{syst}) \pm 0.003\,(\pi_{e2})] \times 10^{-8}\,,$$

McFarlane et al. [PRD 1985]: $B = (1.026 \pm 0.039) \times 10^{-8}$

SM Prediction (PDG):

$B =$ $1.038 - 1.041 \times 10^{-8}$ (90% C.L.)

($1.005 - 1.007 \times 10^{-8}$ excl. rad. corr.)

$\Rightarrow$ Most sensitive test of CVC/radiative corr. in a meson to date!

PDG 2012: $V_{ud} = 0.97425(22)$

PIBETA: $V_{ud} = 0.9748(25)$ or $V_{ud} = 0.9728(30)$.